Solution	Marks
REG-2223-MOCK-SET 5-MATH-CP 1	
Suggested solutions	
1. $\dfrac{m^8n^{-7}}{(m^3n^{-3})^2} = \dfrac{m^8n^{-7}}{m^6n^{-6}}$	1M
$= \dfrac{m^{8-6}}{n^{-6+7}}$	1M
$= \dfrac{m^2}{n}$	1A
2. $\dfrac{3m+2}{n} - p = m$	
$3m + 2 - np = mn$	1M
$m(3-n) = np - 2$	1M
$m = \dfrac{np-2}{3-n}$	1A
3. (a) $49m^2 - 25n^2 = (7m+5n)(7m-5n)$	1A
(b) $49m^2 - 25n^2 - 7m - 5n = (7m+5n)(7m-5n) - (7m+5n)$	1M
$= (7m+5n)(7m-5n-1)$	1A
4. Let the number of students were given two tickets and three tickets be x and y respectively.	
We have $x = 4y$ and $2x + 3y = 220$.	1A+1A
$2(4y) + 3y = 220$	1M
$y = 20$	
Number of students $= 80 + 20 = 100$	1A
5. (a) Percentage profit $= \dfrac{28}{140-28} \times 100\%$	1M
$= 25\%$	1A
(b) Marked price $= \dfrac{140}{1-20\%}$	1M
$= \$175$	1A
6. (a) $\angle POR = 310° - 130° = 180°$	1M
Thus, P, O and R are collinear.	1A
(b) Area $= \dfrac{(3+8)(4)\sin(310° - 280°)}{2}$	1M
$= 11$	1A

Solution	Marks
7. (a) Least possible weight $= 700 - \frac{10}{2} = 695$ g	1M+1A
(b) Least total weight $= 695 \times 40$	1M
$= 27.8$ kg	
> 27.75 kg	1M
The claim is disagreed.	1A
8. $\angle DAC = \angle CBD = 25°$	1A
$\angle ADC = 90°$	1A
$\angle ACD = 180° - 90° - 25° = 65°$	1A
$\angle CDE = 128° - 65° = 63°$	1A
9. (a) Let $f(x) = ax + b\sqrt{x}$, where a and b are non-zero constants.	1A
$\begin{cases} 46 = 4a + 2b \\ 188 = 16a + 4b \end{cases}$	1M
Solving, we have $a = 12$ and $b = -1$.	1A
Thus, $f(x) = 12x - \sqrt{x}$.	
(b) Required change $= [12(9) - \sqrt{9}] - [12(16) - \sqrt{16}]$	1M
$= -83$	1A
10. (a) $28 = 159 - (130 + a)$	
$a = 1$	1A
$145 = \frac{131 + 132 + \ldots + 159}{16}$	1M
$b = 1$	1A
Interquartile range $= 151 - 141 = 10$ cm	1A
(b) (i) For group B, interquartile range $= 168 - 157 = 11$ cm > 10 cm.	
The distribution of heights of students in group B is more dispersed than that in group A.	1A
(ii) Median of the distribution in group B (162 cm) is higher than the maximum of the distribution in group A (159 cm).	1M
The claim is agreed.	1A

Solution	Marks
11. (a) $f(x) = (2x^2 - 1)(x + a) + bx - 9$	1M
$\begin{cases} f(1) = 0 = (2-1)(1+a) + b - 9 \\ f(2) = 1 = (8-1)(2+a) + 2b - 9 \end{cases}$	1M
Solving, we have $a = -4$ and $b = 12$.	1A+1A
(b) $f(x) = (2x^2 - 1)(x - 4) + 12x - 9 = 0$	
$2x^3 - 8x^2 + 11x - 5 = 0$	
$(x - 1)(2x^2 - 6x + 5) = 0$	1M
$x = 1$ or $2x^2 - 6x + 5 = 0$	
For $2x^2 - 6x + 5 = 0$, $\Delta = 6^2 - 4(2)(5) = -4 < 0$. The equation has no real roots.	1M
The claim is disagreed.	1A
12. (a) $\angle ACE = \angle BCD$ *(common ∠)*	
$\angle CAE = \angle CBD$ *(given)*	
$\triangle ACE \sim \triangle BCD$ *(AA)*	
Marking Scheme **Case 1** Any correct proof with correct reasons. — 2 **Case 2** Any correct proof without reasons. — 1	
(b) (i) $\dfrac{BE + 10}{26} = \dfrac{15}{10}$	1M
$BE = 29\text{ cm}$	1A
(ii) $BD = \sqrt{39^2 - 15^2} = 36\text{ cm}$	1M
Required area $= \dfrac{(36)(26 - 15)}{2}$	
$= 198\text{ cm}^2$	1A
(iii) $AE = \sqrt{26^2 - 10^2} = 24\text{ cm}$	
$AB = \sqrt{24^2 + 29^2} = \sqrt{1417}\text{ cm}$	
Let the shortest distance between D and P be h cm.	
$\dfrac{(\sqrt{1417})(h)}{2} = 198$	1M
$h \approx 10.5 > 10$	
There is no such point P.	1A

Solution	Marks
13. (a) $(0,\ 7)$	1A
(b) (i) Let $P(x,\ y)$.	
$\sqrt{x^2+(y-7)^2}=\sqrt{(x-8)^2+(y-1)^2}$	1M
$x^2+y^2-14y+49=x^2+y^2-16x-2y+65$	
$4x-3y-4=0$	1A
The equation of locus of P is $4x-3y-4=0$.	
(ii) Let the midpoint of AE be F. Then $F(4,\ 4)$.	
$AF=\sqrt{4^2+3^2}=5$ and radius of $C=\sqrt{7^2-40}=3<5$.	1M
Thus, F lies outside the circle and C does not have any intersection with Γ.	1A
(c) Required ratio $=AH:AE$	
$=3:2\times 5$	
$=3:10$	1A
14. (a) Volume of prism $=(798)(20)=15\,960\text{ cm}^3$	1A
Volume ratio of two pyramids $=4^{\frac{3}{2}}:25^{\frac{3}{2}}=8:125$	1M
Volume of smaller pyramid $=15\,960\times\dfrac{8}{8+125}$	
$=960\text{ cm}^3$	1A
(b) Let the height of the smaller pyramid be h cm.	
$\dfrac{1}{3}(24)^2(h)=960$	1M
$h=5$	
Total surface area of smaller pyramid $=24^2+4\times\dfrac{24\left[\sqrt{5^2+\left(\frac{24}{2}\right)^2}\right]}{2}$	1M
$=1200\text{ cm}^2$	
Required area $=1200\times\dfrac{25}{4}$	1M
$=7500\text{ cm}^2$	1A
15. (a) Let the mean be μ.	
$\dfrac{71-\mu}{6}=1.5$	1M
$\mu=62$	1A
(b) Score of David $=62-2.5(6)=47$	1M
Range of scores $\geq 71-47=24>23$	
The claim is disagreed.	1A

Solution	Marks
16. (a) Required probability $= \dfrac{C_5^{10}}{C_5^{16}}$	1M
$= \dfrac{3}{52}$	1A
(b) Required probability $= 1 - \dfrac{C_5^{10} + C_4^{10}C_1^6}{C_5^{16}}$	1M
$= \dfrac{17}{26}$	1A
17. Slope $= \dfrac{-1}{3}$	
$\log_2 y = -\dfrac{1}{3}\log_2 x + 1$	1M
$y = 2^{-\frac{1}{3}\log_2 x + 1}$	
$y = 2x^{-\frac{1}{3}}$	
Thus, $a = 2$ and $b = -\dfrac{1}{3}$.	1A+1A
18. (a) Let the time required be n months.	
$250\,000[1 + (95\%) + (95\%)^2 + \ldots + (95\%)^{n-1}] > 2\,000\,000$	
$\dfrac{1 - 0.95^n}{1 - 0.95} > 8$	1M
$0.95^n < 0.6$	
$n \log 0.95 < \log 0.6$	1M
$n > 9.96$	
At least 10 months are required.	1A
(b) Number of barrels that the oil well can produce	
$< \dfrac{250\,000}{1 - 0.95}$	1M
$= 5\,000\,000$	
The claim is agreed.	1A

Solution	Marks
19. (a) $x^2 - 4x + 2 = kx - 3k$	1M
$x^2 + (-4 - k)x + (3k + 2) = 0$	
$\Delta = (-4 - k)^2 - 4(1)(3k + 2)$	1M
$= k^2 - 4k + 8$	
$= (k - 2)^2 + 4$	
> 0	
Thus, L and P intersect at two distinct points.	1
(b) (i) α and β are distinct roots of the equation $x^2 + (-4 - k)x + (3k + 2) = 0$.	
$(\alpha - 4)(\beta - 4) = \alpha\beta - 4(\alpha + \beta) + 16$	
$= (3k + 2) - 4(4 + k) + 16$	1M
$= -k + 2$	1A
(ii) $(\alpha - 4)(\beta - 4) < 0$	1M
$-k + 2 < 0$	
$k > 2$	
y-coordinate of midpoint of $AB = \dfrac{[k\alpha - 3k] + [k\beta - 3k]}{2}$	1M
$= \dfrac{k(4 + k) - 6k}{2}$	
$= \dfrac{k(k - 2)}{2}$	
> 0 (for $k > 2$)	
Thus, it is not possible.	1A

Solution	Marks
20. (a) $AD = \dfrac{\sqrt{30^2 + 40^2}}{2} = 25\text{ cm}$	
$\angle BAC = \cos^{-1}\dfrac{30}{50} = \cos^{-1}\dfrac{3}{5}$	
$BD^2 = 25^2 + 30^2 - 2(25)(30)\cos\angle BAC$	1M
$BD = 25\text{ cm}$	1A
$\angle ABD = \angle BAD = \cos^{-1}\dfrac{3}{5}$	
$AF = 30\sin\angle ABD = 24\text{ cm}$	1A
$\cos\angle BAF = \dfrac{24}{30} = \dfrac{4}{5}$	
$FE = \dfrac{30}{\cos\angle BAF} - 24 = 13.5\text{ cm}$	1A
(b) (i) Required angle is $\angle AFE$.	
$\cos\angle AFE = \dfrac{13.5}{24}$	1M
$\angle AFE \approx 55.8^\circ$	1A
(ii) Since $AF \perp BD$ and $FE \perp BD$,	
the claim is agreed.	1A
(iii) Area of $\triangle BCD = \dfrac{\text{area of } \triangle ABC}{2}$	
$= \dfrac{(30)(40)}{2 \times 2}$	
$= 300\text{ cm}^2$	
Required volume $= \dfrac{1}{3}(300)(24\sin\angle AFE)$	1M
$\approx 1984\text{ cm}^3$	1A
(iv) Area of $\triangle ABD$ is equal to the area of $\triangle BCD$, i.e. 300 cm^2.	
By considering the volume of the tetrahedron,	
$\dfrac{1}{3}(300)(AB\sin\alpha) = \dfrac{1}{3}(300)(BC\sin\beta)$	1M
$\dfrac{\sin\alpha}{\sin\beta} = \dfrac{40}{30} > 1$	
So, $\sin\alpha > \sin\beta$ and therefore $\alpha > \beta$.	1A

END OF PAPER