REG-2223-MOCK-SET 2-MATH-CP 2
Suggested solutions

Answers:

1. A	2. B	3. A	4. C	5. B	6. B	7. A	8. A	9. D	10. D
11. B	12. C	13. C	14. A	15. B	16. B	17. A	18. B	19. C	20. D
21. D	22. C	23. C	24. D	25. D	26. D	27. C	28. A	29. D	30. B
31. B	32. D	33. A	34. B	35. D	36. C	37. D	38. C	39. A	40. A
41. A	42. C	43. C	44. B	45. D					

Suggested Solutions:

1. $\boxed{\text{A}}$

$$\left(\frac{1}{4^{153}}\right)8^{101} = 2^{-2\times153}\times 2^{101\times3} = 2^{-3} = \frac{1}{8}$$

2. $\boxed{\text{B}}$

Coefficient of $q^3 = (-1)(1) = -1$
Coefficient of $p^2q = (1)(1) + (-1)(1) = 0$
Only option B satisfies these.

3. $\boxed{\text{A}}$

$$\begin{aligned}\frac{c}{a-1} - \frac{ab}{1-a} &= 3 \\ c + ab &= 3a - 3 \\ a(b-3) &= -3 - c \\ a &= \frac{3+c}{3-b}\end{aligned}$$

4. $\boxed{\text{C}}$

Compare coefficients of x and constant term,

$$\begin{aligned}-a + 5a &= 9 - 1 \quad \text{and} \quad 5 - 5a^2 = -9 - b \\ 4a &= 8 \qquad\qquad\qquad b = 6 \\ a &= 2\end{aligned}$$

5. $\boxed{\text{B}}$

$$\begin{aligned}-f(2) + f(-2) &= -(2+4-4) + (2-4-4) \\ &= -8\end{aligned}$$

6. $\boxed{\text{B}}$

$y = (qx - p)(x + 1) + 3 = q\left(x - \frac{p}{q}\right)(x + 1) + 3$

I. ✓. Coefficient of $x^2 = q > 0$ since the graph open upwards.

II. ✓. y-intercept $= -p + 3 < 0 \quad \Rightarrow \quad p > 3$

III. ✗. When $x = -1$, $y = 0 + 3 = 3 \neq 0$.

7. $\boxed{\text{A}}$

The inequalities become $x > 6$ or $x > 2$.
Thus, $x > 2$.

8. $\boxed{\text{A}}$

$$p(-k) = -k^3 + k^3 - 4k - 16 = 0$$
$$k = -4$$

$$\text{Remainder} = (-2)^3 - 4(-2)^2 + 4(-2) - 16$$
$$= -48$$

9. $\boxed{\text{D}}$

$$\text{Amount} = 94\,000\left(1 + \frac{4\%}{12}\right)^{3\times 12}$$
$$\approx \$105\,964$$

10. $\boxed{\text{D}}$

Let $\begin{cases} 3b - 4c = 1 \\ 4b - 3c = 2 \end{cases}$. Then $b = \frac{5}{7}$ and $c = \frac{2}{7}$.

We have $a = c \times \frac{2}{1} = \frac{4}{7}$.

$(a + b) : (b + c) = \frac{9}{7} : \frac{7}{7} = 9 : 7$

11. $\boxed{\text{B}}$

Let $z = \frac{kx^3}{\sqrt{y}}$, where k is a non-zero constant.

Then $k = \frac{\sqrt{y}z}{x^3}$.

So, $\frac{yz^2}{x^6} = k^2$ is a constant.

12. $\boxed{\text{C}}$

$$n < \frac{2.5 \times 1000}{39.5}$$

$n < 63.3$

The greatest possible value of n is 63.

13. $\boxed{\text{C}}$

The numbers are formed by +6, +10, +14, . . .

The sequence is 1, 7, 17, 31, 49, 71, 97.

Required number is 97.

14. $\boxed{\text{A}}$

$$34\pi = \pi(r)\left(\frac{17r}{8}\right)$$

$$r = 4$$

$$\text{Volume} = \frac{1}{3}\pi(4)^2\sqrt{8.5^2 - 4^2}$$

$$= 40\pi \text{ cm}^3$$

15. $\boxed{\text{B}}$

$$\sin \angle ODC = \frac{OC}{OD}$$

$$= \frac{1}{2}$$

$$\angle ODC = 30°$$

$\angle DFE = 75° - 30° = 45°$ and $\angle CFB = \angle DFE = 45°$

$\angle CBF = 180° - 90° - 45° = 45°$ and $\angle OAB = \angle OBA = 45°$

$\angle DOA = 75° - 45° = 30°$

Area of sector $= \pi(12)^2 \times \dfrac{30°}{360°} = 12\pi \text{ cm}^2$

16. $\boxed{\text{B}}$

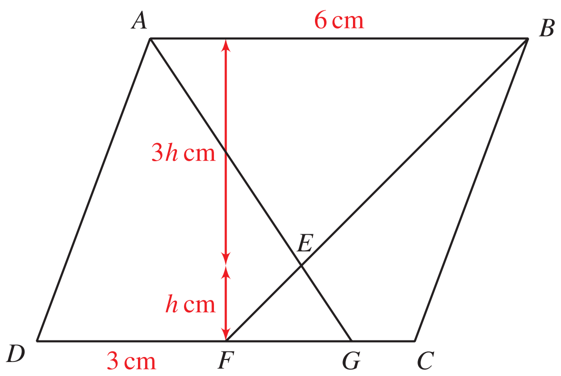
Let $CG = 1$ cm. Then $FG = 2$ cm and $DF = 3$ cm and $AB = 6$ cm.

$\triangle EFG \sim \triangle EBA$ (ratio 1 : 3)

$$1265 = \frac{(3)(4h)}{2} - \frac{(2)(h)}{2}$$

$$h = 253$$

$$\text{Area of } \triangle EBA = \frac{(3 \times 253)(6)}{2} = 2277\text{ cm}^2$$



17. $\boxed{\text{A}}$

Let $\angle EAD = x$.

$\angle ADB = \angle ABD = \frac{x}{2}$ and $\angle CDB = \angle ABD = \frac{x}{2}$.

$\angle DCB = 4\angle BDC = 2x$

$$2x + 2x + \frac{x}{2} = 180°$$

$$x = 40°$$

18. $\boxed{\text{B}}$

$\triangle ADE \sim \triangle ACF$ (AA)

$\frac{AD}{AC} = \frac{AE}{AF} \quad \Rightarrow \quad AD = 10$ cm

$DE = \sqrt{10^2 - 6^2} = 8$ cm and $CF = 8 \times \frac{2}{1} = 16$ cm

$BC = \sqrt{16^2 + 12^2} = 20$ cm

19. $\boxed{\text{C}}$

$BE = \sqrt{20^2 - 16^2} = 12$ cm and $DE = \sqrt{13^2 - 12^2} = 5$ cm

Perimeter $= 5 + 13 + (5 + 16) + 20 = 58$ cm

20. $\boxed{\text{D}}$

$\angle CGD = \frac{90°}{2} = 45°$ and $\angle EDG = \angle CGD = 45°$

$CD = DG = DE$ and $\angle DEC = \angle DCE = \frac{180° - 90° - 45°}{2} = 22.5°$

$\angle ENG = 22.5° + 45° = 67.5°$ and $\angle EMG = \angle ENG = 67.5°$

21. $\boxed{\text{D}}$

Let $AD = 1$.

$\angle AFD = \alpha$, $AF = \dfrac{1}{\sin\alpha}$ and $DF = \dfrac{1}{\tan\alpha}$

$CF = 1 - \dfrac{1}{\tan\alpha}$ and $EF = \dfrac{CF}{\sin(180^\circ - \beta)} = \dfrac{1 - \frac{1}{\tan\alpha}}{\sin\beta}$

$$\begin{aligned}\frac{AF}{EF} &= \frac{1}{\sin\alpha} \div \frac{1 - \frac{1}{\tan\alpha}}{\sin\beta} \\ &= \frac{\sin\beta}{\sin\alpha - \cos\alpha}\end{aligned}$$

22. $\boxed{\text{C}}$

$\angle CAD = \angle BAC \times \dfrac{16}{24} = 42^\circ$

$\angle ACB = 180^\circ - 90^\circ - 63^\circ = 27^\circ$ and $\angle ADE = \angle ACB = 27^\circ$

$\angle AED = 180^\circ - 27^\circ - 42^\circ = 111^\circ$

23. $\boxed{\text{C}}$

There are 6 axes of reflectional symmetry.

24. $\boxed{\text{D}}$

I. ✓. Slope $= -\dfrac{a}{5} < 0 \quad \Rightarrow \quad a > 0$

II. ✓. y-intercept $= \dfrac{b}{5} < -1 \quad \Rightarrow \quad b < -5$

III. ✓. x-intercept $= \dfrac{b}{a} > -2$ and $a > 0 \quad \Rightarrow \quad b > -2a$

25. $\boxed{\text{D}}$

AB is fixed, so the perpendicular distance from P to L is also a constant.

Thus, locus of P is a pair of straight lines, parallel to L and maintain a fixed distance from L.

26. $\boxed{\text{D}}$

The equation is in the form $2x - 3y + C = 0$, where C is a constant.

Put $(5,\ 1)$ into the equation, we have $C = -7$.

27. $\boxed{\text{C}}$

C_2: $x^2 + y^2 + 6x - 8y + \dfrac{33}{2} = 0$
$G_1\ (4,\ 3)$ and $G_2\ (-3,\ 4)$.

I. ✓. Slope of $G_1O \times$ slope of $G_2O = \dfrac{3}{4} \times \dfrac{4}{-3} = -1$

II. ✗. Radius of $C_1 = \sqrt{4^2 + 3^2 - 20} = \sqrt{5}$ and radius of $C_2 = \sqrt{3^2 + 4^2 \dfrac{33}{2}} = \sqrt{8.5} > \sqrt{5}$
So, area of C_1 is smaller than the area of C_2.

III. ✓. $OG_1 = OG_2 = \sqrt{3^2 + 4^2} = 5$

28. $\boxed{\text{A}}$

$$\text{Required probability} = \frac{3+2+2+1}{8 \times 7} = \frac{1}{7}$$

29. $\boxed{\text{D}}$

In the cumulative frequency curve, steeper $\Rightarrow$ more data in the corresponding class.
So, the data is more concentrated in the lower part.
Minimum, lower quartile, median and upper quartile will be closed to each other.

30. $\boxed{\text{B}}$

Mode = 60 $\Rightarrow$ at least three of a, b, c, d are 60
Let $a = b = c = 60$.
$70 = \dfrac{60 + 60 + 60 + d + \ldots + 81}{12}$

$d = 76$
Median = 76

31. $\boxed{\text{B}}$

From $y = f(x)$,
reflect about x-axis $\rightarrow$ $y = -f(x)$
translate leftwards by 2 units $\rightarrow$ $y = -f(x+2)$

32. $\boxed{\text{D}}$

$11010_2 = 26$ and the '1' has a place value 2^9.
So, the answer is D.

33. $\boxed{\text{A}}$

For the point $\left(\frac{1}{3},\ 0\right)$,

$$\log_8 x = \frac{1}{3} \qquad \text{and} \log_4 y = 0$$
$$x = 8^{\frac{1}{3}} = 2 \qquad y = 1$$

Only option A satisfies this.

34. $\boxed{\text{B}}$

Take log on the 'positive' part.

A. $589 \log 111 \approx 1205$

B. $577 \log 123 \approx 1206$

C. $565 \log 135 \approx 1204$

D. $553 \log 147 \approx 1199$

The 'positive' part of the number in B is the greatest. So, it gives the least value when it is negative.

35. $\boxed{\text{D}}$

Take $m = 1$, $i^7 + \dfrac{i^5 - 4}{m - i} = -\dfrac{5}{2} - \dfrac{5}{2}i$.

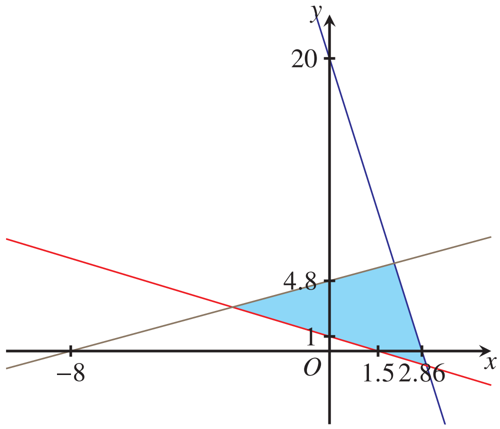
Only option D gives $-\dfrac{5}{2}$ when $m = 1$.

36. $\boxed{\text{C}}$

Line	x-intercept	y-intercept
$7x + y = 20$	$\dfrac{20}{7}$	20
$2x + 3y = 3$	$\dfrac{3}{2}$	1
$5y - 3x = 24$	-8	$\dfrac{24}{5}$

Sketch the graph and shaded region T.

Greatest value of $4y - 3x - 5 \quad \Rightarrow \quad$ top left corner



$(x,\ y)$	$(-3,\ 3)$	$(2,\ 6)$
$4y - 3x - 5$	16	13

Greatest value = 16

37. $\boxed{\text{D}}$

Let the first term and common ratio be a and r respectively.

$$\frac{ar^7}{ar^2} = \frac{6}{192}$$
$$r^5 = \frac{1}{32}$$
$$r = \frac{1}{2}$$

I. ✗.

II. ✓. $a = \dfrac{192}{r^2} = 768.$

$$768\left(\frac{1}{2}\right)^{n-1} > 10^{-2}$$
$$(n-1)\log\frac{1}{2} > \log\frac{1}{76\,800}$$
$$n < 17.2$$

17 terms are greater than 10^{-2}.

III. ✓. $\text{Sum} = \dfrac{768\left(1-\left(\frac{1}{2}\right)^{13}\right)}{1-\frac{1}{2}} \approx 1535.8$

> 1535

38. $\boxed{\text{C}}$

$$4\cos^2\theta - 7\sin\theta - 7 = 0$$
$$4(1-\sin^2\theta) - 7\sin\theta - 7 = 0$$
$$-4\sin^2\theta - 7\sin\theta - 3 = 0$$
$$\sin\theta = -1 \quad \text{or} \quad -\frac{3}{4}$$

$\sin\theta = -1$ has one root and $\sin\theta = -\dfrac{3}{4}$ has two roots.

39. $\boxed{\text{A}}$

$CM = 30 \times \dfrac{7}{3+7} = 21\,\text{cm}$ and $MH = 30 - 21 = 9\,\text{cm}$

$MA = \sqrt{12^2 + 16^2 + 21^2} = 29\,\text{cm}$ and $MG = \sqrt{12^2 + 9^2} = 15\,\text{cm}$

$AG = \sqrt{30^2 + 16^2} = 34\,\text{cm}$

$AG^2 = AM^2 + MG^2 - 2(AM)(MG)\cos\theta$

$\cos\theta = \dfrac{-3}{29}$

40. $\boxed{\text{A}}$

$\angle AGE = 2 \times 66° = 132°$
$\angle CEA = 132° \times \dfrac{6}{5+6} = 72°$
$\angle CAT = \angle CEA = 72°$ and $\angle ATC = 180° - 2 \times 72° = 36°$

41. $\boxed{\text{A}}$

I. ✓. G lies inside $\triangle OAB$, which is in the second quadrant. The x- and y-coordinates are not equal (one positive and one negative).

II. ✓. Let the radius of inscribed circle be r. Then the coordinates of G are $(-r,\ r)$.

$$\begin{aligned} 4r + (-r) &= 3kb \\ r &= kb \end{aligned}$$

Using tangent properties, OB is divided into two segments with lengths $b - r$ and r.
OA is divided into two segments with lengths $10 - r$ and r.

$$\begin{aligned} (10-r) + (b-r) &= \sqrt{10^2 + b^2} \\ [10 + b(1-2k)]^2 &= b^2 + 100 \\ 100 + 20b(1-2k) + b^2(1-2k)^2 &= b^2 + 100 \\ b^2(4k^2 - 4k) + 20b(1-2k) &= 0 \\ b &= -\frac{20(1-2k)}{4k^2 - 4k} \\ &= \frac{5(1-2k)}{k(1-k)} \end{aligned}$$

Required distance $= r = kb = \dfrac{5(1-2k)}{1-k}$

III. ✗. When $k = \dfrac{1}{6}$, $r = \dfrac{5(1-2k)}{1-k} = 4$.
Equation of inscribed circle is $(x+4)^2 + (y-4)^2 = 4^2$.

$$\begin{aligned} (x+4)^2 + (5 - 3x - 4)^2 &= 16 \\ 10x^2 + 2x + 1 &= 0 \end{aligned}$$

$\Delta = 2^2 - 4(10)(1) = -36 < 0$.
The straight line $3x + y = 5$ does not cut the inscribed circle of $\triangle OAB$ and hence is not a tangent.

42. $\boxed{\text{C}}$

Required number $= C_3^{10} 3!9! = 261\,273\,600$

43. $\boxed{\text{C}}$

Required probability $= 1 - \left(1 - \frac{2}{8}\right)\left(1 - \frac{3}{8}\right)\left(1 - \frac{4}{8}\right)$

$= \frac{49}{64}$

44. $\boxed{\text{B}}$

Let the standard deviation of the test scores be σ.

$$\frac{76 - 64}{\sigma} = 1.5$$

$$\sigma = 8$$

Standard score of Anson $= \frac{54 - 64}{8} = -1.25$

45. $\boxed{\text{D}}$

From new to old,

multiply by 4 and then subtract 3 from the resulting number.

Thus, mean of original set $= 4m - 3$ and variance $= 16v$.