Solution	Marks

REG-2223-MOCK-SET 4-MATH-EP(M2)
Suggested solutions

1. $f'(1) = \lim_{h\to 0} \dfrac{(1+h+1)\ln(1+h) - 0}{h}$ 1M

$= \lim_{h\to 0} \dfrac{h+2}{h} \ln(1+h)$

$= \lim_{h\to 0} (2+h)\ln(1+h)^{\frac{1}{h}}$ 1M+1A

$= 2\ln e$

$= 2$ 1A

2. $(1+x)^m(1-2x)^n = \left[1 + mx + \dfrac{m(m-1)}{2}x^2 + \ldots\right]\left[1 - 2nx + \dfrac{n(n-1)}{2}4x^2 + \ldots\right]$ 1M

$= 1 + (m-2n)x + \left[\dfrac{m(m-1)}{2} - 2mn + \dfrac{n(n-1)}{2}\right]x^2 + \ldots$

$$\begin{cases} m - 2n = 0 \\ \dfrac{m(m-1)}{2} - 2mn + 2n(n-1) = -9 \end{cases}$$ 1M

$\dfrac{2n(2n-1)}{2} - 4n^2 + 2n(n-1) = -9$ 1M

$9 - 3n = 0$

$n = 3$

When $n = 3$, $m = 2(3) = 6$. 1A

Solution	Marks
3. (a) When $n = 1$,	
L.H.S. $= \sum_{k=1}^{1}(-1)^k(2k-1)^2 = (-1)(1)^2 = -1$	
R.H.S. $= \dfrac{(-1)^1(2-1)(2+1)+1}{2} = -1 = $ L.H.S.	
It is true for $n = 1$.	1
Assume $\sum_{k=1}^{p}(-1)^k(2k-1)^2 = \dfrac{(-1)^p(2p-1)(2p+1)+1}{2}$, where $p \in \mathbb{Z}^+$.	1
$\sum_{k=1}^{p+1}(-1)^k(2k-1)^2$	
$= \sum_{k=1}^{p}(-1)^k(2k-1)^2 + (-1)^{p+1}(2p+1)^2$	
$= \dfrac{(-1)^p(2p-1)(2p+1)+1}{2} + (-1)^{p+1}(2p+1)^2$	1
$= \dfrac{(-1)^{p+1}(2p+1)[-(2p-1)+2(2p+1)]+1}{2}$	
$= \dfrac{(-1)^{p+1}(2p+1)(2p+3)+1}{2}$	
It is true when $n = p + 1$.	
By M.I., it is true $\forall n \in \mathbb{Z}^+$.	1
(b) $\sum_{k=99}^{200}(-1)^{k+1}(2k-1)^2$	
$= -\sum_{k=1}^{200}(-1)^k(2k-1)^2 + \sum_{k=1}^{98}(-1)^k(2k-1)^2$	1M
$= -\dfrac{(-1)^{200}(400-1)(400+1)+1}{2} + \dfrac{(-1)^{98}(196-1)(196+1)+1}{2}$	
$= -60\,792$	1A

Solution	Marks
4. (a) $\dfrac{\sin x+\sin y}{\cos x+\cos y}=\dfrac{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$	1M
$=\tan\dfrac{x+y}{2}$	1
(b) (i) $\dfrac{\tan\left[2\left(\frac{\theta}{2}-\frac{\pi}{4}\right)\right]}{\tan\left(\frac{\theta}{2}-\frac{\pi}{4}\right)}=\dfrac{\tan\left(\theta-\frac{\pi}{2}\right)}{\tan\frac{\theta-\frac{\pi}{2}}{2}}=\dfrac{-\frac{1}{\tan\theta}}{\frac{\sin\theta+\sin\frac{-\pi}{2}}{\cos\theta+\cos\frac{-\pi}{2}}}$	1M
$=-\dfrac{\cos^2\theta}{\sin\theta(\sin\theta-1)}=\dfrac{(\sin\theta+1)(\sin\theta-1)}{\sin\theta(\sin\theta-1)}$	1M
$=\dfrac{\sin\theta+1}{\sin\theta}=\csc\theta+1$	1
(ii) $\dfrac{\tan\left[2\left(\frac{\theta}{2}-\frac{\pi}{4}\right)\right]}{\tan\left(\frac{\theta}{2}-\frac{\pi}{4}\right)}=3$	
$\csc\theta+1=3$	
$\sin\theta=\dfrac{1}{2}$	1M
$\theta=\dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$	1A
5. (a) $\ell^2=(OC+OP\cos\theta)^2+(OB+OP\sin\theta)^2$	
$=(1+\cos\theta)^2+(1+\sin\theta)^2$	1A+1A
$=2+(\sin^2\theta+\cos^2\theta)+2\sin\theta+2\cos\theta$	
$=3+2(\sin\theta+\cos\theta)$	1
(b) $\dfrac{d\ell^2}{dt}=2(\cos\theta-\sin\theta)\dfrac{d\theta}{dt}$	
$2\ell\dfrac{d\ell}{dt}=2(\cos\theta-\sin\theta)\dfrac{d\theta}{dt}$	1M
$\dfrac{d\ell}{dt}=\dfrac{\cos\theta-\sin\theta}{\ell}\dfrac{d\theta}{dt}$	1A
When $\theta=\dfrac{\pi}{3}$,	
$\ell^2=3+2\left(\sin\dfrac{\pi}{3}+\cos\dfrac{\pi}{3}\right)$	
$\ell=\sqrt{4+\sqrt{3}}$	1A
$\dfrac{d\ell}{dt}=\dfrac{\cos\frac{\pi}{3}-\sin\frac{\pi}{3}}{\ell}(3)$	
≈ -0.46	
Required rate is -0.46 cm/s.	1A

Solution	Marks
6. (a) $P(P^3+P^2+P+I)=P(\mathbf{0})$	1M
$P^4+(P^3+P^2+P)=\mathbf{0}$	
$P^4+(-I)=\mathbf{0}$	1M
$P^4=I$	1
(b) $A^2=\begin{pmatrix}3&2\\-5&-3\end{pmatrix}\begin{pmatrix}3&2\\-5&-3\end{pmatrix}=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$	1A
$A^3=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}\begin{pmatrix}3&2\\-5&-3\end{pmatrix}=\begin{pmatrix}-3&-2\\5&3\end{pmatrix}$	
$A^3+A^2+A+I=\begin{pmatrix}-3&-2\\5&3\end{pmatrix}+\begin{pmatrix}-1&0\\0&-1\end{pmatrix}+\begin{pmatrix}3&2\\-5&-3\end{pmatrix}+\begin{pmatrix}1&0\\0&1\end{pmatrix}$	
$=\mathbf{0}$	1A
$A^{2021}=(A^4)^{505}A$	
$=I^{505}A$	1M
$=A$	
$=\begin{pmatrix}3&2\\-5&-3\end{pmatrix}$	1A

Solution	Marks
7. (a) $\overrightarrow{AB} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$	
$\overrightarrow{AC} = 4\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$	
$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & -1 \\ 4 & 2 & -5 \end{vmatrix}$	
$= -3\mathbf{i} + 11\mathbf{j} + 2\mathbf{k}$	1A
(b) $\overrightarrow{AM} = (\alpha - 3)\mathbf{i} - 22\mathbf{j} + (\beta + 2)\mathbf{k}$	
$\overrightarrow{AM}$ is parallel to $\overrightarrow{AB} \times \overrightarrow{AC}$.	
We have $\dfrac{\alpha - 3}{-3} = \dfrac{-22}{11} = \dfrac{\beta + 2}{2}$.	1M
Solving, we have $\alpha = 9$ and $\beta = -6$.	1A+1A
(c) We have $\overrightarrow{AP} = \dfrac{t}{1+t}\overrightarrow{AB}$ and $\overrightarrow{AQ} = \dfrac{1}{1+t}\overrightarrow{AC}$.	
$\dfrac{3}{16} = \dfrac{\text{volume of } MAPQ}{\text{volume of } MABC}$	
$= \dfrac{\frac{1}{3}(AM)(\text{area of } \triangle APQ)}{\frac{1}{3}(AM)(\text{area of } \triangle ABC)}$	
$= \dfrac{\left\lvert \overrightarrow{AP} \times \overrightarrow{AQ} \right\rvert}{\left\lvert \overrightarrow{AB} \times \overrightarrow{AC} \right\rvert}$	1M
$= \dfrac{\frac{t}{(1+t)^2}\left\lvert \overrightarrow{AB} \times \overrightarrow{AC} \right\rvert}{\left\lvert \overrightarrow{AB} \times \overrightarrow{AC} \right\rvert}$	
$= \dfrac{t}{(1+t)^2}$	1A
$3(1+t)^2 = 16t$	
$3t^2 - 10t + 3 = 0$	
$t = 3 \quad \text{or} \quad \dfrac{1}{3}$	1A

Solution	Marks
8. (a) $g'(x) = e^x \cos x - e^x \sin x$	1M
When $g'(x) = 0$,	
$e^x(\cos x - \sin x) = 0$	
$\tan x = 1$	
$x = \dfrac{\pi}{4}$	
<table><tr><td>x</td><td>$0 < x < \frac{\pi}{4}$</td><td>$\frac{\pi}{4} < x < \pi$</td></tr><tr><td>$g'(x)$</td><td>$+$</td><td>$-$</td></tr></table>	1M
Thus, G attains its maximum value only at $x = \dfrac{\pi}{4}$, and G has only one maximum point.	1
(b) $\displaystyle\int e^{2x}\cos 2x\,dx = \frac{1}{2}e^{2x}\cos 2x + \int e^{2x}\sin 2x\,dx$	1M
$\displaystyle = \frac{1}{2}e^{2x}\cos 2x + \frac{1}{2}e^{2x}\sin 2x - \int e^{2x}\cos 2x\,dx$	1M
$\displaystyle 2\int e^{2x}\cos 2x\,dx = \frac{1}{2}e^{2x}\cos 2x + \frac{1}{2}e^{2x}\sin 2x + \text{constant}$	
$\displaystyle \int e^{2x}\cos 2x\,dx = \frac{1}{4}e^{2x}\cos 2x + \frac{1}{4}e^{2x}\sin 2x + \text{constant}$	1A
(c) Volume $\displaystyle = \pi\int_0^{\frac{\pi}{4}} e^{2x}\cos^2 x\,dx$	1M
$\displaystyle = \frac{\pi}{2}\int_0^{\frac{\pi}{4}} e^{2x}(\cos 2x + 1)\,dx$	
$\displaystyle = \frac{\pi}{2}\left[\frac{1}{4}e^{2x}\cos 2x + \frac{1}{4}e^{2x}\sin 2x\right]_0^{\frac{\pi}{4}} + \frac{\pi}{2}\left[\frac{1}{2}e^{2x}\right]_0^{\frac{\pi}{4}}$	
$\displaystyle = \frac{3\pi}{8}\left(e^{\frac{\pi}{2}} - 1\right)$	1A

Solution	Marks
9. (a) Vertical asymptote is $x = -\dfrac{1}{2}$.	1A
$f(x) = \dfrac{x}{2} - \dfrac{9}{4} + \dfrac{9}{4(2x+1)}$	1M
Thus, the oblique asymptote is $y = \dfrac{x}{2} - \dfrac{9}{4}$.	1A
(b) $f'(x) = \dfrac{1}{2} - \dfrac{9}{2(2x+1)^2}$	1M+1A
(c) $f''(x) = \dfrac{18}{(2x+1)^3}$.	
When $f'(x) = 0$,	
$\dfrac{1}{2} - \dfrac{9}{2(2x+1)^2} = 0$	
$(2x+1)^2 = 9$	
$x = -2 \quad \text{or} \quad 1$	1A
When $x = -2$, $f(-3) = \dfrac{-2}{2} - \dfrac{9}{4} + \dfrac{9}{4(-4+1)} = -4$ and $f''(-2) = \dfrac{18}{(-4+1)^3} = -\dfrac{2}{3} < 0$	1M
When $x = 1$, $f(1) = -1$ and $f''(1) = \dfrac{2}{3} > 0$.	
The maximum point is $(-2,\ -4)$ and the minimum point is $(1,\ -1)$.	1A+1A
(d) $\dfrac{x^2 - 4x}{2x+1} = 0$	
$x(x-4) = 0$	
$x = 0 \quad \text{or} \quad 4$	1A
Required area $= \displaystyle\int_0^4 -\left(\dfrac{x^2-4x}{2x+1}\right) dx$	1M
$= \displaystyle\int_0^4 \left[-\dfrac{x}{2} + \dfrac{9}{4} - \dfrac{9}{4(2x+1)}\right]$	
$= \left[-\dfrac{x^2}{4} + \dfrac{9x}{4} - \dfrac{9}{8}\ln\lvert 2x+1\rvert\right]_0^4$	1M
$= 5 - \dfrac{9}{8}\ln 9$	1A

Solution	Marks
10. (a) (i) (1) $\begin{vmatrix} h & -2 & 2h-1 \\ 3 & -1 & 5 \\ 1 & -3 & 3-h \end{vmatrix} = \begin{vmatrix} h-6 & 0 & 2h-11 \\ 3 & -1 & 5 \\ -8 & 0 & -h-12 \end{vmatrix}$	
$= (h-6)(-h-12) + 8(2h-11)$	
$= h^2 - 10h + 16$	1A
$= (h-2)(h-8)$	
(E) has a unique solution if and only if $\begin{vmatrix} h & -2 & 2h-1 \\ 3 & -1 & 5 \\ 1 & -3 & 3-h \end{vmatrix} \neq 0$	1M
So, $h \neq 2$ and $h \neq 8$.	1
(2) $x = \dfrac{\begin{vmatrix} -2 & -2 & 2h-1 \\ -2k & -1 & 5 \\ k+1 & -3 & 3-h \end{vmatrix}}{(h-2)(h-8)} = \dfrac{18hk - 29k - 35}{(h-2)(h-8)}$	1M+1A
$y = \dfrac{\begin{vmatrix} h & -2 & 2h-1 \\ 3 & -2k & 5 \\ 1 & k+1 & 3-h \end{vmatrix}}{(h-2)(h-8)} = \dfrac{2h^2k - hk - 5h - 5k + 5}{(h-2)(h-8)}$	
$z = \dfrac{\begin{vmatrix} h & -2 & -2 \\ 3 & -1 & -2k \\ 1 & -3 & k+1 \end{vmatrix}}{(h-2)(h-8)} = \dfrac{-7hk - h + 10k + 22}{(h-2)(h-8)}$	1A
(ii) (1) The augmented matrix of (E) is	
$\left(\begin{array}{ccc\|c} 2 & -2 & 3 & -2 \\ 3 & -1 & 5 & -2k \\ 1 & -3 & 1 & k+1 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -1 & \frac{3}{2} & -1 \\ 0 & 2 & \frac{1}{2} & 3-2k \\ 0 & -2 & -\frac{1}{2} & k+2 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & -1 & \frac{3}{2} & -1 \\ 0 & 2 & \frac{1}{2} & 3-2k \\ 0 & 0 & 0 & 5-k \end{array}\right)$	1M
Since (E) is consistent, $k = 5$.	1A
(2) The augmented matrix of (E) is	
$\left(\begin{array}{ccc\|c} 1 & -1 & \frac{3}{2} & -1 \\ 0 & 2 & \frac{1}{2} & -7 \\ 0 & 0 & 0 & 0 \end{array}\right)$	
Let $z = t$, where $t \in \mathbb{R}$. Then $y = \dfrac{-t-14}{4}$ and $x = \dfrac{-7t-18}{4}$	
The solution set is $\left\{\left(\dfrac{-7t-18}{4}, \dfrac{-t-14}{4}, t\right) : t \in \mathbb{R}\right\}$.	1A
(b) The system is equivalent to (E) when $h = 2$ and $k = 5$.	

Solution	Marks
The solution is $\left\{\left(\frac{-7t-18}{4},\ \frac{-t-14}{4},\ t\right) : t \in \mathbb{R}\right\}$.	1M
$$2x + y^2 + z < 1$$	
$$\frac{-7t-18}{2} + \left(\frac{-t-14}{4}\right)^2 + t < 1$$	
$$\frac{t^2}{16} - \frac{3t}{4} + \frac{9}{4} < 0$$	
$$\frac{1}{6}(t-6)^2 < 0$$	1M
The inequality has no solution as $\frac{1}{6}(t-6)^2$ is non-negative for all real values of t.	
Thus, there is no real solution of the system satisfying $2x + y^2 + z < 1$.	1A

Solution	Marks
11. (a) $\dfrac{d}{d\theta}\ln(\tan\theta) = \dfrac{\sec^2\theta}{\tan\theta} = \dfrac{2}{2\sin\theta\cos\theta} = 2\csc 2\theta$	1M
$\int \csc 2\theta \, d\theta = \dfrac{1}{2}\ln(\tan\theta) + \text{constant}$	1A+1
(b) (i) $\sqrt{2}\sin\left(\dfrac{\pi}{4}+\theta\right) = \sqrt{2}\left[\sin\dfrac{\pi}{4}\cos\theta + \cos\dfrac{\pi}{4}\sin\theta\right]$	1M
$= \sin\theta + \cos\theta$	1
(ii) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sin\theta+\cos\theta} = \frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}} \csc\left(\frac{\pi}{4}+\theta\right) d\theta$	
Let $u = \dfrac{\pi}{8}+\dfrac{\theta}{2}$. Then $du = \dfrac{1}{2}\,d\theta$.	
$\displaystyle\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sin\theta+\cos\theta} = \sqrt{2}\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \csc 2u \, du$	1M
$= \dfrac{1}{\sqrt{2}}\Big[\ln(\tan u)\Big]_{\frac{\pi}{8}}^{\frac{3\pi}{8}}$	
$= \dfrac{1}{\sqrt{2}}\left[\ln\left(\tan\left(\dfrac{\pi}{2}-\dfrac{\pi}{8}\right)\right) - \ln\left(\tan\dfrac{\pi}{8}\right)\right]$	1M
$= -\sqrt{2}\ln\tan\dfrac{\pi}{8}$	
$= -\sqrt{2}\ln\left(\sqrt{2}-1\right)$	1A
(c) Let $\phi = \dfrac{\pi}{2}-\theta$. Then $d\phi = -\,d\theta$	1M
$\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sin\theta+\cos\theta}\,d\theta = -\int_{\frac{\pi}{2}}^{0} \frac{\sin\left(\frac{\pi}{2}-\phi\right)}{\sin\left(\frac{\pi}{2}-\phi\right)+\cos\left(\frac{\pi}{2}-\phi\right)}\,d\phi$	
$\displaystyle= \int_0^{\frac{\pi}{2}} \frac{\cos\phi}{\sin\phi+\cos\phi}\,d\phi = \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta+\cos\theta}\,d\theta$	1
(d) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sin\theta+\cos\theta}\,d\theta = \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sin\theta+\cos\theta}\,d\theta + \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta+\cos\theta}\,d\theta\right]$	1M
$= \dfrac{1}{2}\Big[\theta\Big]_0^{\frac{\pi}{2}} = \dfrac{\pi}{4}$	
$\displaystyle\int_0^{\frac{\pi}{2}} \frac{2\sin\theta+5}{\sin\theta+\cos\theta}\,d\theta = 2\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\sin\theta+\cos\theta}\,d\theta + 5\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sin\theta+\cos\theta}$	1M
$= \dfrac{\pi}{2} - 5\sqrt{2}\ln\left(\sqrt{2}-1\right)$	1A

Solution	Marks
12. (a) (i) $\overrightarrow{OA}\cdot\overrightarrow{OC} = c\left(\dfrac{\mathbf{a}\cdot\mathbf{a}}{\lvert\mathbf{a}\rvert}+\dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{b}\rvert}\right)$	1M
$\lvert\mathbf{a}\rvert\lvert\mathbf{c}\rvert\cos\angle AOC = c\lvert\mathbf{a}\rvert\left(1+\dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}\right)$	1M
$\angle AOC = \dfrac{c}{\lvert\mathbf{c}\rvert}\left(1+\dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}\right)$	
$\overrightarrow{OB}\cdot\overrightarrow{OC} = c\left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert}+\lvert\mathbf{b}\rvert\right)$	
$\lvert\mathbf{b}\rvert\lvert\mathbf{c}\rvert\cos\angle BOC = c\lvert\mathbf{b}\rvert\left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}+1\right)$	
$\cos\angle BOC = \dfrac{c}{\lvert\mathbf{c}\rvert}\left(1+\dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}\right)$	
$= \cos\angle AOC$	
Since $0 \le \angle AOC \le \pi$ and $0 \le \angle BOC \le \pi$, we have $\angle AOC = \angle COB$.	1
(ii) $\overrightarrow{BC} = c\left(\dfrac{\mathbf{a}}{\lvert\mathbf{a}\rvert}+\dfrac{\mathbf{b}}{\lvert\mathbf{b}\rvert}\right)-\mathbf{b}$	1M
$= \dfrac{\lvert\mathbf{b}\rvert\mathbf{a}+\lvert\mathbf{a}\rvert\mathbf{b}}{\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert+\lvert\mathbf{a}-\mathbf{b}\rvert}-\mathbf{b}$	
$= \dfrac{\lvert\mathbf{b}\rvert\mathbf{a}-\lvert\mathbf{b}\rvert\mathbf{b}-\lvert\mathbf{a}-\mathbf{b}\rvert\mathbf{b}}{\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert+\lvert\mathbf{a}-\mathbf{b}\rvert}$	
$= \dfrac{\lvert\mathbf{a}-\mathbf{b}\rvert\lvert-\mathbf{b}\rvert}{\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert+\lvert\mathbf{a}-\mathbf{b}\rvert}\left(\dfrac{\mathbf{a}-\mathbf{b}}{\lvert\mathbf{a}-\mathbf{b}\rvert}+\dfrac{-\mathbf{b}}{\lvert-\mathbf{b}\rvert}\right)$	
$= l\left(\dfrac{\mathbf{a}-\mathbf{b}}{\lvert\mathbf{a}-\mathbf{b}\rvert}+\dfrac{-\mathbf{b}}{\lvert-\mathbf{b}\rvert}\right)$ where $l = \dfrac{\lvert\mathbf{a}-\mathbf{b}\rvert\lvert-\mathbf{b}\rvert}{\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert+\lvert\mathbf{a}-\mathbf{b}\rvert}$	1A
Note that $\overrightarrow{BA} = \mathbf{a}-\mathbf{b}$ and $\overrightarrow{BO} = -\mathbf{b}$. By (a)(i), we have $\angle ABC = \angle CBO$.	1M
Since $\angle ABC = \angle CBO$ and $\angle AOC = \angle COB$, C is the incentre of $\triangle OAB$.	1
(b) $\overrightarrow{PQ} = (3\mathbf{i}-5\mathbf{k})-(\mathbf{i}+\mathbf{j}-3\mathbf{k}) = 2\mathbf{i}-\mathbf{j}-2\mathbf{k}$ and $\overrightarrow{PR} = 4\mathbf{i}-6\mathbf{j}+12\mathbf{k}$.	1M
Let $\mathbf{a} = \overrightarrow{PQ}$ and $\mathbf{b} = \overrightarrow{PR}$.	
$\mathbf{a}-\mathbf{b} = -2\mathbf{i}+5\mathbf{j}-14\mathbf{k}$, $\lvert\mathbf{a}\rvert = \sqrt{2^2+1^2+2^2} = 3$, $\lvert\mathbf{b}\rvert = 14$ and $\lvert\mathbf{a}-\mathbf{b}\rvert = 15$	1M
$\overrightarrow{PI} = \dfrac{(3)(14)}{3+14+15}\left(\dfrac{2\mathbf{i}-\mathbf{j}-2\mathbf{k}}{3}+\dfrac{4\mathbf{i}-6\mathbf{j}+12\mathbf{k}}{14}\right)$	1M
$= \dfrac{5}{4}\mathbf{i}-\mathbf{j}+\dfrac{1}{4}\mathbf{k}$	1A
$\overrightarrow{OI} = \overrightarrow{OP}+\overrightarrow{PI} = \dfrac{9}{4}\mathbf{i}-\dfrac{11}{4}\mathbf{k}$	1A

END OF PAPER