**REG-MEN-2223-ASM-SET 2-MATH**
**Suggested solutions**

**Conventional Questions**

1. (a) Let the base radius and height of the cylinder be $r$ cm and $h$ cm respectively.

$$2\pi r h = \frac{1}{2}(2\pi r h + 2\pi r^2) \qquad \text{1M}$$
$$h = r$$

$$\pi r^2 h = 216\pi$$
$$r^3 = 216$$
$$r = 6 \qquad \text{1A}$$

Base radius = 6 cm

(b) Percentage change $= \dfrac{2(6)(12)}{2\pi(6)^2 + 2\pi(6)(6)} \times 100\%$ 1M+1M

$\approx 31.8\%$ 1A

2. (a) Volume of cylinder $= \pi(14)^2(182)$

$= 35672\pi \text{ cm}^3$ 1A

Ratio of volume of two cones $= 9^{\frac{3}{2}} : 16^{\frac{3}{2}} = 27 : 64$. 1M

Volume of the larger cone $= 35\,672\pi \times \dfrac{64}{27+64}$

$= 25088\pi \text{ cm}^3$ 1A

(b) Let the base radius of the larger cone be $r$ cm.

$$\frac{1}{3}\pi r^2(96) = 25\,088\pi \qquad \text{1M}$$
$$r = 28 \quad \text{or} \quad -28 \text{ (rejected)}$$

Curved surface area of the larger cone

$= \pi(28)\sqrt{28^2 + 96^2}$ 1M

$= 2800\pi \text{ cm}^2$

Required area $= 2800\pi \times \dfrac{9}{16}$

$= 1575\pi \text{ cm}^2$ 1A

3. (a) $$\frac{OM}{ON} = \frac{AM}{ND}$$

$$\frac{13.5 - ON}{ON} = \frac{5}{4}$$ 1M

$$ON = 6\text{ cm}$$

Required volume $= \frac{1}{3}\pi(4)^2(6)$ 1M

$= 32\pi\text{ cm}^3$ 1A

(b) Since the volume of milk is greater than the volume of the lower cone, there are $36\pi - 32\pi = 4\pi\text{ cm}^3$ of milk in the upper cone.

Let the height of milk in the upper cone be $h$ cm.

$$\left(\frac{h}{6}\right)^3 = \frac{4\pi}{32\pi}$$ 1M

$$h = 3$$

Curved surface area of the lower cone

$= \pi(4)(\sqrt{4^2 + 6^2})$ 1M

$= 8\sqrt{13}\pi$

Total wet curved surface area

$= 8\sqrt{13}\pi \times \left(\frac{3}{6}\right)^2 + 8\sqrt{13}\pi$ 1M

$= 10\sqrt{13}\pi\text{ cm}^2$

$> 36\pi\text{ cm}^2$

The claim is agreed. 1A

4. (a) Let the radius of the sphere be $r$ cm.

$$4\pi r^2 = 144\pi$$

$$r = 6 \quad \text{or} \quad -6 \text{ (rejected)}$$

Required volume $= \dfrac{4\pi}{3}(6)^3$ 1M

$= 288\pi \text{ cm}^3$ 1A

(b) Volume of water $= \pi(16)^2(14) - 288\pi$ 1M

$= 3296\pi \text{ cm}^3$

Required depth $= \dfrac{3296\pi}{\pi(16)^2}$ 1M

$= \dfrac{103}{8}$ cm 1A

(c) Base radius of cone $= \dfrac{48\pi}{2\pi} = 24$ cm

Slant height of cone $= \dfrac{720\pi}{\pi(24)} = 30$ cm

Volume of cone $= \dfrac{\pi}{3}(24)^2\sqrt{30^2 - 24^2}$ 1M+1M

$= 3456\pi \text{ cm}^3$

$> 3296\pi \text{ cm}^3$

The water will not overflow. 1A

5. (a) Required radius $= \sqrt{8^2 - 4^2} = 4\sqrt{3}$ cm 1A

(b) Volume of the cone $= \dfrac{1}{3}\pi(4\sqrt{3})^2 h = 16\pi h \text{ cm}^3$. 1A

$V = 16\pi h - 16\pi h \times \left(\dfrac{h-4}{h}\right)^3$ 1M

$= 16\pi h \times \dfrac{h^3 - (h-4)^3}{h^3}$

$= 16\pi h \times \dfrac{4[h^2 + h(h-4) + (h-4)^2]}{h^3}$

$= \dfrac{64\pi(3h^2 - 12h + 16)}{h^2}$ 1

(c) Volume of the hemisphere $= \dfrac{2}{3}\pi(8)^3 = \dfrac{1024\pi}{3} \text{ cm}^3$ 1A

$\dfrac{19}{48} \times \dfrac{1024\pi}{3} = \dfrac{64\pi(3h^2 - 12h + 16)}{h^2}$ 1M+1A

$\dfrac{19h^2}{9} = 3h^2 - 12h + 16$

$0 = \dfrac{8h^2}{9} - 12h + 16$

$h = 12 \quad \text{or} \quad \dfrac{3}{2}$ (rejected) 1A

Height of the cone is 12 cm.

6. (a) Volume of water $= \frac{\pi}{3}(9)^3 + 12\pi(9)^2 + 144\pi(9) = 2511\pi \text{ cm}^3$ 1A

If the spheres are just fully submerged, depth of water is $2r$ cm.

$$\frac{\pi}{3}(2r)^3 + 12\pi(2r)^2 + 144\pi(2r) = 2 \times \frac{4}{3}\pi r^3 + 2511\pi$$ 1M+1M

$$48r^2 + 288r - 2511 = 0$$

$$16r^2 + 96r - 837 = 0$$

$$r \approx 4.83 \quad \text{or} \quad -10.8 \text{ (rejected)}$$ 1A

(b) If it can be fully submerged, the depth of water is at least 10 cm.

The corresponding volume $= \frac{\pi}{3}(10)^3 + 12\pi(10)^2 + 144\pi(10)$

$= \frac{8920\pi}{3} \text{ cm}^3$

Volume of sphere and water $= \frac{4}{3}\pi(5)^3 + 2511\pi$ 1M

$= \frac{8033\pi}{3} \text{ cm}^3$

$< \frac{8920\pi}{3} \text{ cm}^3$ 1M

Thus, the sphere cannot be fully submerged.

The claim is disagreed. 1A

 **END OF PAPER**