Solution	Marks
REG-2223-MOCK-SET 3-MATH-CP 1	
Suggested solutions	
1. $\dfrac{(m^{-2}n^5)^3}{m^6n^{-7}} = \dfrac{m^{-6}n^{15}}{m^6n^{-7}}$	1M
$= \dfrac{n^{15+7}}{m^{6+6}}$	1M
$= \dfrac{n^{22}}{m^{12}}$	1A
2. (a) $2x^2 - 3x - 2 = (2x+1)(x-2)$	1A
(b) $6x^2y + 3xy - 2x^2 + 3x + 2 = 3xy(2x+1) - (2x+1)(x-2)$	1M
$= (2x+1)(3xy - x + 2)$	1A
3. (a) 11	1A
(b) 10.16	1A
(c) 10.2	1A
4. (a) $\dfrac{3x-7}{4} < 2x+5$	
$-\dfrac{5x}{4} < \dfrac{27}{4}$	
$x > -\dfrac{27}{5}$	1A
$x + 2 \geq 0$	
$x \geq -2$	
Thus, $x > -\dfrac{27}{5}$.	1M
(b) $4x + 13 < 9$	
$x < -1$	1A
Thus, $-\dfrac{27}{5} < x < -1$.	1M
Required sum $= (-5) + (-4) + (-3) + (-2) = -14$	1A
5. (a) $P(-3,\ -4)$ and $Q(3,\ 4)$	1A+1A
(b) $P'(-4,\ 3)$	
Slope of $OP' = \dfrac{3-0}{-4-0} = -\dfrac{3}{4}$ and slope of $OQ' = \dfrac{-4}{3} \neq -\dfrac{3}{4}$.	1M
They are not collinear.	1A
6. $2(-k)^2 + k(-k) - 6 = k$	1M
$k^2 - k - 6 = 0$	
$k = 3$ or -2	1A+1A

Solution	Marks
7. (a) Least possible capacity $= 350 - \frac{1}{2} = 349.5\text{ mL}$	1M+1A
(b) Least possible total capacity $= 349.5 \times 6$	
$= 2097\text{ mL}$	
$> 2050\text{ mL}$	1M
It is not possible.	1A
8. (a) Let the number of packages A and packages B bought by Hailey be a and b respectively.	
$\begin{cases} 12a + 20b = 444 \\ a = b(1 - 20\%) \end{cases}$	1A 1A
$12(0.8b) + 20b = 444$	1M
$b = 15$	
Required number $= 15 + 0.8(15) = 27$	1A
(b) Required ratio $= 12(12) : 20(15)$	
$= 12 : 25$	1A
9. $\angle ADB = \angle BDC = 32°$	1A
$\angle ABD = 90°$	1A
$\angle BAD = 180° - 90° - 32° = 58°$	1A
$\angle BCD = 180° - 58° = 122°$	1M+1A
10. (a) Let $C = ad + bn$, where a and b are non-zero constants.	1A
$\begin{cases} 58\,500 = 3a + 25b \\ 78\,000 = 5a + 20b \end{cases}$	1M
Solving, we have $a = 12\,000$ and $b = 900$.	1A
Total cost $= 12\,000(4) + 900(33) = \$77\,700$	1A
(b) Original amount paid by students $= \frac{77700}{30} = \$2590$	
New amount $= \frac{12\,000(5) + 900(33 + 2)}{30} = \$3050$	1A
Percentage increase $= \frac{3050 - 2590}{2590} \approx 17.8\% > 15\%$	1A
The amount paid by each student is increased by more than 15%.	

Solution	Marks
11. (a) Range $= 13.1 - 1.8$	1M
$= 11.3 \text{ g}/100\text{mL}$	1A
Interquartile range $= 9.2 - 5.4$	
$= 3.8 \text{ g}/100\text{mL}$	1A
(b) New mean $= \dfrac{7.2 \times 20 + 2.4 + 4.6 + 7.5 + 10.4 + 13.4}{20 + 5}$	1M
$= 7.292 \text{ g}/100\text{mL}$	1A
New median is the 13th datum in ascending order.	
New median is 7.5 g/100mL.	1M+1A
12. (a) (i) Let $P(x,\ y)$.	
$\dfrac{y+1}{x-5} \times \dfrac{y-5}{x+3} = -1$	1M
$(x+3)(x-5) + (y+1)(y-5) = 0$	
$x^2 + y^2 - 2x - 4y - 20 = 0$	1A
The equation of locus of P is $x^2 + y^2 - 2x - 4y - 20 = 0$.	
(ii) Locus of P is a circle centred at $(1,\ 2)$ with radius 5, excluding points A and B.	1A+1A
Remarks: The fact that the locus excludes points A and B can be omitted.	
(b) Centre of C is at $(9,\ 8)$.	
Distance between centres $= \sqrt{(9-1)^2 + (8-2)^2} = 10$.	1M
Sum of radii $= \sqrt{25} + 5 = 10 =$ distance between centres	1M
The circles touch each other externally.	
The claim is disagreed.	1A
13. (a) Volume $= 4^2(6) - \dfrac{1}{3}\left(\dfrac{(4)(4)}{2}\right)\left(\dfrac{6}{2}\right)$	1M+1A
$= 88 \text{ cm}^3$	1A
(b) Total surface area of $ABCDEFGH$	
$= 2[4^2 + (4)(6)(2)]$	
$= 128 \text{ cm}^2$	1A
Difference in volume $= 88 \times \left[\left(\dfrac{512}{128}\right)^{\frac{3}{2}} - 1\right]$	1M+1A
$= 616 \text{ cm}^3 > 600 \text{ cm}^3$	1A
The difference in volume is not less than 600 cm^3.	

Solution	Marks
14. (a) $f(x) = (8x^2 + ax + 8)(3x^2 + 7x + r) + bx + c$	1M
$= 24x^4 + (56 + 3a)x^3 + \ldots$	
$56 + 3a = 47$	1M
$a = -3$	1A
(b) (i) Let $g(x) = A(8x^2 + ax + 8) + bx + c$, where A is a constant.	1M
$f(x) - g(x) = [(8x^2 + ax + 8)(3x^2 + 7x + r) + bx + c] - [A(8x^2 + ax + 8) + bx + c]$	
$= (8x^2 + ax + 8)(3x^2 + 7x + r - A)$	
Thus, $f(x) - g(x)$ is divisible by $8x^2 + ax + 8$.	1
(ii) $f(x) - g(x) = 0$	
$(8x^2 - 3x + 8)(3x^2 + 7x + r - A) = 0$	
$8x^2 - 3x + 8 = 0$ or $3x^2 + 7x + r - A = 0$	1M
For $8x^2 - 3x + 8 = 0$,	
$\Delta = (-3)^2 - 4(8)(8) = -247 < 0$. The equation has no real roots.	1M
For $3x^2 + 7x + r - A = 0$, the equation has at most 2 real roots.	
Thus, $f(x) - g(x) = 0$ has at most 2 real roots.	
The claim is disagreed.	1A
15. (a) Required probability $= \dfrac{C_3^{20}C_2^{15}}{C_5^{35}}$	1M
$= \dfrac{4275}{11\,594}$	1A
(b) Required probability $= \dfrac{C_3^{20}C_2^{10}}{C_3^{20}C_2^{15} + C_4^{20}C_1^{15} + C_5^{20}}$	1M
$= \dfrac{900}{3647}$	1A
16. (a) Let the mean and standard deviation of the distribution be μ and σ respectively.	
$\begin{cases} \dfrac{60 - \mu}{\sigma} = 1.25 \\ \dfrac{44 - \mu}{\sigma} = 0.25 \end{cases}$	1M
Solving, we have $\mu = 40$ and $\sigma = 16$.	1A+1A
(b) New standard score of Carol $= \dfrac{44(1 + 10\%) - 40(1 + 10\%)}{16(1 + 10\%)}$	1M
$= 0.25$	
The claim is not correct.	1A

Solution	Marks
17. (a) Required amount	
$= 3.24 \times 10^{10}[1 + (1 + 5\%) + (1 + 5\%)^2 + \ldots + (1 + 5\%)^{14}]$	1A
$= \dfrac{3.24 \times 10^{10}(1.05^{15} - 1)}{1.05 - 1}$	1M
$\approx \$6.99 \times 10^{11}$	1A
(b) $3.24 \times 10^{10}(1 + 1.05 + 1.05^2 + \ldots + 1.05^{n-1}) > 10^{12}$	
$\dfrac{3.24 \times 10^{10}(1.05^n - 1)}{1.05 - 1} > 10^{12}$	1M
$1.05^n > \dfrac{206}{81}$	
$n \log 1.05 > \log \dfrac{206}{81}$	1M
$n > 19.1$	
The least value of n is 20.	1A
18. (a) D is circumcentre of $\triangle ABC$. So, BC is a diameter.	1
$\angle NAB = \angle ACB$ *(∠ in alt. segment)*	
$\angle CBA = \angle NAB$ *(alt. ∠s, MN//BC)*	
$= \angle ACB$	
$AC = AB$ *(sides opp. equal ∠s)*	
$\angle CAB = 90°$ *(∠ in semicircle)*	
Thus, $\triangle ABC$ is a right-angled isosceles triangle.	

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

Solution	Marks
(b) (i) $C(-2, 4)$	1A
D is the midpoint of BC, at $(1, 3)$.	
The equation of circle is	
$(x - 1)^2 + (y - 3)^2 = (-2 - 1)^2 + (4 - 3)^2$	1M
$(x - 1)^2 + (y - 3)^2 = 10$	1A
(ii) Slope of required tangent = slope of MN = slope of BC = $\dfrac{4 - 2}{-2 - 4} = -\dfrac{1}{3}$.	1M
D is the midpoint of A and E. So, $E(2, 6)$.	
Required equation is	
$y - 6 = -\dfrac{1}{3}(x - 2)$	1M
$y = -\dfrac{x}{3} + \dfrac{20}{3}$	1A

Solution	Marks
19. (a) $\dfrac{145}{\sin 42^\circ} = \dfrac{AC}{\sin(180^\circ - 30^\circ - 42^\circ)}$	1M+1A
$AC \approx 206\,\text{m}$	1A
$AB^2 = 240^2 + AC^2 - 2(240)(AC)\cos 25^\circ$	1M
$AB \approx 102\,\text{m}$	1A
(b) Let T' be the projection of T on the plane ABC. T' lies on AC.	
Angle of elevation of T from $P = \tan^{-1}\dfrac{TT'}{PT'}$.	
The angle of elevation is greater when PT' is shorter.	1M
$240^2 = AC^2 + AB^2 - 2(AC)(AB)\cos\angle CAB$	1M
$\angle CAB \approx 96.4^\circ > 90^\circ$	
Thus, $\triangle T'AB$ is an obtuse-angled triangle.	1M
When P moves from B to A, PT' decreases gradually.	1M
Thus, the angle of elevation is the greatest when P is at A.	
The claim is agreed.	1A

END OF PAPER