Solution	Marks
REG-2223-MOCK-SET 6-MATH-CP 1	
Suggested solutions	
1. $\dfrac{(x^{-2}y)^{-3}}{x^{-2}y} = \dfrac{x^6y^{-3}}{x^{-2}y}$	1M
$= \dfrac{x^{6+2}}{y^{1+3}}$	1M
$= \dfrac{x^8}{y^4}$	1A
2. (a) $2m^2 - 3mn + n^2 = (2m-n)(m-n)$	1A
(b) $2m^2 - 3mn + n^2 - mx + nx = (2m-n)(m-n) - x(m-n)$	1M
$= (2m-n-x)(m-n)$	1A
3. $r + r(1-p) = 4p$	
$r + r - pr = 4p$	1M
$p(-r-4) = -2r$	1M
$p = \dfrac{2r}{r+4}$	1A
4. (a) Selling price $= 1400(1 + 35\%)$	1M
$= \$1890$	1A
(b) Percentage discount $= \dfrac{2100 - 1890}{2100} \times 100\%$	1M
$= 10\%$	1A
5. (a) Mean $= \dfrac{2(6) + 3(16) + 4(13) + 5(15)}{50}$	1M
$= 3.74$	1A
(b) Required probability $= \dfrac{6+16}{50}$	1M
$= \dfrac{11}{25}$	1A
6. (a) $3x - 9 > 7 - 2x$	
$x > 4$	1A
$\dfrac{15-4x}{3} + 2 < x$	
$x > 3$	1A
Thus, $x > 4$.	1M
(b) 4	1A

Solution	Marks
7. (a) The coordinates of P' are $(-4,\ -3)$.	1A
The coordinates of Q' are $(-6,\ -2)$.	1A
(b) Slope of $P'Q = \dfrac{-6+3}{2+4} = -\dfrac{1}{2}$	1M
Slope of $QQ' = \dfrac{-2+6}{-6-2} = -\dfrac{1}{2}$ = slope of $P'Q$	
Thus, P' lies on the line QQ'.	1A
8. $\angle BAD = 180° - 108°$	1M
$= 72°$	
$\angle DCE + 80° + 72° = 180°$	1M
$\angle DCE = 28°$	1A
$\angle ADE = \angle DAE = \angle DCE = 28°$	1M
$\angle ABE = \angle ADE = 28°$	1M
$\angle EBC + 28° + 108° = 180°$	
$\angle EBC = 44°$	1A

9. (a) $\angle BCD = \angle FCE$ *(vert. opp. ∠s)*

$\angle CBD = \angle CFE$ *(alt. ∠s, BD//EG)*

$\triangle BCD \sim \triangle FCE$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\angle BCE = \angle DCF$ *(vert. opp. ∠s)*

$BC = DC$ *(property of square)*

$\triangle BCD \sim \triangle FCE$ *(proved)*

$\dfrac{BC}{FC} = \dfrac{CD}{CE}$ *(corr. sides, ~ △s)*

$CE = CF$

$\triangle BCE \cong \triangle DCF$ *(SAS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

Solution	Marks
10. (a) Let $C = a + b\sqrt{n}$, where a and b are non-zero constants.	1A
$\begin{cases} 212\,000 = a + b\sqrt{10\,000} \\ 224\,000 = a + b\sqrt{40\,000} \end{cases}$	1M
Solving, we have $a = 200\,000$ and $b = 120$.	1A
Required cost $= 200\,000 + 120\sqrt{62\,500}$	
$= \$230\,000$	1A
(b) New cost $= 200\,000 + 120\sqrt{250\,000}$	1M
$= \$260\,000$	
Increase in cost $= 260\,000 - 230\,000$	
$= \$30\,000$	
$< \$50\,000$	
The claim is agreed.	1A
11. (a) Let $f(x) = (x^2 - 4)(Ax + B) + 4x + k$, where A and B are constants.	1A
$f(2) = 0 + 4(2) + k = 0$	1M
$k = -8$	1A
(b) We have	
$\begin{cases} f(-4) = (16 - 4)(-4A + B) - 16 - 8 = 0 \\ f(0) = (-4)(B) - 8 = 40 \end{cases}$	1M
Solving, we have $A = -\dfrac{7}{2}$ and $B = -12$.	1A
$f(x) = (x^2 - 4)\left(-\dfrac{7}{2}x - 12\right) + 4x - 8 = 0$	
$-\dfrac{1}{2}(x+2)(x-2)(7x+24) + 4(x-2) = 0$	
$\dfrac{1}{2}(x-2)[-(x+2)(7x+24) + 8] = 0$	1M
$\dfrac{1}{2}(x-2)(-7x^2 - 38x - 40) = 0$	
$-\dfrac{1}{2}(x-2)(7x+10)(x+4) = 0$	
$x = 2 \quad \text{or} \quad -4 \quad \text{or} \quad -\dfrac{10}{7}$	
The claim is disagreed.	1A

Solution	Marks
12. (a) $36 = \dfrac{21+23+24+\ldots+52}{14}$	1M
$a+b=7$	
Since $0 \le a \le b \le 5$, $(a,\ b)=(2,\ 5)$ or $(3,\ 4)$.	1A+1A
(b) (i) Median is the least when the left scouts are of weight greater than 35 kg.	
When $(a,\ b)=(2,\ 5)$, new median = 35 kg.	1M
When $(a,\ b)=(3,\ 4)$, new median = 34.5 kg.	
Required median = 34.5 kg.	1A
(ii) Mean is the greatest when the left scouts are of weight 21 kg and 23 kg.	1M
Required mean $= \dfrac{36\times 14-21-23}{12}$	
$= \dfrac{115}{3}$ kg	1A
13. (a) Volume $= 4^2(6) - \dfrac{1}{3}\left(\dfrac{(4)(4)}{2}\right)\left(\dfrac{6}{2}\right)$	1M+1A
$= 88\text{ cm}^3$	1A
(b) Total surface area of $ABCDEFGH$	
$= 2[4^2+(4)(6)(2)]$	
$= 128\text{ cm}^2$	1A
Difference in volume $= 88 \times \left[\left(\dfrac{512}{128}\right)^{\frac{3}{2}} - 1\right]$	1M+1A
$= 616\text{ cm}^3 > 600\text{ cm}^3$	1A
The difference in volume is not less than 600 cm^3.	
14. (a) (i) $\sqrt{(x-7)^2+(y+3)^2} = \sqrt{(x-1)^2+(y+1)^2}$	1M+1A
$-12x+4y+56=0$	
$3x-y-14=0$	1A
The equation of Γ is $3x-y-14=0$.	
(ii) Γ is the perpendicular bisector of AB.	1A
(b) (i) 3	1A
(ii) When $\triangle AQB$ is an acute-angled triangle,	
$\angle AQB = \dfrac{1}{2}\angle AMB = 30°$.	1M+1A
When $\triangle AQB$ is an obtuse-angled triangle,	
$\angle AQB = 180° - 30° = 150°$.	1A

Solution	Marks
15. We have	
$\begin{cases} 0 = m^2 - n \\ -12 = m - n \end{cases}$	1M
$0 + 12 = m^2 - m$	1M
$0 = m^2 - m - 12$	
$m = 4 \quad \text{or} \quad -3 \text{ (rejected)}$	
When $m = 4$, $n = m + 12 = 16$.	1A
$4^x - 16 > 2021$	
$4^x > 2037$	
$x \log 4 > \log 2037$	1M
$x > 5.50$	1A
16. (a) Required probability $= \dfrac{C_3^8 C_1^4}{C_4^{12}}$	1M
$= \dfrac{224}{495}$	1A
(b) Required probability $= \dfrac{C_4^8}{C_4^{12}} + \dfrac{224}{495}$	1M
$= \dfrac{98}{165}$	1A
17. (a) We have $\alpha_n + \beta_n = \sqrt{\log_3 2^n}$ and $\alpha_n \beta_n = \log_9 \sqrt{2^n}$.	1A
$\alpha_n^2 + \beta_n^2 = (\alpha_n + \beta_n)^2 - 2\alpha_n\beta_n$	1M
$= \log_3 2^n - 2 \log_9 \sqrt{2^n}$	
$= \log_3 2^n - \dfrac{2 \times \frac{n}{2} \log_3 2^n}{\log_3 9}$	1M
$= n \log_3 2 - \dfrac{n}{2} \log_3 2$	
$= \dfrac{n \log_3 2}{2}$	1A
(b) $\alpha_1^2 + \beta_1^2 + \alpha_2^2 + \beta_2^2 + \ldots + \alpha_k^2 + \beta_k^2 = \log_3 32^k$	
$\dfrac{\log_3 2}{2} + \dfrac{2 \log_3 2}{2} + \ldots + \dfrac{k \log_3 2}{2} = 5k \log_3 2$	1M
$\dfrac{k(k+1)}{2} \times \dfrac{\log_3 2}{2} = 5k \log_3 2$	1M
$\dfrac{k(k+1)}{4} = 5k$	
$\dfrac{k^2}{4} - \dfrac{19k}{4} = 0$	
$k = 19 \quad \text{or} \quad 0 \text{ (rejected)}$	1A

Solution	Marks
18. (a) $\dfrac{\sin \angle ABD}{19} = \dfrac{\sin 80^\circ}{30}$	1M
$\angle ABD \approx 38.6^\circ$ or 141° (rejected)	1A
(b) (i) $25^2 = 19^2 + 30^2 - 2(19)(30)\cos \angle CDA$	1M
$\angle CDA \approx 56.1^\circ$	1A
(ii) Let E be a point on BD such that $AE \perp BD$.	
Let F be a point on CD such that $EF \perp BD$.	
Required angle is $\angle AEF$.	1A
$AE = 19 \sin 80^\circ \approx 18.7\text{ cm}$	1M
$DE = 19 \cos 80^\circ \approx 3.30\text{ cm}$	
$\angle EDF = \angle ABD \approx 38.6^\circ$	
$EF = DE \tan EDF \approx 2.63\text{ cm}$	
$DF = \dfrac{DE}{\cos \angle EDF} \approx 4.22\text{ cm}$	
$AF^2 = DF^2 + 19^2 - 2(DF)(19)\cos \angle CDA$	1M
$AF \approx 17.0\text{ cm}$	
In $\triangle AEF$,	
$AF^2 = AE^2 + EF^2 - 2(AE)(EF)\cos \angle AEF$	
$\angle AEF \approx 46.6^\circ$	1A
Required angle is 46.6°.	

Solution	Marks
19. (a) $f(3)=\dfrac{1}{k+2}[3^2+(2k-2)3-5k-1]$	
$=\dfrac{1}{k+2}(k+2)$	
$=1$	
The graph of $y=f(x)$ passes through A.	1
(b) (i) $g(x)=f(-x)-2$	1M
$=\dfrac{1}{k+2}[x^2-(2k-2)x-7k-5]$	
$=\dfrac{1}{k+2}[(x-2(k-1)x+(k-1)^2)-k^2-5k-6]$	1M
$=\dfrac{1}{k+2}[(x-(k-1)^2)-k^2-5k-6]$	
$=\dfrac{1}{k+2}[x-(k-1)]^2-\dfrac{(k+2)(k+3)}{k+2}$	
$=\dfrac{1}{k+2}[x-(k-1)]^2-k-3$	
The coordinates of M are $(k-1,\ -k-3)$.	1A
(ii) AN is a diameter of the circumcircle of $\triangle ANM$.	
So, $\angle AMN=90°$.	1M
$\dfrac{(-k-3)+9}{(k-1)-1}\times\dfrac{1-(-k-3)}{3-(k-1)}=-1$	1M
$-k^2+2k+24=k^2-6k+8$	
$-2k^2+8k+16=0$	
$k=2+2\sqrt{3}$ or $2-2\sqrt{3}$ (rejected)	1A
(iii) The coordinates of P are $(-3,\ -1)$.	1A
The coordinates of Q are $\left(1+2\sqrt{3},\ -5-2\sqrt{3}\right)$.	
Circumcentre S lies on AN. So, S is the midpoint of AN.	
The coordinates of S are $(2,\ -4)$.	1A
Slope of $PS=\dfrac{-1+4}{-3-2}=\dfrac{-3}{5}$	1M
Slope of $PQ=\dfrac{-5-2\sqrt{3}+1}{1+2\sqrt{3}+3}=-1\neq\dfrac{-3}{5}$	
P, Q, S are not collinear.	
The claim is disagreed.	1A

END OF PAPER