

M2REG-DIFF-2223-ASM-SET 2-MATH

Suggested solutions

$$1. \frac{dy}{dx} = \frac{4(1-5x) - (2+4x)(-5)}{(1-5x)^2} \quad 1M$$

$$= \frac{14}{(1-5x)^2} \quad 1A$$

$$2. \frac{dy}{dx} = \frac{4(2x^2+x) - (4x-1)(4x+1)}{(2x^2+x)^2} \quad 1M$$

$$= \frac{-8x^2+4x+1}{(2x^2+x)^2} \quad 1A$$

$$3. \quad y = \frac{x^2+10x+25}{x-11} \quad 1M$$

$$\frac{dy}{dx} = \frac{(2x+10)(x-11) - (x^2+10x+25)(1)}{(x-11)^2} \quad 1M$$

$$= \frac{x^2-22x-135}{(x-11)^2} \quad 1A$$

$$4. \quad y = 4 \left(\frac{2x^2+1}{5-3x^2} \right)^3 \cdot \frac{4x(5-3x^2) - (2x^2+1)(-6x)}{(5-3x^2)^2} \quad 1M$$

$$= 4 \left(\frac{2x^2+1}{5-3x^2} \right)^3 \cdot \frac{26x}{(5-3x^2)^2}$$

$$= \frac{104x(2x^2+1)^3}{(5-3x^2)^5} \quad 1A$$

$$5. \quad \frac{dy}{dx} = \frac{3x^2(5x^6+6)^{\frac{1}{2}} - (x^3+1) \left(\frac{1}{2} \right) (5x^6+6)^{-\frac{1}{2}} (30x^5)}{5x^6+6} \quad 1M$$

$$= \frac{3x^2(5x^6+6) - 15x^5(x^3+1)}{(5x^6+6)^{\frac{3}{2}}}$$

$$= \frac{-15x^5+18x^2}{(5x^6+6)^{\frac{3}{2}}} \quad 1A$$

$$6. \quad \frac{d}{dx}(x^3 - y^3 + x^2y^4) = \frac{d}{dx}(50)$$

$$3x^2 - 3y^2 \frac{dy}{dx} + 2xy^4 + 4x^2y^3 \frac{dy}{dx} = 0 \quad 1M$$

$$(3y^2 - 4x^2y^3) \frac{dy}{dx} = 3x^2 + 2xy^4$$

$$\frac{dy}{dx} = \frac{3x^2 + 2xy^4}{3y^2 - 4x^2y^3} \quad 1A$$

$$7. \quad \frac{xy - 1}{x^2y^2 + 1} = 1$$

$$\frac{d}{dx}(xy - 1) = \frac{d}{dx}(x^2y^2 + 1)$$

$$y + x \frac{dy}{dx} = 2xy^2 + 2x^2y \frac{dy}{dx}$$

1M

$$(x - 2x^2y) \frac{dy}{dx} = 2xy^2 - y$$

$$\frac{dy}{dx} = \frac{y(2xy - 1)}{x(1 - 2xy)}$$

$$= -\frac{y}{x}$$

1A

$$8. \quad (x + y)(2y - x) = xy$$

$$-x^2 + xy + 2y^2 = xy$$

$$\frac{d}{dx}(-x^2 + 2y^2) = \frac{d}{dx}(0)$$

$$-2x + 4y \frac{dy}{dx} = 0$$

1M

$$\frac{dy}{dx} = \frac{x}{2y}$$

1A

$$9. \quad (a) \quad \frac{4x}{y^2} + 2xy = 25$$

$$\frac{d}{dx}(4x + 2xy^3) = \frac{d}{dx}(25y^2)$$

$$4 + 2y^3 + 6xy^2 \frac{dy}{dx} = 50y \frac{dy}{dx}$$

1M

$$(6xy^2 - 50y) \frac{dy}{dx} = -2y^3 - 4$$

$$\frac{dy}{dx} = \frac{y^3 + 2}{25y - 3xy^2}$$

1A

$$(b) \quad \frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$$

$$= \frac{25y - 3xy^2}{y^3 + 2}$$

1A

$$\left. \frac{dx}{dy} \right|_{(5, 2)} = \frac{25(2) - 3(5)(2)^2}{2^3 + 2}$$

$$= -1$$

1A

10. (a) $\frac{d}{dy}(\sqrt{x}) = \frac{d}{dx}\left(y^2 - 5\sqrt{y} - \frac{12}{y}\right)$
 $\frac{1}{2}x^{-\frac{1}{2}}\frac{dx}{dy} = 2y - \frac{5}{2}y^{-\frac{1}{2}} + 12y^{-2}$ 1M
 $\frac{dx}{dy} = x^{\frac{1}{2}}(4y - 5y^{-\frac{1}{2}} + 24y^{-2})$ 1A
- (b) $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$
 $= \frac{1}{x^{\frac{1}{2}}(4y^2 - 5y^{-\frac{1}{2}} + 24y^{-2})}$ 1A
 $\left.\frac{dy}{dx}\right|_{(9, 4)} = \frac{1}{9^{\frac{1}{2}}[4(4)^2 - 5(4)^{-\frac{1}{2}} + 24(4)^{-2}]}$
 $= \frac{1}{45}$ 1A
11. (a) $x^2 + y^2 = 30$
 $2x + 2y\frac{dy}{dx} = 0$ 1M
 $\frac{dy}{dx} = -\frac{x}{y}$ 1A
 $\left.\frac{dy}{dx}\right|_{(p, q)} = \frac{-p}{q} = -\frac{1}{2}$
 $q = 2p$ 1
- (b) $p^2 + (2p)^2 = 30$ 1M
 $p^2 = 6$
 $p = \pm\sqrt{6}$
 Thus, $(p, q) = (\sqrt{6}, 2\sqrt{6})$ or $(-\sqrt{6}, -2\sqrt{6})$. 1A+1A