REG-EOC-2223-ASM-SET 3-MATH
Suggested solutions

Multiple Choice Questions

1. $\boxed{\text{A}}$

   Required equation is

   $$(x+2)^2 + (y-5)^2 = (2+2)^2 + (1-5)^2$$
   $$x^2 + y^2 + 4x - 10y - 3 = 0$$

2. $\boxed{\text{A}}$

   Required equation is

   $$(x+2)^2 + (y-4)^2 = (3+2)^2 + (5-4)^2$$
   $$x^2 + y^2 + 4x - 8y - 6 = 0$$

3. $\boxed{\text{C}}$

   Radius $= 0 - (-4) = 4$. The equation is

   $$(x+4)^2 + (y-3)^2 = 4^2$$
   $$x^2 + y^2 + 8x - 6y + 9 = 0$$

4. $\boxed{\text{D}}$

   Required equation is

   $$(x+5)^2 + (y-12)^2 = (0+5)^2 + (0-12)^2$$
   $$(x+5)^2 + (y-12)^2 = 169$$

5. $\boxed{\text{C}}$

   Centre $(-10,\ 12)$. The equation is in the form $x^2 + y^2 + 20x - 24y + F = 0$, where F is a constant.
   The coordinates of midpoint of AB are $(-10,\ 0)$.
   Thus, the x-coordinates of A and B are $-10 \pm 16 = 6$ or -26.

   $$(6)^2 + (0)^2 + 20(6) - 24(0) + F = 0$$
   $$F = -156$$

   C: $x^2 + y^2 + 20x - 24y - 156 = 0$

6. $\boxed{\text{C}}$

   The circle passes through points $(5 \pm 12,\ 0)$, i.e., $(-7,\ 0)$ and $(17,\ 0)$.
   Centre $(5,\ -7) \Rightarrow$ The circle is in the form $x^2 + y^2 - 10x + 14y + F = 0$.
   Substitute $(-7,\ 0)$, $F = -119$.

7. $\boxed{\text{C}}$

Radius $= \sqrt{\left(\frac{8}{2}\right)^2 + 3^2} = 5$

Required equation is

$$(x+2)^2 + (y-3)^2 = 5^2$$
$$x^2 + y^2 + 4x - 6y - 12 = 0$$

8. $\boxed{\text{C}}$

Centre is the midpoint of AB.

Coordinates of centre are $\left(-\frac{3}{2},\ 2\right)$.

Required equation is

$$\left(x + \frac{3}{2}\right)^2 + (y-2)^2 = \left(0 + \frac{3}{2}\right)^2 + (1-2)^2$$
$$x^2 + y^2 + 3x - 4y + 3 = 0$$

9. $\boxed{\text{A}}$

Centre is the midpoint of AB.

The coordinates of the centre are $(-1,\ -1)$.

Required equation is

$$(x+1)^2 + (y+1)^2 = (2+1)^2 + (3+1)^2$$
$$x^2 + y^2 + 2x + 2y - 23 = 0$$

10. $\boxed{\text{D}}$

Distance between the two points $= \sqrt{(0+8)^2 + (0+6)^2} = 10 = 2(5)$

The line segment joining the two points is therefore a diameter of the required circle.

The coordinates of the centre are $(-4,\ -3)$.

Required equation is

$$(x+4)^2 + (y+3)^2 = 5^2$$
$$x^2 + y^2 + 8x + 6y = 0$$

11. $\boxed{\text{C}}$

The centre is the midpoint of AC, the coordinates are $(7,\ 5)$.

Required equation is $x^2 + y^2 - 14x - 10y + F = 0$, where F is a constant.

$$8^2 + 8^2 - 14(8) - 10(8) + F = 0$$
$$F = 64$$

12. $\boxed{\text{A}}$

Since $\angle AOB = 90°$, where O is the origin.
Centre of the circle is the midpoint of AB.
The coordinates of the centre are $\left(\frac{-3}{2},\ 2\right)$.
Required equation is

$$\left(x + \frac{3}{2}\right)^2 + (y - 2)^2 = \left(0 + \frac{3}{2}\right)^2 + (0 - 2)^2$$
$$x^2 + y^2 + 3x - 4y = 0$$

13. $\boxed{\text{C}}$

Centre is the midpoint of $(4,\ 0)$ and $(0,\ -6)$.
Coordinates of centre are $(2,\ -3)$.
Required equation is

$$(x - 2)^2 + (y + 3)^2 = (0 - 2)^2 + (0 + 3)^2$$
$$x^2 + y^2 - 4x + 6y = 0$$

14. $\boxed{\text{B}}$

Line segment joining $(0,\ 8)$ and $(-6,\ 0)$ is a diameter.
Coordinates of centre are $(-3,\ 4)$.
The equation is $x^2 + y^2 + 6x - 8y = 0$ (passes through origin).

15. $\boxed{\text{B}}$

x-coordinate of centre $= \dfrac{1 + 5}{2} = 3$
y-coordinate of centre $= \dfrac{1 + (-3)}{2} = -1$
Radius $= 5 - 3 = 2$
Required equation is $(x - 3)^2 + (y + 1)^2 = 4$.

16. $\boxed{\text{B}}$

Let the coordinates of C be $(h,\ 0)$, where $h < 0$.
Since $y = 3$ is a tangent to the circle, radius of the circle is 3.
Since $x = 1$ is a tangent to the circle, we have

$$1 - h = 3$$
$$h = -2$$

Required equation is $(x + 2)^2 + y^2 = 9$.

17. $\boxed{\text{B}}$

Radius = 3

Required equation is

$$(x+5)^2 + (y+3)^2 = 3^2$$
$$x^2 + y^2 + 10x + 6y + 25 = 0$$

18. $\boxed{\text{C}}$

y-coordinate of centre = 2

x-coordinate of centre = $\dfrac{1+4}{2} = \dfrac{5}{2}$

Radius = $\dfrac{5}{2}$

Required equation is

$$\left(x - \frac{5}{2}\right)^2 + (y-2)^2 = \left(\frac{5}{2}\right)^2$$
$$x^2 + y^2 - 5x - 4y + 4 = 0$$

19. $\boxed{\text{A}}$

y-coordinate of centre = $\dfrac{(-1)+(-9)}{2} = -5$

Radius of circle = $0 - (-5) = 5$

Let the coordinates of centre be $(h,\ -5)$.

$$\sqrt{(h-0)^2 + (-5+1)^2} = 5$$
$$h = -3 \quad \text{or} \quad \text{(rejected)}$$

20. $\boxed{\text{A}}$

x-coordinate of centre = $\dfrac{2+8}{2} = 5$

Radius = 5 and y-coordinate of centre = 4

Required equation is

$$(x-5)^2 + (y-4)^2 = 16$$
$$x^2 + y^2 - 10x - 8y + 16 = 0$$

21. $\boxed{\text{D}}$

Let the radius be r.
The coordinates of the centre are $(r,\ -r)$.
Centre lies on L.

$$\begin{aligned}(r) + 2(-r) + 4 &= 0 \\ r &= 4\end{aligned}$$

Required equation is

$$\begin{aligned}(x-4)^2 + (y+4)^2 &= 4^2 \\ x^2 + y^2 - 8x + 8y + 16 &= 0\end{aligned}$$

22. $\boxed{\text{B}}$

Let the coordinates of centre be $(h,\ 0)$.
The equation of C is $(x-h)^2 + y^2 = (-h)^2$.

$$\begin{aligned}(-2-h)^2 + (-4)^2 &= h^2 \\ h^2 + 4h + 20 &= h^2 \\ h &= -5\end{aligned}$$

Required equation is

$$\begin{aligned}(x+5)^2 + y^2 &= 5^2 \\ x^2 + y^2 + 10x &= 0\end{aligned}$$

23. $\boxed{\text{A}}$

Let the coordinates of the centre be $(1,\ k)$.
The equation of C is $(x-1)^2 + (y-k)^2 = k^2$.

$$\begin{aligned}(5-1)^2 + (8-k)^2 &= k^2 \\ k^2 - 16k + 80 &= k^2 \\ k &= 5\end{aligned}$$

Required equation is

$$\begin{aligned}(x-1)^2 + (y-5)^2 &= 5^2 \\ x^2 + y^2 - 2x - 10y + 1 &= 0\end{aligned}$$

24. $\boxed{\text{C}}$

Let the radius of the circle be r.
The coordinates of the centre are $(-r,\ r)$.
Equation of C is $(x+r)^2+(y-r)^2=r^2$.

$$\frac{r-12}{-r-0}=\frac{12-0}{0+24}$$
$$2r-24=-r$$
$$r=8$$

Required equation is

$$(x+8)^2+(y-8)^2=8^2$$
$$x^2+y^2+16x-16y+64=0$$

25. $\boxed{\text{B}}$

Let the coordinates of centre be $(h,\ 0)$.

$$\sqrt{(h-0)^2+(5-0)^2}=\sqrt{(h-6)^2+(1-0)^2}$$
$$h^2+25=h^2-12h+37$$
$$h=1$$

Required equation is

$$(x-1)^2+y^2=(0-1)^2+5^2$$
$$x^2+y^2-2x-25=0$$

26. $\boxed{\text{A}}$

A. ✓.

B. ✗. Centre $(1,\ -7)$ does not lie on L.

C. ✗. Centre $\left(\frac{5}{2},\ \frac{7}{2}\right)$ does not lie on L.

D. ✗. $(3,\ 4)$ does not lie on the circle.

27. $\boxed{\text{D}}$

A. ✗. Centre $\left(-\frac{5}{2},\ 5\right)$ does not lie on $x+y-1=0$.

B. ✗. Centre $\left(\frac{19}{2},\ -4\right)$ does not lie on $x+y-1=0$.

C. ✗. Centre $\left(\frac{17}{2},\ -\frac{19}{2}\right)$ does not lie on $x+y-1=0$.

D. ✓. Centre $\left(-\frac{17}{2},\ \frac{19}{2}\right)$ lie on $x+y-1=0$,
and both $(0,\ 0)$ and $(3,\ 4)$ satisfy the equation.

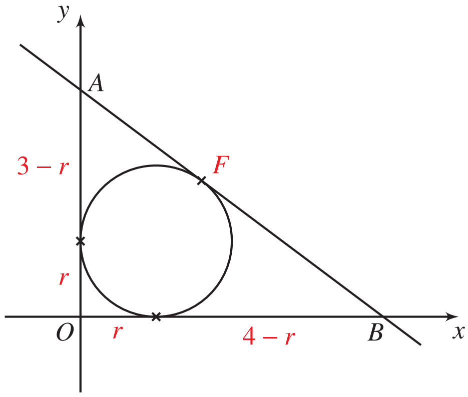
28. $\boxed{\text{C}}$

Let F be the point of contact of AB and the circle. Let r be the radius.

$AF = 3 - r$ and $BF = 4 - r$

$$(3 - r) + (4 - r) = \sqrt{3^2 + 4^2}$$
$$r = 1$$

The equation of circle is $(x - 1)^2 + (y - 1)^2 = 1$.



29. $\boxed{\text{A}}$

Radius of circle is 2. Let the centre of C_3 be $(h,\ k)$. Note that the polygon formed by joining the centres is an equilateral triangle.

By considering the line segment joining centres of C_1 and C_3,

$$h - 2 = (2 + 2)\cos 60° \quad \text{and} \quad k - 2 = (2 + 2)\sin 60°$$
$$h = 4 \qquad\qquad k = 2 + 2\sqrt{3}$$

Only option A has its centre at $\left(4,\ 2 + 2\sqrt{3}\right)$.

30. $\boxed{\text{B}}$

$$y\text{-coordinate of centre} = \frac{2\sqrt{3} + 8\sqrt{3}}{2}$$
$$= 5\sqrt{3}$$

Centre of circle is the centroid/orthocentre/circumcentre/incentre of $\triangle CAB$.

(Note: four centres coincide when it is equilateral triangle.)

A. ✗. Centre $\left(-5,\ 5\sqrt{3}\right)$ should not lie on the line AB.

B. ✓.

C. ✗. y-coordinate of centre $= -5\sqrt{3} \neq 5\sqrt{3}$

D. ✗. y-coordinate of centre $= -5\sqrt{3} \neq 5\sqrt{3}$

Conventional Questions

31. (a) $(x-7)^2+(y+3)^2=(-8-7)^2+(5+3)^2$ 1M

$(x-7)^2+(y+3)^2=289$ 1A

(b) $JK=\sqrt{(7+1)^2+(-3-25)^2}=\sqrt{848}>$ radius of C. 1M

So, J lies outside the circle. 1A

(c) (i) J, K and P are collinear. 1A

(ii) Slope of the line $=\dfrac{25+3}{-1-7}=-\dfrac{7}{2}$.

Required equation is

$$y-25=-\frac{7}{2}(x+1)$$ 1M

$$7x+2y-43=0$$ 1A

32. (a) (i) Consider $\triangle ABC$ and $\triangle ODA$,

$AB = AD$	*(given)*
$\angle ACB = \angle ACD$	*(equal chords, equal $\angle$s)*
$\angle OAD = \angle ACD$	*($\angle$ in alt. segment)*
$= \angle ACB$	
$\angle ADO = \angle ABC$	*(ext. $\angle$, cyclic quad.)*
$\triangle ABC \sim \triangle ODA$	*(AA)*
$\dfrac{AB}{BC} = \dfrac{OD}{AD}$	*(corr. sides, $\sim \triangle$s)*
$AB^2 = BC \cdot OD$	

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(ii) Since $\angle ACD = \angle ACB$,

$$\cos\angle ACD = \cos\angle ACB$$

$$\frac{CD^2 + AC^2 - AD^2}{2(CD)(AC)} = \frac{AC^2 + BC^2 - AB^2}{2(AC)(BC)} \qquad \text{1M}$$

$$2\left(CD^2 + AC^2 - AD^2\right) = AC^2 + BC^2 - AB^2$$

$$2CD^2 + 2AC^2 - 2AD^2 = AC^2 + (2CD)^2 - AD^2 \qquad \text{1M}$$

$$AC^2 = 2CD^2 + AD^2 \qquad \text{1}$$

(b) (i) Since $\angle BCD = 90°$, BD is a diameter.
Centre $= \left(\dfrac{15+27}{2}, \ \dfrac{0+24}{2}\right) = (21, \ 12)$.
The equation of the circle is

$$(x-21)^2 + (y-12)^2 = (15-21)^2 + (0-12)^2 \qquad \text{1M}$$

$$(x-21)^2 + (y-12)^2 = 180 \qquad \text{1A}$$

(ii) $AB^2 = BC \cdot OD$

$AB = \sqrt{24 \cdot 15}$

$= 6\sqrt{10}$ 1A

$AC^2 = 2CD^2 + AD^2$

$AC^2 = 2(12)^2 + (6\sqrt{10})^2$ 1M

$AC = 18\sqrt{2}$ 1A

 END OF PAPER