

M2REG-AOD-2223-ASM-SET 4-MATH

Suggested solutions

1. (a) Vertical asymptote is $x = 1$.

1A

$$\begin{aligned} f(x) &= \frac{3x^2 - 3x + 1}{x - 1} \\ &= 3x + \frac{1}{x - 1} \end{aligned}$$

1M

Oblique asymptote is $y = 3x$.

1A

(b) $f'(x) = 3 - \frac{1}{(x - 1)^2}$

1A

$$\begin{aligned} f'(2) &= 3 - \frac{1}{(2 - 1)^2} \\ &= 2 \end{aligned}$$

1A

Required equation is

$$y - 7 = 2(x - 2)$$

$$2x - y + 3 = 0$$

1A

2. (a) $9 = 8a + 4b + 1$

1M

$$\frac{dy}{dx} = 3ax^2 + 2bx$$

$$0 = 3a(4) + 2b(2)$$

1M

Solving, we have $a = -2$ and $b = 6$.

1A

(b) $\frac{dy}{dx} = -6x^2 + 12x$

$$\frac{d^2y}{dx^2} = -12x + 12$$

$$\left. \frac{d^2y}{dx^2} \right|_{(2, 9)} = -12(2) + 12 = -12 < 0$$

1M

Thus, $(2, 9)$ is a maximum point. The claim is agreed.

1A

(c) When $\frac{d^2y}{dx^2} = 0$, $x = 1$.

x	$x < 1$	$x > 1$
$\frac{d^2y}{dx^2}$	+	-

1M

Point of inflexion is $(1, 5)$.

1A

3. (a) Vertical asymptote is $x = 4$. 1A

$$f(x) = \frac{x^2 + 2x - 8}{x - 4}$$

$$= x + 6 + \frac{16}{x - 4}$$

1M

Oblique asymptote is $y = x + 6$. 1A

(b) $f'(x) = \frac{(x - 4)(2x + 2) - (x^2 + 2x - 8)}{(x - 4)^2}$ 1M

$$= \frac{x^2 - 8x}{(x - 4)^2}$$

1A

- (c) When $f'(x) = 0$, $x = 0$ or 8 . 1A

x	$x < 0$	$0 < x < 4$	$4 < x < 8$	$x > 8$
$f'(x)$	+	-	-	+

1M

The maximum point is $(0, 2)$ and the minimum point is $(8, 18)$. 1A+1A

(d) $\frac{x^2 + 2x - 8}{x - 4} = 0$

$$x^2 + 2x - 8 = 0$$

$$x = -4 \quad \text{or} \quad 2$$

1A

Slope of tangent = $\frac{(-4)^2 - 8(-4)}{(-4 - 4)^2}$

$$= \frac{3}{4}$$

1A

Required equation is

$$y - 0 = \frac{3}{4}(x + 4)$$

$$3x - 4y + 12 = 0$$

1A

4. (a) $5(1) + 9y - 11 = 0$ 1M

$$y = \frac{2}{3}$$

The point of contact is $\left(1, \frac{2}{3}\right)$.

$$\frac{2}{3} = \frac{1+a}{1+b}$$
 1M

$$a = \frac{2b-1}{3}$$

$$\frac{dy}{dx} = \frac{(x^2 + bx) - (x+a)(2x+b)}{(x^2 + bx)^2}$$
 1M

$$= \frac{-x^2 - 2ax - ab}{(x^2 + bx)^2}$$

$$\frac{-5}{9} = \frac{-1 - 2a - ab}{(1+b)^2}$$
 1M

$$9 + 18a + 9ab = 5(b+1)^2$$

$$9 + 18\left(\frac{2b-1}{3}\right) + 9\left(\frac{2b-1}{3}\right)b = 5(b^2 + 2b + 1)$$

$$b^2 - b - 2 = 0$$

$$b = 2 \quad \text{or} \quad -1 \text{ (rejected)}$$
 1A

$$a = \frac{2(2) - 1}{3} = 1$$
 1A

(b) $\frac{dy}{dx} = -\frac{x^2 + 2x + 2}{(x^2 + 2x)^2}$

$$-\frac{x^2 + 2x + 2}{(x^2 + 2x)^2} = 0$$
 1M

$$x^2 + 2x + 2 = 0$$

$$\Delta = 2^2 - 4(1)(2) = -4 < 0$$

The equation has no real roots.

The curve has no stationary point. 1A

(c) x -intercept = -1 ; y -intercept does not exist. 1A

(d) $\frac{d^2y}{dx^2} = -\frac{(2x+2)(x^2+2x)^2 - (x^2+2x+2)(2)(x^2+2x)(2x+2)}{(x^2+2x)^4}$ 1M

$$= \frac{2(x+1)(x^2+2x+4)}{(x^2+2x)^3}$$

When $\frac{d^2y}{dx^2} = 0$,

$$(x+1)(x^2+2x+4) = 0$$

$$x = -1 \quad \text{or} \quad x^2 + 2x + 4 = 0$$

For $x^2 + 2x + 4 = 0$, $\Delta = 2^2 - 4(1)(4) = -12 < 0$.

The equation has no real roots.

x	$x < -1$	$-1 < x < 0$	$x > 0$
$\frac{d^2y}{dx^2}$	+	-	+

1M

Point of inflexion is $(-1, 0)$.

1A

(e) Vertical asymptotes are $x = 0$ and $x = -2$. 1A

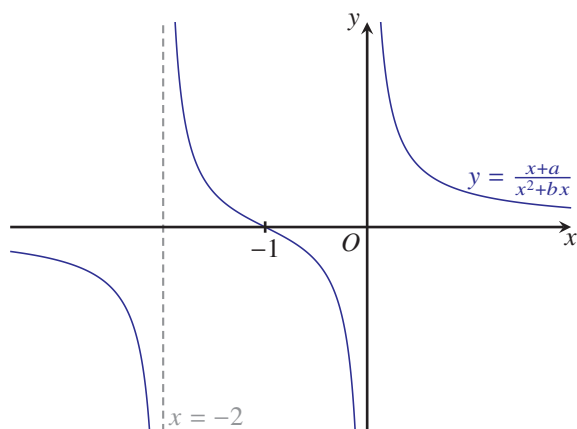
$$y = \frac{x+1}{x^2+2x}$$

Horizontal asymptote is $y = 0$.

1A

(f) (Correct shape) 1A

(All correct) 1A



5. (a) x -intercept = -1 1A
 y -intercept = -1 1A

(b) $\frac{dy}{dx} = 2(x+1)(x-1)^{-1} - (x+1)^2(x-1)^{-2}$ 1M

$$= \frac{(x-3)(x+1)}{(x-1)^2}$$

When $\frac{dy}{dx} = 0$, $x = 3$ or -1 . 1M

x	$x < -1$	$-1 < x < 1$	$1 < x < 3$	$x > 3$
$\frac{dy}{dx}$	+	-	-	+

1M

Maximum point is $(-1, 0)$ and minimum point is $(3, 8)$.

1A

- (c) Vertical asymptote is $x = 1$. 1A

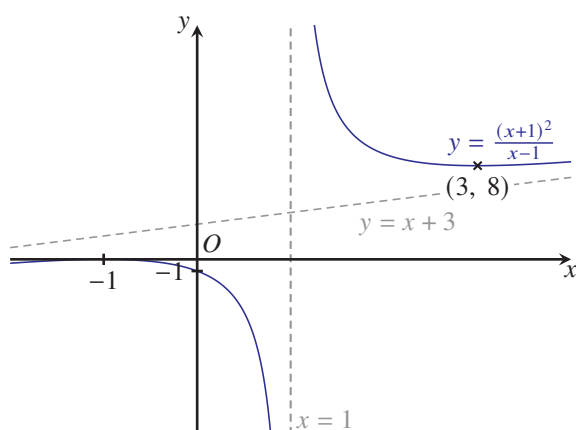
$$y = x + 3 + \frac{4}{x-1}$$

1M

Oblique asymptote is $y = x + 3$. 1A

- (d) (Correct shape) 1A

(All correct) 1A



6. (a) Vertical asymptote is $x = 1$. 1A

$$f(x) = x + 2 + \frac{3x - 2}{(x - 1)^2} \quad 1M$$

Oblique asymptote is $y = x + 2$. 1A

(b) $f'(x) = 1 + \frac{3(x - 1)^2 - (3x - 2)(2)(x - 1)}{(x - 1)^4}$ 1M

$$= 1 + \frac{-3x + 1}{(x - 1)^3} \quad 1A$$

$$1 + \frac{-3x + 1}{(x - 1)^3} = 0$$

$$(x - 1)^3 - 3x + 1 = 0$$

$$x^2(x - 3) = 0$$

$$x = 0 \quad \text{or} \quad 3 \quad 1A$$

(c)

x	$x < 0$	$0 < x < 1$	$1 < x < 3$	$x > 3$
$f'(x)$	+	+	-	+

1M

Minimum point is $\left(3, \frac{27}{4}\right)$ and no maximum point. 1A

(d) $f''(x) = \frac{-3(x - 1)^3 - (-3x + 1)(3)(x - 1)^2}{(x - 1)^3}$

$$= \frac{6x}{(x - 1)^4}$$

1

(e) When $f''(x) = 0$, $x = 0$.

x	$x < 0$	$0 < x < 1$	$x > 1$
$f''(x)$	-	+	+

1M

Point of inflexion is $(0, 0)$. 1A

(f) (Correct shape) 1A

(All correct) 1A

