REV-TRIG-2223-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. $\boxed{\text{D}}$

   $7\sin^2 x = \sin x$

   $\sin x = 0 \quad \text{or} \quad \frac{1}{7}$

   When $\sin x = 0$, $x = 0°$ or $180°$ or $360°$; when $\sin x = \frac{1}{7}$, there are two roots.
   There are total 5 roots.

2. $\boxed{\text{D}}$

   $3\tan^2 x = \tan x$

   $\tan x(3\tan x - 1) = 0$

   $\tan x = 0 \quad \text{or} \quad \frac{1}{3}$

   When $\tan x = 0$, $x = 0°$ or $180°$ or $360°$
   When $\tan x = \frac{1}{3}$, there are 2 roots (in quadrants I and III)
   Altogether 5 roots.

3. $\boxed{\text{B}}$

   $2\tan\theta = \sin\theta$

   $2\sin\theta = \sin\theta\cos\theta$

   $\sin\theta(2 - \cos\theta) = 0$

   $\sin\theta = 0 \quad \text{or} \quad \cos\theta = 2$ (rejected)
   There are 3 roots.

4. $\boxed{\text{D}}$

   We have $\sin\theta = 0$ or $\tan\theta = -3$.
   When $\sin\theta = 0$, $\theta = 0°$ or $180°$ or $360°$.
   When $\tan\theta = -3$, there are two roots.
   Thus, there are 5 roots.

5. $\boxed{\text{D}}$

   $\sin\theta = \sin^3\theta$

   $0 = \sin\theta(\sin^2\theta - 1)$

   $\sin\theta = 0 \quad \text{or} \quad \pm 1$
   With reference to the graph of sine, there are 5 roots.

6. $\boxed{\text{A}}$

$$\frac{3}{\tan^2(90^\circ - \alpha)} - \frac{5}{\sin(270^\circ + \alpha)} = \frac{1}{\cos^2 \alpha}$$

$$3\tan^2 \alpha + \frac{5}{\cos \alpha} = \frac{1}{\cos^2 \alpha}$$

$$3\sin^2 \alpha + 5\cos \alpha = 1$$

$$-3\cos^2 \alpha + 5\cos \alpha + 2 = 0$$

$$\cos \alpha = -\frac{1}{3} \quad \text{or} \quad 2$$

There are two roots for $\cos \alpha = -\frac{1}{3}$. Required number of roots is 2.

7. $\boxed{\text{A}}$

$3\cos^2 \theta + \cos \theta - 4 = 0$

$$\cos \theta = 1 \quad \text{or} \quad -\frac{4}{3} \text{ (rejected)}$$

$$\theta = 0^\circ$$

There is 1 root.

8. $\boxed{\text{B}}$

$5\sin^2 \theta + \sin \theta - 4 = 0$

$$\sin \theta = -1 \quad \text{or} \quad \frac{4}{5}$$

$\sin \theta = -1 \quad \Rightarrow \quad \theta = 270^\circ; \quad \sin \theta = \frac{4}{5} \quad \Rightarrow \quad \theta \approx 53.1^\circ$ or 127°.

$\Rightarrow \quad 3$ roots

9. $\boxed{\text{C}}$

A. 3 roots

B. 3 roots

C. 4 roots

D. 2 roots

10. $\boxed{\text{A}}$

$9\cos^4 \theta - 13\cos^2 \theta + 4 = 0$

$$\cos^2 \theta = 1 \quad \text{or} \quad \frac{4}{9}$$

$$\cos \theta = \pm 1 \quad \text{or} \quad \pm\frac{2}{3}$$

When $\cos \theta = \pm 1$, $\theta = 180^\circ$

When $\cos \theta = \pm\frac{2}{3}$, there are 3 roots (one root per quadrant).

There are total 4 roots.

11. $\boxed{\text{C}}$

$$\sin\theta\tan\theta = 1$$
$$\frac{\sin^2\theta}{\cos\theta} = 1$$
$$1-\cos^2\theta = \cos\theta$$
$$-\cos^2\theta - \cos\theta + 1 = 0$$
$$\cos\theta = \frac{1 \pm \sqrt{1^2 - 4(-1)(1)}}{-2}$$
$$= \frac{-1 \mp \sqrt{5}}{2}$$

Since $\sin\theta\tan\theta = 1$, $\sin\theta$ and $\tan\theta$ are both positive or both negative.

In any of the case, $\cos\theta > 0$.

Thus, $\cos\theta = \frac{\sqrt{5}-1}{2}$.

12. $\boxed{\text{C}}$

$$4\cos^2\theta - 7\sin\theta - 7 = 0$$
$$4(1-\sin^2\theta) - 7\sin\theta - 7 = 0$$
$$-4\sin^2\theta - 7\sin\theta - 3 = 0$$
$$\sin\theta = -1 \quad \text{or} \quad -\frac{3}{4}$$

$\sin\theta = -1$ has one root and $\sin\theta = -\frac{3}{4}$ has two roots.

13. $\boxed{\text{C}}$

$$5(1-\sin^2 x) = \sin x + 1$$
$$0 = 5\sin^2 x + \sin x - 4$$
$$\sin x = \pm\frac{2}{\sqrt{5}}$$

When $\sin x = \frac{2}{\sqrt{5}}$, there are two roots; when $\sin x = -\frac{2}{\sqrt{5}}$, there are also two roots.

There are total 4 roots.

14. $\boxed{\text{A}}$

$$\cos x = \tan x$$
$$\cos^2 x = \sin x$$
$$-\sin^2 x - \sin x + 1 = 0$$
$$\sin x \approx -1.62 \text{ (rejected)} \quad \text{or} \quad 0.618$$

There are 2 roots and hence 2 intersections.

15. $\boxed{\text{D}}$

$$2\cos^2\theta = 2 - \sin\theta$$
$$2(1 - \sin^2\theta) = 2 - \sin\theta$$
$$-2\sin^2\theta + \sin\theta = 0$$
$$\sin\theta = 0 \quad \text{or} \quad \frac{1}{2}$$

When $\sin\theta = 0$, $\theta = 0°$ or $180°$ or $360°$.

When $\sin\theta = \frac{1}{2}$, $\theta = 30°$ or $150°$.

There are 5 roots.

16. $\boxed{\text{C}}$

$$\frac{1 + \cos\theta}{\sin\theta} = 3\tan\theta$$
$$\cos\theta(1 + \cos\theta) = 3\sin^2\theta$$
$$\cos\theta + \cos^2\theta = 3(1 - \cos^2\theta)$$
$$4\cos^2\theta + \cos\theta - 3 = 0$$
$$\cos\theta = \frac{3}{4} \quad \text{or} \quad -1$$

There are two roots for $\cos\theta = \frac{3}{4}$ and one root for $\cos\theta = -1$.

Thus, there are 3 roots.

17. $\boxed{\text{C}}$

$$\cos^2 x - \sin x = 1$$
$$1 - \sin^2 x - \sin x = 1$$
$$\sin x(\sin x + 1) = 0$$
$$\sin x = 0 \quad \text{or} \quad -1$$

There are three roots for $\sin x = 0$ and one root for $\sin x = -1$.

Thus, there are 4 roots.

18. $\boxed{\text{B}}$

The graph passes through $(0,\ 2)$.

A. ✓. When $x = 0$, $y = 2 - 0 = 2$.

B. ✓. When $x = 0$, $y = 2 - 0 = 2$.

C. ✗. When $x = 0$, $y = 1 - 1 = 0 \neq 2$.

D. ✓. When $x = 0$, $y = 1 + 1 = 2$.

The graph passes through $(45,\ 1)$.

A. ✗. When $x = 45$, $y = 2 - \sin 22.5° \neq 1$.

B. ✓. When $x = 45$, $y = 2 - \sin 90° = 1$.

C.

D. ✗. When $x = 45$, $y = 1 + \cos 22.5° \neq 1$.

19. $\boxed{\text{C}}$

The graph passes through $(60,\ 0)$.

A. ✗. When $x = 60$, $y = \sin 90° - 2 = -1 \neq 0$.

B. ✗. When $x = 60$, $y = \sin 30° - 2 = -\frac{3}{2} \neq 0$.

C. ✓. When $x = 60$, $y = \sin 90° - 1 = 0$.

D. ✗. When $x = 60$, $y = \sin 30° - 1 = -\frac{1}{2} \neq 0$.

20. $\boxed{\text{B}}$

The graph passes through $(120,\ 3)$.

A. ✗. When $x = 120$, $y = -3\sin(120° - 30°) = -3 \neq 3$.

B. ✓. When $x = 120$, $y = 3\sin(120° - 30°) = 3$.

C. ✗. When $x = 120$, $y = -3\sin(120° + 30°) = -\frac{3}{2} \neq 3$.

D. ✗. When $x = 120$, $y = 3\sin(120° + 30°) = \frac{3}{2} \neq 3$.

21. $\boxed{\text{D}}$

Check the equations by the points on the graph:

	$(0,\ 7)$	$(90,\ 1)$
A.	✗	
B.	✗	
C.	✓	✗
D.	✓	✓

22. $\boxed{\text{D}}$

The graph passes through the following points:

	$\underline{(75,\ 0)}$
A.	✗
B.	✗
C.	✗
D.	✓

23. $\boxed{\text{A}}$

The graph passes through the following points:

	$\underline{(b,\ 2)}$	$\underline{(0,\ \ominus)}$
A.	✓	✓
B.	✗	
C.	✗	
D.	✓	✗

$\ominus$ represents a negative number (the y-intercept).

24. $\boxed{\text{B}}$

The graph passes through $(210,\ -4)$.

	$\underline{(210,\ -4)}$
A.	$y = -2\cos 240° + 2 \neq -4$
B.	$y = 2\cos 180° - 2 = -4$
C.	$y = 2\cos 240° + 2 \neq 4$
D.	$y = -2\cos 180° - 2 = 0 \neq -4$

25. $\boxed{\text{A}}$

If p is positive, then the maximum value and the minimum value of $y = p\sin 3x° + q$ are $p + q$ and $-p + q$ respectively.

Solve $\begin{cases} p + q = 1 \\ -p + q = -3 \end{cases}$, we have $p = 2$ and $q = -1$.

26. $\boxed{\text{C}}$

The curve passes through $(0,\ 1)$. Only option C satisfies this.

27. $\boxed{\text{C}}$

The graph passes through $(0,\ 3)$.

A. ✗. When $x = 0$, $y = \sin 30° + 3 = 3.5 \neq 3$

B. ✗. When $x = 0$, $y = 2\sin 30° + 1 = 2 \neq 3$

C. ✓. When $x = 0$, $y = 2\sin 30° + 2 = 3$

D. ✗. When $x = 0$, $y = 3\sin 30° + 4 = 5.5 \neq 3$

28. $\boxed{\text{C}}$

The graph passes through $(0°,\ 1)$ and $(90°,\ 3)$.

	$(0°,\ 1)$	$(90°,\ 3)$
A.	✓	✗
B.	✗	
C.	✓	✓
D.	✗	

29. $\boxed{\text{B}}$

The graph passes through $(146,\ 0)$.

A. ✗. When $x = 146$, $y = -3\sin 112° \neq 0$.

B. ✓. When $x = 146$, $y = -3\sin 180° = 0$.

C. ✗. When $x = 146$, $y = -3\sin 100° \neq 0$.

D. ✗. When $x = 146$, $y = -3\sin 192° \neq 0$.

30. $\boxed{\text{C}}$

The graph passes through $(0,\ k)$, where $-1 < k < 0$.

A. ✗. $f(0) = -1$

B. ✓. $f(0) = -\frac{1}{2}$, which lies between -1 and 0.

C. ✓. $f(0) = -\frac{1}{2}$, which lies between -1 and 0.

D. ✗. $f(0) = -1$

The graph passes through $\left(90,\ -\frac{1}{2}\right)$.

B. ✗. $f(90) = \frac{\sin 45°}{2} - \frac{1}{2} \neq -\frac{1}{2}$

C. ✓. $f(90) = 0 - \frac{1}{2} = -\frac{1}{2}$

END OF PAPER