REG-2223-MOCK-SET 3-MATH-CP 2

Answers:

1. C	2. D	3. B	4. C	5. B	6. A	7. C	8. D	9. C	10. A
11. C	12. A	13. B	14. D	15. D	16. A	17. B	18. A	19. B	20. D
21. B	22. C	23. A	24. B	25. D	26. A	27. D	28. C	29. B	30. D
31. B	32. C	33. D	34. C	35. D	36. A	37. C	38. C	39. B	40. A
41. D	42. A	43. B	44. A	45. D					

Suggested Solutions:

1. C

$$\begin{aligned}(3 \cdot 9^{n-1})^2 &= (3 \cdot 3^{2n-2})^2 \\ &= (3^{2n-1})^2 \\ &= 3^{4n-2}\end{aligned}$$

2. D

$$\begin{aligned}\frac{x+y}{x} &= \frac{1-y}{y} \\ xy + y^2 &= x - xy \\ x(2y-1) &= -y^2 \\ x &= \frac{y^2}{1-2y}\end{aligned}$$

3. B

Consider the coefficient of each term in the expansions.

	Constant term	x term
A.	✗	
B.	✓	✓
C.	✓	✗
D.	✗	

4. C

$$\begin{aligned}\left(\frac{\pi}{5}\right)^3 &\approx 0.248\,050\,213 \\ &= 0.2481 \quad \text{(correct to 4 significant figures)}\end{aligned}$$

5. B

$$2f(-1)+3 = 2\left[(-1)^{2012}+2012(-1)+2012\right]+3$$
$$= 2(1-2012+2012)+3$$
$$= 5$$

6. A

$0 = k^3 - k(k^2) + 2k - 4$

$k = 2$

Remainder $= (-k)^3 - k(-k)^2 + 2(-k) - 4$
$$= -2k^3 - 2k - 4$$
$$= -24$$

7. C

The inequalities become $x < \frac{1}{4}$ and $x \le -\frac{1}{6}$.

Thus, $x \le -\frac{1}{6}$.

The greatest value of x is -1.

8. D

$$x(x-k) = x-1$$
$$x^2 + (-k-1)x + 1 = 0$$
$$\Delta = (-k-1)^2 - 4(1)(1) = 0$$
$$k^2 + 2k - 3 = 0$$
$$k = -3 \quad \text{or} \quad 1$$

9. C

The coordinates of vertex are $(-a,\ b)$.

I. ✓. From the vertex, we have $-a < -1$. Therefore, $a > 1$.

II. ✗. From the vertex, we have $b < 0$.

III. ✓. y-intercept $= (0+a)^2 + b = a^2 + b < 1$

10. A

$$\text{Cost} = \frac{6400-420}{1+15\%}$$
$$= \$5200$$

11. C

Let the actual area be A cm^2.

$$\frac{A}{25} = 4000^2$$
$$A = 4 \times 10^8$$

Actual area $= \dfrac{4 \times 10^8}{100^2} = 40\,000\ \text{m}^2$

12. A

Let $x = \dfrac{k\sqrt{z}}{y}$, where k is a non-zero constant.

Then $k = \dfrac{xy}{\sqrt{z}}$.

Thus, $\dfrac{x^2y^2}{z} = k^2$ is a constant.

13. B

The numbers are formed by +3, +4, +5, . . .

The sequence is 4, 7, 11, 16, 22, 29, 37, . . .

The required number is 37.

14. D

$AE = 12 - 5 = 7$ cm

$DE = \sqrt{12^2 + 5^2} = 13$ cm

$AD = \sqrt{12^2 + 12^2} \approx 17.0$ cm

Required perimeter $= AD + 13 + 7 \approx 37.0$ cm

15. D

$\angle ABE = 180^\circ - 132^\circ = 48^\circ$

$\angle AEB = \dfrac{180^\circ - 48^\circ}{2} = 66^\circ$

$\angle EAD = \angle AEB = 66^\circ$

$\angle AED = 180^\circ - 2 \times 66^\circ = 48^\circ$

$\angle DEC = 180^\circ - 66^\circ - 48^\circ = 66^\circ$

16. A

Let the height and base radius of the cone be h and r respectively.

$$\frac{1}{3}\pi r^2 h = 2 \times \frac{2}{3}\pi r^3$$

$$h = 4r$$

$$h : r = 4 : 1$$

17. B

Ratio of base radius $= \sqrt{\frac{4}{9}}$

$= \frac{2}{3}$

Required ratio $= \frac{2^3}{3^3 - 2^3}$

$= \frac{8}{19}$

18. A

$\triangle ABC \sim \triangle BEC$

Area of $\triangle ABD = 18 \times \frac{1}{3} = 6\,\text{cm}^2$

Let the area of $\triangle BEC$ be $x\,\text{cm}^2$.

$$\frac{x}{x + 18 + 6} = \left(\frac{BE}{AB}\right)^2$$

$$x = 8$$

Area of $\triangle ABC = 18 + 6 + 8 = 32\,\text{cm}^2$

19. B

$AC = \sqrt{1^2 + 1^2} = \sqrt{2}\,\text{cm}$

$AD = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2\,\text{cm}$

Required area $= \frac{1^2}{2} + \frac{(\sqrt{2})^2}{2} + \frac{2^2}{2}$

$= \frac{7}{2}\,\text{cm}^2$

20. D

I. ✓. $DF = DP = DR$

II. ✓. Note that $\triangle DRS$ is also an equilateral triangle.
So, $\angle DRS = 60°$.
Since $DF = DR$, $\angle DRF = \angle DFR = \dfrac{\angle PDF}{2} = 30°$.
Thus, RF is the angle bisector of $\angle DRS$.

III. ✓. Note that $\triangle DPQ$ is also an equilateral triangle.
$\angle DPQ = \angle DPF = 60°$ and $\angle PRQ = \angle PRF = 30°$.
Thus, $\triangle PQR \cong \triangle PFR \quad (ASA)$

21. B

$\angle CBE = 90° - \alpha$ and $\angle BCE = 180° - 90° - (90° - \alpha) = \alpha$

$$CE = \frac{BE}{\tan\alpha} = \frac{\left(\frac{AB}{\cos\alpha}\right)}{\tan\alpha} = \frac{AB}{\cos\alpha \times \frac{\sin\alpha}{\cos\alpha}} = \frac{AB}{\sin\alpha}$$

22. C

$\angle ABC = 180° - 68° = 112°$
$\angle ABD = 90°$
$\angle CBD = 112° - 90° = 22°$
$\angle BDC = \angle CBD = 22°$
$\angle ADB = 68° - 22° = 46°$

23. A

Note that $\angle ACB = 90°$ and AB is a diameter of the circle.

$$\text{Required area} = \frac{1}{2} \times \left(\frac{25}{2}\right)^2 \pi - \frac{(7)(24)}{2}$$
$$\approx 161 \text{ cm}^2$$

24. B

$$(2,\ 120°) = \left(-1,\ \sqrt{3}\right) \longrightarrow \left(-1,\ -\sqrt{3}\right)$$

25. D

I. ✓. Slope of $L_1 = -\frac{1}{A} < 0$. So, $A > 0$.

II. ✓. Slope of $L_2 = -\frac{1}{C}$.

$$\left(\frac{-1}{A}\right)\left(\frac{-1}{C}\right) = -1$$

$$AC = -1$$

III. ✓. x-intercept of $L_1 = B$ and x-intercept of $L_2 = D$.

Thus, $B > D$.

26. A

$\frac{a}{c} = \frac{b}{5} = \frac{2}{-1}$

Thus, $a = -2c$ and $b = -10$.

$2a + 3b + 4c = 2(a + 2c) + 3b = -30$

27. D

$C\colon x^2 + y^2 - 3x + y - \frac{13}{2} = 0$

I. ✗. Centre $\left(\frac{3}{2}, -\frac{1}{2}\right)$

II. ✓. $AB = \sqrt{(2-1)^2 + (1+2)^2} = \sqrt{10}$

Radius $= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \frac{13}{2}} = 3 < AB$

III. ✓. Slope of $AB = \frac{1+2}{2-1} = 3$

Slope of $AG = \frac{-2 + \frac{1}{2}}{1 - \frac{3}{2}} = 3$, where G is the centre.

Thus, three points are collinear.

28. C

Required probability $= \frac{3+5}{6^2}$

$= \frac{2}{9}$

29. B

50% of the data lies between lower quartile and upper quartile.

30. D

We have $m = n = 5$.

I. ✓.

II. ✓. Mean $= \dfrac{1+2+5+12+\ldots+5}{10} = 5.2$

III. ✓. Range $= 12 - 1 = 11$

31. B

$$1 - \frac{ab}{a^2-b^2} - \frac{b}{b-a} = \frac{(a^2-b^2) - ab + b(a+b)}{(a+b)(a-b)}$$
$$= \frac{a^2}{a^2-b^2}$$

32. C

For the point $(0,\ -1)$,

$$\log_7 x = 0 \quad \text{and} \quad \log_7 y = -1$$
$$x = 1 \qquad\qquad y = 7^{-1}$$

We have $7^{-1} = a(1)^b$, and $a = \dfrac{1}{7}$.

For the point $(2,\ 0)$,

$$\log_7 x = 2 \quad \text{and} \quad \log_7 y = 0$$
$$x = 49 \qquad\qquad y = 1$$

We have $1 = \dfrac{1}{7}(49)^b$, and $b = \dfrac{1}{2}$.

33. D

Each digit in hexadecimal number corresponds to four digits in its binary representation. $F6_{16} = 11110110_2$ and $14_{16} = 10100_2$

Thus, $14F6_{16} = 1010011110110_2$.

34. C

Put $k = 1$, using the calculator CMPLX mode,

$$\frac{5k+10i}{1-2i} = \frac{5+10i}{1-2i} = -3+4i$$

Imaginary part = 4

Only option C gives 4 when $k = 1$.

35. D

α is a root of the equation.

$$2\alpha^2 + 4\alpha - 1 = 0$$
$$\alpha^2 = -2\alpha + \frac{1}{2}$$

$$\begin{aligned}\alpha^2 - 2\beta &= \left(-2\alpha + \frac{1}{2}\right) - 2\beta \\ &= -2(\alpha + \beta) + \frac{1}{2} \\ &= -2(-2) + \frac{1}{2} \\ &= \frac{9}{2}\end{aligned}$$

36. A

$$\begin{aligned}\text{General term} &= (2^{n+1} - 2) - (2^n - 2) \\ &= 2^n(2 - 1) \\ &= 2^n\end{aligned}$$

I. ✓. $\dfrac{T(n+1)}{T(n)} = \dfrac{2^{n+1}}{2^n} = 2 = \text{constant}$.

II. ✗. Second term $= 2^2 = 4 \neq 6$.

III. ✗.

37. C

Let K be a point on VB such that $AK \perp VB$. We have $CK \perp VB$ also.
Required angle is $\angle AKC$.

$AC = \sqrt{12^2 + 12^2} = 12\sqrt{2}\text{ cm}$

$VA = VB = VC = \sqrt{8^2 + \left(\frac{12\sqrt{2}}{2}\right)^2} = \sqrt{136}\text{ cm}$

In $\triangle VAB$,

$$\cos\angle ABV = \frac{\left(\frac{12}{2}\right)}{VB}$$
$$\angle ABV \approx 59.0°$$

$AK = CK = 12\sin\angle ABV \approx 10.3\text{ cm}$

In $\triangle AKC$,

$$AC^2 = AK^2 + CK^2 - 2(AK)(CK)\cos\angle AKC$$
$$\angle AKC \approx 111°$$

38. C

$g(x) = -\frac{1}{2}f(x)$

The graph of $y = f(x)$ is reduced along the y-axis to $\frac{1}{2}$ times the original and then is reflected about the x-axis to the graph of $y = g(x)$.
The answer is C.

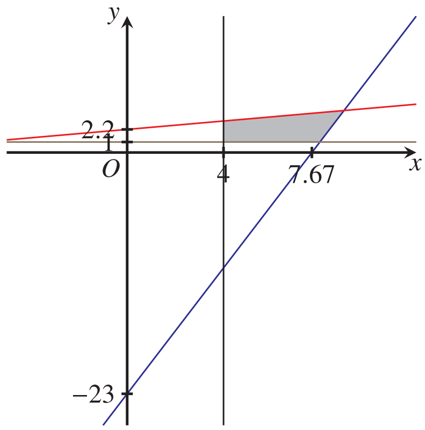
39. B

Line	***x*-intercept**	***y*-intercept**
$x = 4$	4	
$y = 1$		1
$3x - y = 23$	7.67	−23
$x - 5y + 11 = 0$	−11	2.2

Sketch the graph using the intercepts.

The value of $y - 4x + 20$ is larger when x is small and y is large, i.e., top left corners.

The top left corners are $(4,\ 3)$ and $(9,\ 4)$.



$(x,\ y)$	$(4,\ 3)$	$(9,\ 4)$
$y - 4x + 20$	7	−12

Required value = 7

40. A

$\angle BCA = 90°$ and $\angle CBA = 180° - 90° - 42° = 48°$

$\angle OBC = \dfrac{48°}{2} = 24°$

$\angle BOC = 180° - 90° - 24° = 66°$

41. D

x-coordinate of vertex $= \dfrac{-k}{2(1)} = -\dfrac{k}{2}$

Midpoint of $PR = \left(-\dfrac{k}{2},\ 0\right)$

Consider the x-coordinate of centroid,

$$-2 = \frac{0(1) + \left(-\frac{k}{2}\right)(2)}{1 + 2}$$

$$k = 6$$

42. A

Required number $= C_5^{25} - C_5^{15} - C_5^{10}$

$= 49\,875$

43. B

Solve the system $\begin{cases} 2x + y - 5 = 0 \\ x^2 + y^2 - kx + 6y - 10 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
2	2	+

Required range does not contain 2 and 2 is not a boundary value of the range.

The answer is B.

44. A

$$\sin\theta = 5\tan\theta$$

$$\sin\theta = \frac{5\sin\theta}{\cos\theta}$$

$$\sin\theta\left(1 - \frac{5}{\cos\theta}\right) = 0$$

$$\sin\theta = 0 \quad \text{or} \quad \cos\theta = 5 \text{ (rejected)}$$

$$\theta = 0° \quad \text{or} \quad 180°$$

45. D

Required probability $= \dfrac{\frac{4}{5} \times \frac{2}{3}}{\frac{4}{5} \times \frac{2}{3} + \frac{1}{5} \times \frac{1}{3}}$

$= \dfrac{8}{9}$

END OF PAPER