REG-2223-MOCK-SET 5-MATH-CP 2

Answers:

1. D	2. D	3. D	4. C	5. C	6. B	7. A	8. A	9. D	10. B
11. D	12. A	13. A	14. C	15. D	16. C	17. A	18. A	19. C	20. A
21. D	22. A	23. B	24. C	25. B	26. B	27. D	28. B	29. C	30. B
31. A	32. C	33. A	34. D	35. D	36. A	37. B	38. C	39. D	40. B
41. C	42. B	43. B	44. D	45. C					

Suggested Solutions:

1. D

$$2^{2n} \cdot 9^n = 2^{2n} \cdot 3^{2n}$$
$$= 6^{2n}$$

2. D

Check the coefficient of each term.

	$-q$	p	q^2
A.	✗		
B.	✗		
C.	✓	✓	✗
D.	✓	✓	✓

3. D

$$\frac{a+x}{x+b} = \frac{c}{d}$$
$$ad + dx = cx + bc$$
$$x(d-c) = bc - ad$$
$$x = \frac{ad - bc}{c - d}$$

4. C

$$\frac{1}{2x-5} - \frac{1}{2x+5} = \frac{(2x+5)-(2x-5)}{(2x-5)(2x+5)}$$
$$= \frac{10}{4x^2 - 25}$$

5. C

Solve $\begin{cases} 2u + 3v = -6 \\ 3u - 4v = 25 \end{cases}$, we have $u = 3$ and $v = -4$.

6. B

The inequalities become $x < 1$ or $x \leq -1$.
Thus, $x < 1$.

7. A

I. ✓. You may take m and n be some random numbers to verify it. The detail proof is written below.

$$\begin{aligned} m^3 - n^3 &= (m-n)(m^2 + mn + n^2) \\ &= (m-n)\left[\left(m + \frac{n}{2}\right)^2 + \frac{3n^2}{4}\right] \end{aligned}$$

Since $\left(m + \frac{n}{2}\right)^2 + \frac{3n^2}{4} > 0$ and $m - n > 0$, we have $m^3 - n^3 > 0$ and $m^3 - n^3$.

II. ✓. $k^3 < 0$ and therefore $\frac{m}{k^3} < \frac{n}{k^3}$.

III. ✗. Take $m = 2$, $n = k = -1$.
$m + nk = 2 + 1 = 3$ and $n + mk = -1 - 2 = -3$ but $m + nk < n + mk$.

8. A

$$\begin{aligned} f(k+2) &= 2(k+2)^2 + 3(k+2) - 2 \\ &= 2k^2 + (8+3)k + (8+6-2) \\ &= 2k^2 + 11k + 12 \end{aligned}$$

9. D

Compare the coefficients of x^2.

$$A = 2$$

Compare the coefficients of x.

$$\begin{aligned} -2A + B &= -5 \\ B &= -1 \end{aligned}$$

10. B

Let the cost and the discount per cent be $\$x$ and $r\%$ respectively.

$$x(1 + 10\%)(1 - r\%) = x(1 - 23\%)$$
$$r = 30$$

11. D

$$\text{Interest} = 4000\left(1 + \frac{4\%}{4}\right)^{5\times4} - 4000$$
$$\approx \$881$$

12. A

Let the scale be $1 : x$.

$$\frac{400 \times 100^2}{4} = x^2$$
$$x = 1000$$

13. A

$$\left(1,\ -\sqrt{3}\right) \longrightarrow \left(\sqrt{3},\ 1\right) = (2,\ 30°)$$

14. C

Let $y = \dfrac{k}{x^2}$, where k is a non-zero constant.

$$\frac{y_2}{y_1} = \frac{1}{(1.25)^2}$$
$$= 0.64$$

y is decreased by 36%.

15. D

Let $z = as + \dfrac{b}{t^2}$, where a and b are non-zero constants.

$$\begin{cases} 13 = a + b \\ -5 = 3a + \dfrac{b}{4} \end{cases}$$

Solving, we have $a = -3$ and $b = 16$.

Required value $= -3(-2) + \dfrac{16}{(-4)^2} = 7$

16. C

$$\frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta} = \frac{1-\sin^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{\cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{\cos\theta}{\sin\theta}$$
$$= \frac{1}{\tan\theta}$$

17. A

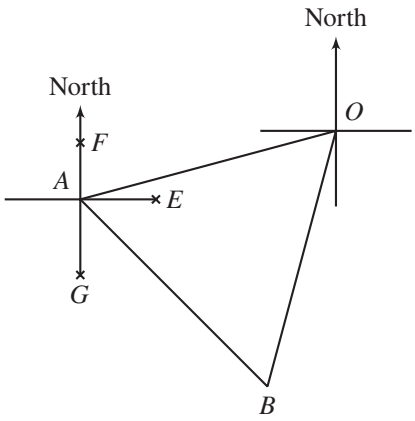
Refer to the figure.

$\angle FAO = 74°$ and $\angle OAE = 90° - 74° = 16°$

$\angle EAB = 62° - 16° = 46°$

$\angle GAB = 90° - 46° = 44°$

Required bearing is N44°W.



18. A

Assign reasonable values to the intercepts.

$ax + y + b = 0$:

$(0,\ 2) \quad \rightarrow \quad b = -2$

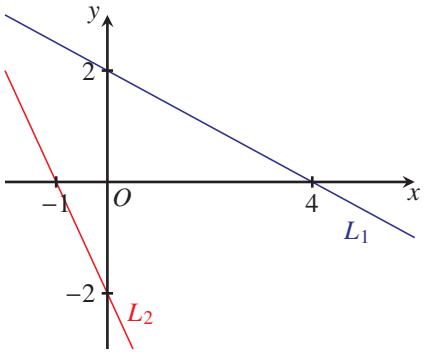
$(4,\ 0) \quad \rightarrow \quad a = \frac{1}{2}$

$cx + y + d = 0$:

$(0,\ -2) \quad \rightarrow \quad d = 2$

$(-1,\ 0) \quad \rightarrow \quad c = 2$

The result follows.



19. C

Open downwards $\quad \rightarrow \quad a < 0$

The slope at y-intercept $= b > 0$

y-intercept $= c < 0$

Thus, $b > 0$ and $ac > 0$.

20. A

$(-2)^3 - k(-2)^2 + 8(-2) - 4 = 0$

$k = -7$

21. D

Let the height of frustum be h cm.

$$\left(\frac{h}{30}\right)^3 = \frac{\frac{1}{3}\pi(12)^2(30) - 1260\pi}{\frac{1}{3}\pi(12)^2(30)}$$

$$\frac{h}{30} = \sqrt[3]{\frac{1}{8}}$$

$$h = 15$$

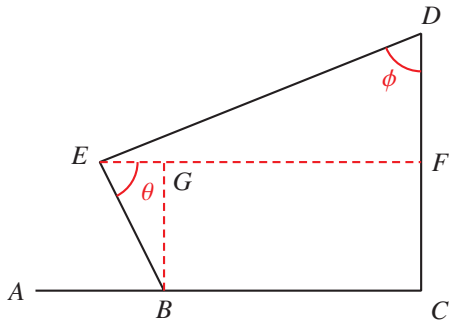
22. A

Let F be a point on CD such that $EF \perp CD$.

Let G be a point on EF such that $BG \perp EF$.

$BC = EF - EG$

$= DE \sin\phi - BE \cos\theta$



23. B

Let $BE = 1$ cm. Then $CE = 2$ cm and $AD = 6$ cm.

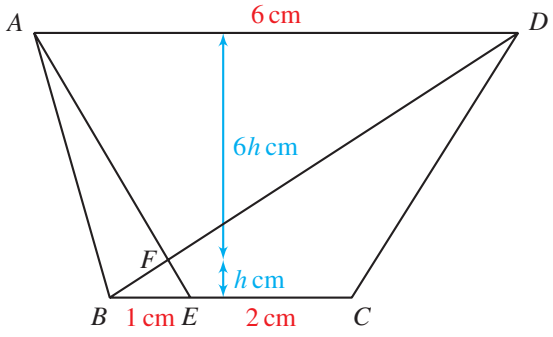
$\triangle ADF \sim \triangle EBF$ (ratio 6 : 1)

$$6 = \frac{(1)(7h)}{2} - \frac{(1)(h)}{2}$$

$$h = 2$$

Required area $= \dfrac{(3)(7h)}{2} - \dfrac{(1)(h)}{2}$

$= 20\,\text{cm}^2$



24. C

Let $\angle PRQ = \theta$.

Since $PQ = QR = RS$, we have $\angle QSR = \angle SQR = \theta$.

Since $\overset{\frown}{PS} : \overset{\frown}{RS} = 2 : 1$, we have $\angle PRS = 2\theta$.

In $\triangle QRS$,

$$\theta + (\theta + 2\theta) + \theta = 180°$$
$$\theta = 36°$$

$\angle PES = 2\theta + \theta = 108°$

25. B

Let $\angle PRM = x$.

Since $ON = NR$, we have $\angle NOR = x$.

$\angle QMR = \dfrac{\angle NOR}{2} = \dfrac{x}{2}$

In $\triangle MQR$,

$$x + \frac{x}{2} = 36°$$
$$x = 24°$$

26. B

Since $\overset{\frown}{PS} = \overset{\frown}{SR}$, we have $\angle POS = \angle ROS = \dfrac{136°}{2} = 68°$.

Since $OP = OS$, we have $\angle SPO = \angle PSO = \dfrac{180° - 68°}{2} = 56°$.

27. D

Let each exterior angle be x.

$$x + (x + 140°) = 180°$$
$$x = 20°$$

I. ✗. $n = \dfrac{360°}{20°} = 18$

II. ✓.

III. ✓.

28. B

$$\frac{m}{m+40} = \frac{2}{m}$$

$$m^2 = 2m + 80$$

$$m^2 - 2m - 80 = 0$$

$$m = 10 \quad \text{or} \quad -8 \text{ (rejected)}$$

29. C

The data is concentrated near larger weights.

The maximum, upper quartile and median should appear closed to each other.

The answer is C.

30. B

I. ✓. Median = 4.

Since all numbers are not less than 4, mean ≥ 4.

II. ✗. Mode = median = 4

III. ✓.

31. A

The three expressions are $2^2m^2n^5$, $2 \cdot 3m^3n^3$ and $2^3m^5n^4$.

The H.C.F. is $2m^2n^3$.

32. C

$$y = 3x^2$$

$$\log y = 2\log x + \log 3$$

I. ✓.

II. ✗. There is no y-intercept indeed, we have only the log y-intercept.

III. ✓. $y = 3x^2$

$\log_3 y = 2\log_3 x + \log_3 3$‘

Slope of the line is also 2.

33. A

$\alpha + \beta = -\frac{1}{2}$ and $\alpha\beta = -\frac{k}{2}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{k}{2}\right)$

$= \frac{1 + 4k}{4}$

34. D

$6 = 110_2$ and only option D satisfies this.

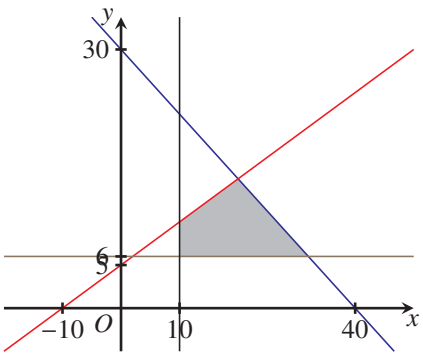
35. D

Line	x-intercept	y-intercept
$3x + 4y = 120$	40	30
$x - 2y + 10 = 0$	-10	5
$y = 6$		6
$x = 10$	10	

Sketch the graph using the intercepts.

Value of $2x + 3y + 20$ is greater when x and y are larger, i.e., the top right corners.

The coordinates of the top right corners are $(20,\ 15)$ and $(32,\ 6)$.



$(x,\ y)$	$(20,\ 15)$	$(32,\ 6)$
$2x + 3y + 20$	105	102

Required value = 105

36. A

x-coordinate of vertex $= \frac{-30}{2(-5)} = 3$

$38 = -5(3)^2 + 30(3) + k$

$k = -7$

37. B

$$\frac{9i^{13}+8i^{14}+7i^{15}+6i^{16}+5i^{17}}{1+i} = \frac{9i-8-7i+6+5i}{1+i}$$
$$= \frac{-2+7i}{1+i}$$
$$= \frac{5}{2}+\frac{9}{2}i$$

Imaginary part $= \frac{9}{2} = 4.5$

38. C

$$7\cos^2 x = \cos x + 6$$

$$7\cos^2 x - \cos x - 6 = 0$$

$$\cos x = 1 \quad \text{or} \quad -\frac{6}{7}$$

When $\cos x = 1$, $x = 0°$ (exclude $360°$)

When $\cos x = -\frac{6}{7}$, $x = 180° - \cos^{-1}\frac{6}{7}$ or $180° + \cos^{-1}\frac{6}{7}$

There are 3 roots.

39. D

Solve the system $\begin{cases} x - 2y + 1 = 0 \\ x^2 + y^2 - 6x + k = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
0	2	+

Required range does not contain $k = 0$ and 0 is not a boundary value of the required range.

The answer is D.

40. B

The coordinates of the vertices of the triangle are $(0,\ 0)$, $(6,\ 0)$ and $(0,\ 8)$.

Let the radius of inscribed circle be r.

By considering the area of the triangle,

$$\frac{(6)(8)}{2} = \frac{(6)(r)}{2} + \frac{(8)(r)}{2} + \frac{(\sqrt{6^2+8^2})(r)}{2}$$

$$r = 2$$

The coordinates of the incentre are $(2,\ 2)$.

41. C

$y = f(x) \quad \longrightarrow \quad y = -f(x) \quad \longrightarrow \quad y = -f(x-2)$

Reflect about the x-axis. Translate rightwards by 2 units.

The answer is C.

42. B

Let the mean and standard deviation be $\overline{x}$ marks and σ marks respectively.

$$\begin{cases} \dfrac{78-\overline{x}}{\sigma} = 1 \\ \dfrac{66-\overline{x}}{\sigma} = -0.5 \end{cases}$$

Solving, we have $\overline{x} = 70$ and $\sigma = 8$.

43. B

$$\begin{aligned} \text{Required number} &= C_3^{18} \times C_4^{22} \\ &= 5\,969\,040 \end{aligned}$$

44. D

$$\begin{aligned} \text{Required probability} &= \frac{14-5}{14} \\ &= \frac{9}{14} \end{aligned}$$

45. C

I. ✓. Both ranges are equal to $f - a$.

II. ✗. Take $P = \{1, 2, 3, 99, 1000, 1001\}$. Median of $P = 51$ and $x = 351$.
Median of $Q = 99 - 3 = 96 > 51$

III. ✓. Standard deviation of Q is equal to the standard deviation of the set $\{a, b, c, d, e, f, x\}$, which has a smaller standard deviation than P.