REG-RF-2223-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. $\boxed{\text{B}}$

$$\begin{aligned}\frac{a^2+a-2}{a^2+7a+10} &= \frac{(a+2)(a-1)}{(a+2)(a+5)} \\ &= \frac{a-1}{a+5}\end{aligned}$$

2. $\boxed{\text{C}}$

$$\begin{aligned}\frac{\frac{2}{a}+\frac{1}{b}}{\frac{a}{b}-\frac{4b}{a}} &= \frac{\frac{2}{a}+\frac{1}{b}}{\frac{a}{b}-\frac{4b}{a}} \times \frac{ab}{ab} \\ &= \frac{2b+a}{a^2-4b^2} \\ &= \frac{2b+a}{(a+2b)(a-2b)} \\ &= \frac{1}{a-2b}\end{aligned}$$

3. $\boxed{\text{B}}$

$$\begin{aligned}\frac{9x-\frac{4}{x}}{3+\frac{2}{x}} &= \frac{9x-\frac{4}{x}}{3+\frac{2}{x}} \times \frac{x}{x} \\ &= \frac{9x^2-4}{3x+2} \\ &= \frac{(3x+2)(3x-2)}{3x+2} \\ &= 3x-2\end{aligned}$$

4. $\boxed{\text{D}}$

$$\begin{aligned}1+\frac{3x}{x+\frac{2}{x}} &= 1+\frac{3x}{x+\frac{2}{x}} \times \frac{x}{x} \\ &= 1+\frac{3x^2}{x^2+2} \\ &= \frac{(x^2+2)+3x^2}{x^2+2} \\ &= \frac{4x^2+2}{x^2+2}\end{aligned}$$

5. $\boxed{\text{A}}$

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\left(\frac{a}{b}+1\right)\left(1-\frac{b}{a}\right)} = \frac{\frac{1}{a^2}-\frac{1}{b^2}}{\left(\frac{a}{b}+1\right)\left(1-\frac{b}{a}\right)} \times \frac{a^2b^2}{a^2b^2}$$
$$= \frac{b^2-a^2}{ab(a+b)(a-b)}$$
$$= \frac{(b+a)(b-a)}{ab(a+b)(a-b)}$$
$$= -\frac{1}{ab}$$

6. $\boxed{\text{B}}$

$$\frac{4x^2-25}{2x^2-3x-5} \div \frac{2x+5}{x^2+3x+2} = \frac{(2x+5)(2x-5)}{(2x-5)(x+1)} \times \frac{(x+1)(x+2)}{2x+5}$$
$$= x+2$$

7. $\boxed{\text{A}}$

$$\frac{1}{1-x} - \frac{x+5}{x^2-1} - \frac{2}{x+1} = \frac{-(x+1)-(x+5)-2(x-1)}{(x+1)(x-1)}$$
$$= \frac{-4x-4}{(x+1)(x-1)}$$
$$= \frac{-4(x+1)}{(x+1)(x-1)}$$
$$= \frac{4}{1-x}$$

8. $\boxed{\text{B}}$

$$\frac{x}{x-y} - \frac{y}{y+x} + \frac{2xy}{y^2-x^2} = \frac{x(x+y)-y(x-y)-2xy}{(x-y)(x+y)}$$
$$= \frac{x^2-2xy+y^2}{(x-y)(x+y)}$$
$$= \frac{(x-y)^2}{(x-y)(x+y)}$$
$$= \frac{x-y}{x+y}$$

9. $\boxed{\text{B}}$

$$\frac{3}{x^2-4x-5} + \frac{1}{1-x^2} = \frac{3}{(x-5)(x+1)} + \frac{1}{(1-x)(1+x)}$$
$$= \frac{3(1-x)+(x-5)}{(x-5)(x+1)(1-x)}$$
$$= \frac{-2x-2}{(x-5)(x+1)(1-x)}$$
$$= \frac{2}{(x-5)(x-1)}$$

10. $\boxed{\text{A}}$

$$\begin{aligned}\frac{2x}{x^2-9}-\frac{x-2}{x^2-x-6}&=\frac{2x}{(x-3)(x+3)}-\frac{x-2}{(x-3)(x+2)}\\&=\frac{2x(x+2)-(x-2)(x+3)}{(x-3)(x+3)(x+2)}\\&=\frac{x^2+3x+6}{(x-3)(x+3)(x+2)}\end{aligned}$$

11. $\boxed{\text{A}}$

$$\begin{aligned}\frac{3x+1}{3x^2+8x-3}-\frac{1}{7x-3x^2-2}&=\frac{3x+1}{(3x-1)(x+3)}-\frac{1}{-(3x-1)(x-2)}\\&=\frac{(3x+1)(x-2)+(x+3)}{(3x-1)(x+3)(x-2)}\\&=\frac{3x^2-4x+1}{(3x-1)(x+3)(x-2)}\\&=\frac{(x-1)(3x-1)}{(3x-1)(x+3)(x-2)}\\&=\frac{x-1}{(x+3)(x-2)}\end{aligned}$$

12. $\boxed{\text{D}}$

$$\begin{aligned}\frac{x}{x^2-4}-\frac{1}{2x-4}&=\frac{2x-(x+2)}{2(x-2)(x+2)}\\&=\frac{x-2}{2(x-2)(x+2)}\\&=\frac{1}{2(x+2)}\end{aligned}$$

13. $\boxed{\text{D}}$

$$\begin{aligned}\frac{1}{(x-1)(x-2)}+\frac{3}{(x-1)(x+2)}&=\frac{x+2+3(x-2)}{(x-1)(x-2)(x+2)}\\&=\frac{4(x-1)}{(x-1)(x-2)(x+2)}\\&=\frac{4}{(x-2)(x+2)}\end{aligned}$$

14. $\boxed{\text{B}}$

$$\begin{aligned}1-\frac{ab}{a^2-b^2}-\frac{b}{b-a}&=\frac{(a^2-b^2)-ab+b(a+b)}{(a+b)(a-b)}\\&=\frac{a^2}{a^2-b^2}\end{aligned}$$

15. $\boxed{\text{D}}$

$$\begin{aligned}\frac{1}{6x^2-45x+81}-\frac{1}{x-3}&=\frac{1}{3(x-3)(2x-9)}-\frac{1}{x-3}\\&=\frac{1-3(2x-9)}{3(x-3)(2x-9)}\\&=\frac{2(3x-14)}{-3(x-3)(2x-9)}\end{aligned}$$

16. $\boxed{\text{C}}$

$$\begin{aligned}\frac{1}{x^2-2x+1}-\frac{1}{x^2+x-2}&=\frac{1}{(x-1)^2}-\frac{1}{(x+2)(x-1)}\\&=\frac{(x+2)-(x-1)}{(x-1)^2(x+2)}\\&=\frac{3}{(x-1)^2(x+2)}\end{aligned}$$

17. $\boxed{\text{A}}$

$$\begin{aligned}\frac{x^2-3x}{x^3+27}-\frac{1}{x+3}&=\frac{x^2-3x}{(x+3)(x^2-3x+9)}-\frac{x^2-3x+9}{(x+3)(x^2-3x+9)}\\&=\frac{-9}{x^3+27}\end{aligned}$$

18. $\boxed{\text{C}}$

$$\begin{aligned}\frac{4}{x^2-x-12}+\frac{x}{x+3}&=\frac{4+x(x-4)}{(x+3)(x-4)}\\&=\frac{(x-2)^2}{(x+3)(x-4)}\end{aligned}$$

19. $\boxed{\text{B}}$

$(a+2)^2$, $(a+2)(a-2)$, $(a+2)(a^2-2a+4)$
L.C.M. $=(a-2)(a+2)^2(a^2-2a+4)$

20. $\boxed{\text{D}}$

The three expressions are $(x-3)^2$, $(x-3)(x-1)$ and $(x+3)(x-3)$.
L.C.M. $=(x-3)^2(x-1)(x+3)$

21. $\boxed{\text{D}}$

$8x^3-1=(2x-1)(4x^2+2x+1)$
$4x^2-1=(2x+1)(2x-1)$
$6x^2+3x-3=3(2x-1)(x+1)$
L.C.M. $=3(x+1)(2x-1)(2x+1)(4x^2+2x+1)$

22. $\boxed{\text{A}}$

The three expressions are $2^2m^2n^5$, $2\cdot 3m^3n^3$ and $2^3m^5n^4$.
The H.C.F. is $2m^2n^3$.

23. $\boxed{\text{C}}$

$2x^2+5x-3=(x+3)(2x-1)$ and $x^4+27x=x(x+3)(x^2-3x+9)$
L.C.M. $=x(x+3)(2x-1)(x^2-3x+9)$

24. $\boxed{\text{A}}$

H.C.F. $=xy^2$

25. $\boxed{\text{C}}$

$3x^4y^2z \quad 2^2xy^5z \quad (2)(3)x^2y^3$
L.C.M. = $2^2(3)x^4y^5z = 12x^4y^5z$

26. $\boxed{\text{C}}$

3^2a^2b, $(2)^2(3)a^4b^3$, $(3)(5)a^6$
L.C.M. = $(2)^2(3)^2(5)a^6b^3 = 180a^6b^3$

27. $\boxed{\text{C}}$

L.C.M. is obtained by taking the maximum degree of each factor.
L.C.M. is x^2y^3z.

28. $\boxed{\text{A}}$

Consider the degree of variables x, y and z in the three expressions. From the expressions of H.C.F. and L.C.M., we have the following fact:

Variable	x	y	z
Minimum degree	2	2	1
Maximum degree	3	4	5

Thus, the third expression is x^2y^4z.

29. $\boxed{\text{B}}$

A. ✗. L.C.M. will be $4a^4b^4c^6$ instead.

B. ✓.

C. ✗. H.C.F. will contain a factor 2.

D. ✗. H.C.F. will contain a factor 2.

30. $\boxed{\text{C}}$

Consider indices of a, max $\{1, 4, r\} = 6 \quad \Rightarrow \quad r = 6$
Consider indices of b, min $\{3, 3, s\} = 1 \quad \Rightarrow \quad s = 1$

Conventional Questions

31. $\dfrac{x^2+7x+6}{x^2-x-2}\times\dfrac{x-2}{x-3}\times\dfrac{x^2-x-12}{x-4}$

$=\dfrac{(x+1)(x+6)}{(x-2)(x+1)}\times\dfrac{x-2}{x-3}\times\dfrac{(x-4)(x+3)}{x-4}$ 1M+1A

$=\dfrac{(x+6)(x+3)}{x-3}$ 1A

32. $\dfrac{a}{1+a}+\dfrac{b}{1-b}=\dfrac{\left(\frac{2x+1}{2x-1}\right)}{1+\frac{2x+1}{2x-1}}+\dfrac{\left(\frac{2x-1}{2x+1}\right)}{1-\frac{2x-1}{2x+1}}$ 1M

$=\dfrac{2x+1}{2x-1}\div\dfrac{(2x-1)+(2x+1)}{2x-1}+\dfrac{2x-1}{2x+1}\div\dfrac{(2x+1)-(2x-1)}{2x+1}$ 1M

$=\dfrac{2x+1}{4x}+\dfrac{2x-1}{2}$ 1M

$=\dfrac{(2x+1)+2x(2x-1)}{4x}$

$=\dfrac{4x^2+1}{4x}$ 1A

33. $\dfrac{A}{2x-1}+\dfrac{B}{3x-2}=\dfrac{A(3x-2)+B(2x-1)}{(2x-1)(3x-2)}$

$=\dfrac{(3A+2B)x+(-2A-B)}{(2x-1)(3x-2)}$ 1M

Comparing coefficients,

$\begin{cases}3A+2B=-1 & \text{1M}\\ -2A-B=-1 & \text{1M}\end{cases}$

Solving, we have $A=3$ and $B=-5$. 1A+1A

34. (a) Put $x=1$, $3(1)^3-6(1)+4(1)-1=0$. 1M

So, $x-1$ is a factor of $3x^3-6x^2+4x-1$. 1

Put $x=-1$, $-6(-1)^3+4(-1)-2=0$

So, $x+1$ is a factor of $-6x^3+4x-2$. 1

(b) $\dfrac{3x^3-6x^2+4x-1}{-6x^3+4x-2}=\dfrac{(x-1)(3x^2-3x+1)}{(x+1)(-6x^2+6x-2)}$ 1M

$=\dfrac{(x-1)(3x^2-3x+1)}{-2(x+1)(3x^2-3x+1)}$ 1M

$=-\dfrac{x-1}{2(x+1)}$ 1A

35. (a) (i) $f\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^3+h\left(-\frac{1}{2}\right)^2-k\left(-\frac{1}{2}\right)-2=0$ 1M

$$\frac{h}{4}+\frac{k}{2}=\frac{9}{4}$$

$$h=9-2k$$ 1A

(ii) $f(x)=2x^3+(9-2k)x^2-kx-2$

$=(2x+1)(x^2+(4-k)x-2)$ 1M

Thus, $Q(x)=x^2+(4-k)x-2$ 1A

(b) $x^2+2x-3=(x+3)(x-1)$. The only possible common linear factors are $x+3$ and $x-1$.

$f(1)=2+h-k-2$ 1M

$=h-k$

If $x-1$ is a common factor of $f(x)$ and x^2+2x-3, then $h-k=0$.
Solving, we have $h=k=3$. 1A

$f(-3)=2(-3)^3+h(-3)^2-k(-3)-2$

$=9h+3k-56$

If $x+3$ is a common factor, then $9h+3k-56=0$.
Solving, we have $h=\frac{17}{3}$ and $k=\frac{5}{3}$. 1A
Therefore, there are only one pair of integers h and k such that $f(x)$ and x^2+2x-3 have a common linear factor.
The claim is disagreed. 1A

36. (a) Let $f(x)=3x^3-x^2-12x+4$.

$f(2)=3(2)^3-2^2-12(2)+4$

$=0$ 1M

Thus, $x-2$ is a factor of $3x^3-x^2-12x+4$. 1

(b) $f(x)=(x-2)(3x^2+5x-2)$ 1M

$=(x-2)(3x-1)(x+2)$ 1A

(c) Let the quotient be $q(x)$, where $q(x)$ is a polynomial.

$3x^3-x^2-7x+5=(ax^2+bx+2)q(x)+5x+1$ 1M

$3x^3-x^2-12x+4=(ax^2+bx+2)q(x)$

$(x-2)(3x-1)(x+2)=(ax^2+bx+2)q(x)$

Since the constant term of ax^2+bx+2 is 2,
$ax^2+bx+2=(x-2)(3x-1)=3x^2-7x+2$; or 1M
$ax^2+bx+2=-(x+2)(3x-1)=-3x^2-5x+2$; or
$ax^2+bx+2=-\frac{1}{2}(x+2)(x-2)=-\frac{1}{2}x^2+2$.
Thus, $(a,\ b)=(3,\ -7)$ or $(-3,\ -5)$ or $\left(-\frac{1}{2},\ 0\right)$. 1A+1A

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