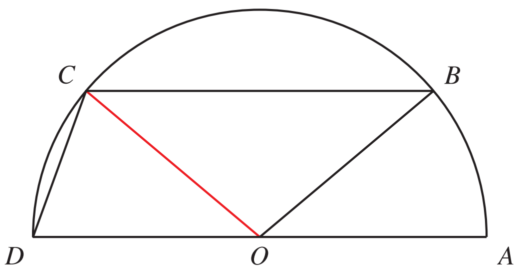
REV-POC-2223-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. $\boxed{\text{B}}$

Join OC (to use the property from equal arcs)

Reflex $\angle BOD = 2 \times 110° = 220°$
$\angle AOB = 220° - 180° = 40°$
$\angle COD = \angle AOB = 40°$
$\angle COB = 180° - 2 \times 40° = 100°$
$\angle OBC = \angle OCB = \dfrac{180° - 100°}{2} = 40°$



2. $\boxed{\text{C}}$

$\angle POS = \angle ROS = \dfrac{136°}{2} = 68°$
$\angle SPO = \angle PSO = \dfrac{180° - 68°}{2} = 56°$

3. $\boxed{\text{B}}$

$\angle AOC = \angle BOD$
$\angle AOD = \angle BOC = \dfrac{180° - 48°}{2} = 66°$
$\angle ABD = \dfrac{180° - 66° - 48°}{2} = 33°$

4. $\boxed{\text{D}}$

I. ✓. Note that $\angle AOB = \angle BOC = \angle COD = \ldots = \angle FOA$.
$\angle EOF = \dfrac{360°}{6} = 60°$

II. ✓. Since $\overset{\frown}{AB} = \overset{\frown}{DE}$, $AB = DE$.

III. ✓. Since all arcs are equal, all chords are also equal.
It is a regular hexagon.

5. $\boxed{\text{C}}$

$\angle PQR = 90°$
$\angle PRQ = \dfrac{180° - 90°}{2} = 45°$
$\angle PQS = 90° - 18° = 72°$
$\angle SRP = \angle SQR = 72°$
$\angle QRS = 72° + 45° = 117°$

6. $\boxed{\text{C}}$

$\angle AOP = \angle POQ = \angle QOC = \dfrac{180° - 90°}{3} = 30°$

$\angle CPQ = \dfrac{1}{2}\angle COP = 15°$

$\angle AQP = \dfrac{1}{2}\angle AOP = 15°$

$\angle ARC = \angle PRQ = 180° - 15° - 15° = 150°$

7. $\boxed{\text{C}}$

$\angle FDE = 145° - 15° = 130°$

$\angle ADC = 180° - 130° = 50°$

$\angle AOC = 2\angle ADC = 100°$

$\angle BOC = \angle AOB = \dfrac{100°}{2} = 50°$

$\angle OCE = \angle BOC - \angle CEO = 50° - 15° = 35°$

8. $\boxed{\text{C}}$

$\angle PRQ : \angle QPR : \angle PQR = \overset{\frown}{PQ} : \overset{\frown}{QR} : \overset{\frown}{PR} = 2 : 3 : 4$

$\angle PQR = 180° \times \dfrac{4}{2+3+4} = 80°$

9. $\boxed{\text{C}}$

Let O be the centre.

$\angle OBA = \angle BAC = 36°$

$\angle AOB = 180° - 36° - 36° = 108°$

$\dfrac{\overset{\frown}{AC}}{\overset{\frown}{AB}} = \dfrac{\angle AOC}{\angle AOB}$

$\dfrac{\overset{\frown}{AC}}{9} = \dfrac{180°}{108°}$

$\overset{\frown}{AC} = 15\text{ cm}$

10. $\boxed{\text{D}}$

$\dfrac{\angle ABD}{\angle BAC} = \dfrac{\overset{\frown}{AD}}{\overset{\frown}{BC}}$

$\dfrac{\angle ABD}{28°} = \dfrac{1}{2}$

$\angle ABD = 14°$

$\angle ADB = 90°$

$\angle CAD = 180° - 90° - 28° - 14° = 48°$

11. $\boxed{\text{B}}$

Let $\angle ABD = 3\theta$. Then $\angle ACB = 3\theta$, $\angle BAC = 2\theta$ and $CBD = 4\theta$.

$$2\theta + (3\theta + 4\theta) + 3\theta = 180^\circ$$
$$\theta = 15^\circ$$

$\angle BFC = 2\theta + 3\theta = 75^\circ$

$\angle ACE = 75^\circ - 40^\circ = 35^\circ$

12. $\boxed{\text{A}}$

$\angle POQ = 360^\circ - 100^\circ - 90^\circ - 50^\circ = 120^\circ$

$$\frac{\overset{\frown}{SR}}{\overset{\frown}{PQ}} = \frac{\angle SOR}{\angle POQ} = \frac{90^\circ}{120^\circ}$$

$\overset{\frown}{SR} : \overset{\frown}{PQ} = 3 : 4$

13. $\boxed{\text{D}}$

$\angle AOB : \angle BOC : \angle COA = 5 : 4 : 3$

$\angle AOB = 360^\circ \times \dfrac{5}{5+4+3} = 150^\circ$

$\angle OBA = \dfrac{180^\circ - 150^\circ}{2} = 15^\circ$

14. $\boxed{\text{C}}$

Let $\angle PRQ = \theta$.

Since $PQ = QR = RS$, we have $\angle QSR = \angle SQR = \theta$.

Since $\overset{\frown}{PS} : \overset{\frown}{RS} = 2 : 1$, we have $\angle PRS = 2\theta$.

In $\triangle QRS$,

$$\theta + (\theta + 2\theta) + \theta = 180^\circ$$
$$\theta = 36^\circ$$

$\angle PES = 2\theta + \theta = 108^\circ$

15. $\boxed{\text{D}}$

$\angle AOB = 24^\circ \times 2 = 48^\circ$

$\angle BOC = 48^\circ \times \dfrac{3}{2} = 72^\circ$

$\angle AOC = 48^\circ + 72^\circ = 120^\circ$

$\angle OAC = \dfrac{180^\circ - 120^\circ}{2} = 30^\circ$

16. $\boxed{\text{B}}$

$\angle ABD = 15^\circ + 25^\circ = 40^\circ$

$\angle ADB = 90^\circ$

$\angle BAD = 180^\circ - 90^\circ - 40^\circ = 50^\circ$

$$\frac{\overset{\frown}{AD}}{\overset{\frown}{BQD}} = \frac{\angle ABD}{\angle BAD} = \frac{40^\circ}{50^\circ} = \frac{4}{5}$$

17. $\boxed{\text{C}}$

Let $\angle ADB = \theta$.

Then $\angle BDC = 2\theta$, $\angle CAD = 2\theta$ and $\angle ACD = 3\theta$.

$$\angle CAD + \angle ACD + \angle ADC = 180°$$
$$2\theta + 3\theta + (\theta + 2\theta) = 180°$$
$$\theta = 22.5°$$

$x = 180° - 2\theta - \theta = 112.5°$

18. $\boxed{\text{B}}$

Reflex $\angle AOD = 2\angle AED = 200°$

$\angle AOB = 200° - 130° = 70°$

$\angle ACB = \dfrac{\angle AOB}{2} = 35°$

19. $\boxed{\text{C}}$

Let $\angle ADB = x$.

Since $AB = BC = CD$, we have $\angle ADB = \angle BDC = \angle CBD = x$.

$$\angle ABC + \angle ADC = 180°$$
$$(75° + x) + 2x = 180°$$
$$x = 35°$$

20. $\boxed{\text{C}}$

$\angle EBC = \angle EDF = 76°$

$\angle AEC = \angle EBC \times \dfrac{1+1}{3+1} = 38°$

$\angle ABC = 180° - \angle AEC = 142°$

21. $\boxed{\text{C}}$

$$\angle BAD : \angle BCD = (3+5) : (3+4)$$
$$= 8 : 7$$

Since $\angle BAD + \angle BCD = 180°$, $\angle BAD = 180° \times \dfrac{8}{8+7} = 96°$.

22. $\boxed{\text{B}}$

$\angle DCE = \dfrac{44°}{2} = 22°$

$$\angle BCE + \angle BAE = 180°$$
$$(y - 22°) + x = 180°$$
$$x + y = 202°$$

23. $\boxed{\text{B}}$

Let $\angle FAG = x$.

$$\angle FEG = \angle FAE + \angle AFE$$
$$= x + 12^\circ$$
$$\angle ABG = 180^\circ - \angle BAG - \angle AGB$$
$$= 180^\circ - x - 18^\circ$$
$$= 162^\circ - x$$
$$\angle ABC = \angle FEG$$
$$30^\circ + 162^\circ - x = x + 12$$
$$x = 90^\circ$$

24. $\boxed{\text{D}}$

Let $\angle BAD = x$.

$$\angle CDF = \angle BAD + \angle AED = x + 32^\circ$$
$$\angle BCD = \angle CDF + \angle AFB = (x + 32^\circ) + 26^\circ = x + 58^\circ$$
$$\angle BAD + \angle BCD = 180^\circ$$
$$x + (x + 58^\circ) = 180^\circ$$
$$x = 61^\circ$$

25. $\boxed{\text{A}}$

$$\angle BAD = 180^\circ - 72^\circ = 108^\circ$$
$$\angle OAD = 108^\circ - 58^\circ = 50^\circ$$
$$\angle ODA = \angle OAD = 50^\circ$$
$$\angle AOD = 180^\circ - 50^\circ - 50^\circ = 80^\circ$$

26. $\boxed{\text{C}}$

Let $\angle ACB = 2x$.

Then $\angle BAC = 3x$ and $\angle CAD = 4x$.

$$\angle ACD = 90^\circ$$
$$\angle BAD + \angle BCD = 180^\circ$$
$$(2x + 4x) + (90^\circ + 3x) = 180^\circ$$
$$x = 10^\circ$$
$$\angle BCD = 90^\circ + 2x = 110^\circ$$

27. $\boxed{\text{A}}$

$$\angle BCD = \angle DEF = 105^\circ$$
$$\angle BAD = 180^\circ - \angle BCD = 180^\circ - 105^\circ = 75^\circ$$

28. $\boxed{\text{D}}$

$\angle CBD = \angle CDB$

$\angle CBD + \angle CDB = 56°$

$\angle CDB = 28°$

Since $\angle BAC = \angle CDB = 28°$, A, B, C and D are concyclic.

$\angle BCA = \angle ADB = 36°$

29. $\boxed{\text{B}}$

I. ✓. Note that $12^2 + 12.6^2 = 302.76 = 17.4^2$.

We have $\angle BAC = 90°$ and BC is a diameter of the circle.

Radius $= \dfrac{BC}{2} = 8.7\,\text{cm}$

II. ✗. Note that $\dfrac{\overset{\frown}{AC}}{\overset{\frown}{AB}} \neq \dfrac{AC}{AB} = \dfrac{21}{20}$ in general.

Let O be the centre of the circle.

$12.6^2 = 8.7^2 + 8.7^2 - 2(8.7)(8.7)\cos\angle AOC$

$\angle AOC \approx 92.8°$

$12^2 = 8.7^2 + 8.7^2 - 2(8.7)(8.7)\cos\angle AOB$

$\angle AOB \approx 87.2°$

$\dfrac{\overset{\frown}{AC}}{\overset{\frown}{AB}} = \dfrac{\angle AOC}{\angle AOB} \approx 1.06 \neq \dfrac{21}{20}$

III. ✓. BC is a diameter and $\angle BAC = 90°$.

$\cos\angle ACB = \dfrac{AC}{BC} = \dfrac{21}{29}$

30. $\boxed{\text{B}}$

I. ✓. Since G is the orthocentre of $\triangle ABC$, $\angle AQG = \angle ARG = 90°$.

$AQGR$ is a cyclic quadrilateral *(opp. $\angle$s supp.)*.

II. ✓. Since G is the orthocentre of $\triangle ABC$, $\angle CQB = \angle CRB = 90°$.

$BCQR$ is a cyclic quadrilateral *(converse of $\angle$s in the same segment)*.

III. ✗. Note that the circle passing through C, Q and R is circle $BCQR$.

Since P is a point on the line segment BC, P lies inside the circle $BCQR$.

$CPRQ$ cannot be a cyclic quadrilateral.

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