Solution	Marks
REG-2223-MOCK-SET 2-MATH-CP 1	
Suggested solutions	
1. $2(3a - 11) = 3a - 5b$	
$6a - 22 = 3a - 5b$	1M
$3a = 22 - 5b$	1M
$a = \frac{22 - 5b}{3}$	1A
2. $\frac{m^6 n^{-3}}{(m^5 n^{-4})^2} = \frac{m^6 n^{-3}}{m^{10} n^{-8}}$	1M
$= \frac{n^{-3+8}}{m^{10-6}}$	1M
$= \frac{n^5}{m^4}$	1A
3. $\frac{5}{3k+2} - \frac{4}{2k+7} = \frac{5(2k+7) - 4(3k+2)}{(3k+2)(2k+7)}$	1M+1A
$= \frac{-2k+27}{(3k+2)(2k+7)}$	1A
4. (a) $25x^2 - 4 = (5x+2)(5x-2)$	1A
(b) $5x^2y - 17xy + 6y = y(5x^2 - 17x + 6)$	
$= y(x-3)(5x-2)$	1A
(c) $5x^2y - 17xy + 6y - 25x^2 + 4 = y(x-3)(5x-2) - (5x+2)(5x-2)$	1M
$= (5x-2)(xy - 3y - 5x - 2)$	1A
5. (a) $-3(x-4) \geq \frac{5x+3}{6}$	
$-\frac{23x}{6} \geq -\frac{23}{2}$	
$x \leq 3$	1A
(b) $6x + 24 > 0$	
$x > -4$	1A
So, $-4 < x \leq 3$.	1M
There are 7 integers.	1A
6. (a) Cost $= \frac{7000}{1 + 40\%}$	1M
$= \$5000$	1A
(b) Percentage profit $= \frac{7000(1 - 12\%) - 5000}{5000} \times 100\%$	1M
$= 23.2\%$	1A

Solution	Marks
7. Let the present age of Irene be x. Then the present age of Peter is $\frac{4x}{3}$.	1A
$\dfrac{x-7}{\frac{4x}{3}-7}=\dfrac{2}{3}$	1M+1A
$3x-21=\dfrac{8x}{3}-14$	
$x=21$	1A
She is 21 years old now.	
8. (a) $57=\dfrac{41+47+\ldots+(70+a)}{12}$	1M
$a=5$	1A
(b) Range $=75-41=34$ kg	1A
Interquartile range $=66-49.5=16.5$ kg	1A
Standard deviation ≈ 10.7 kg	1A
9. (a) Let $\angle AOE=x$.	
Then $\angle AEO=x$ and $\angle DBE=\dfrac{x}{2}$.	1A
$x+\dfrac{x}{2}=48^\circ$	
$x=32^\circ$	1A
(b) $\angle OAB=32^\circ+32^\circ=64^\circ$ and $\angle AOB=180^\circ-2\times 64^\circ=52^\circ$.	1A
$\dfrac{\overset{\frown}{AB}}{AE}=\dfrac{2(AO)\pi\times\frac{52^\circ}{360^\circ}}{AO}$	1M
$\approx 0.908<1$	
So, $\overset{\frown}{AB}<AE$. The claim is agreed.	1A
10. (a) Let $f(x)=a+bx^2$, where a and b are non-zero constants.	1A
$\begin{cases} f(-1)=206=a+b \\ f(3)=254=a+9b \end{cases}$	1M
Solving, we have $a=200$ and $b=6$.	1A
Thus, $f(x)=200+6x^2$.	
(b) $200+6x^2=80x$	
$6x^2-80x+200=0$	
$x=\dfrac{10}{3}$ or 10	1A+1A

Solution	Marks
11. (a) $(c+1)+4+a=8+(b-a)+c$	1M
$2a-b=3$	
Since $a>5$ and $b<11$, $(a,\ b)=(6,\ 9)$.	1A+1A
(b) (i) 1	1A
(ii) 6	1A
(c) Required probability $=\dfrac{3+1}{2+4+6+8+3+1}$	1M
$=\dfrac{1}{6}$	1A
12. (a) $f(-1)=0=-4(-1)^3+(a+2)(-1)^2+2(-1)-3b$	1M
$a-3b=-4$	
$f(2)=9=-4(2)^3+(a+2)(2)^2+2(2)-3b$	1M
$4a-3b=29$	
Solving, we have $a=11$ and $b=5$.	1A
(b) $f(x)=-4x^3+13x^2+2x-15$	
$=(-x^2+2x+3)(4x-5)$	1M
So, $g(x)=4x-5$.	
$kx(4x-5)=(-x^2+2x+3)(4x-5)$	1M
$(4x-5)(x^2+(k-2)x-3)=0$	1M
$x=\dfrac{5}{4}$ or $x^2+(k-2)x-3=0$	
$\Delta=(k-2)^2-4(1)(-3)=(k-2)^2+12>0$ for all real values of k.	
$x^2+(k-2)x-3=0$ has two distinct real roots.	
The claim is agreed.	1A

Solution	Marks
13. (a) Let the radius of the sphere be r cm.	
$4\pi r^2 = 144\pi$	
$r = 6$ or -6 (rejected)	
Required volume $= \frac{4\pi}{3}(6)^3$	1M
$= 288\pi\text{ cm}^3$	1A
(b) Volume of water $= \pi(16)^2(14) - 288\pi$	1M
$= 3296\pi\text{ cm}^3$	
Required depth $= \frac{3296\pi}{\pi(16)^2}$	1M
$= \frac{103}{8}$ cm	1A
(c) Base radius of cone $= \frac{48\pi}{2\pi} = 24$ cm	
Slant height of cone $= \frac{720\pi}{\pi(24)} = 30$ cm	
Volume of cone $= \frac{\pi}{3}(24)^2\sqrt{30^2 - 24^2}$	1M+1M
$= 3456\pi\text{ cm}^3$	
$> 3296\pi\text{ cm}^3$	
The water will not overflow.	1A

Solution	Marks
14. (a) (i) $BC = CD$ *(prop. of square)*	
$\angle BCE = 45^\circ = \angle DCE$ *(prop. of square)*	
$CE = CE$ *(common side)*	
$\triangle BCE \cong \triangle DCE$ *(SAS)*	

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

Solution	Marks
(ii) $\angle GEB = \angle BEF$ *(common $\angle$)*	
$AF // CD$ *(prop. of square)*	
$\angle BFE = \angle CDE$ *(alt. $\angle$., $AF//CD$)*	
$\angle GBE = \angle CDE$ *(corr. $\angle$s, $\cong$ $\triangle$s)*	
$= \angle BFE$	
$\triangle BEG \sim \triangle FEB$ *(AA)*	

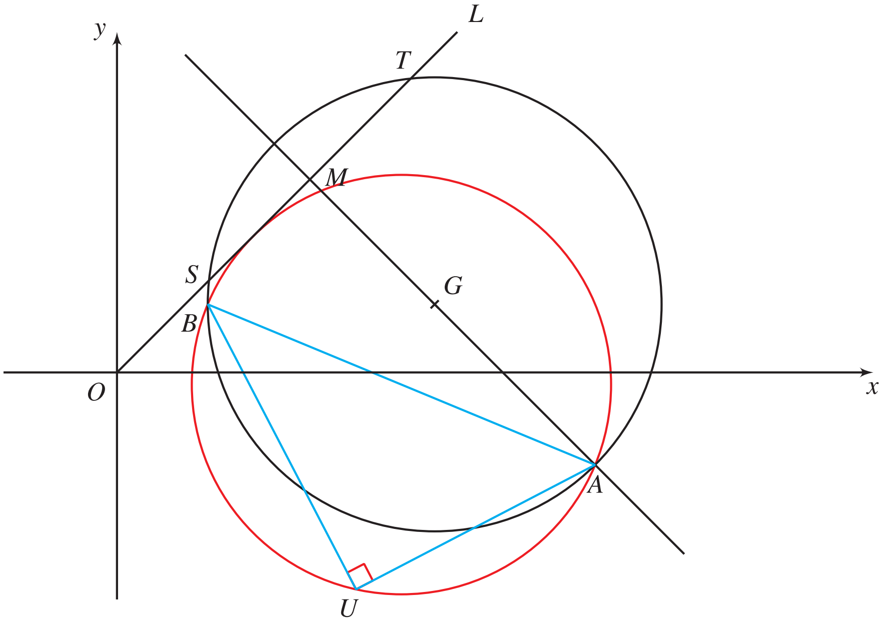
Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

Solution	Marks
(b) Since $\triangle BEG \sim \triangle FEB$, $\frac{BE}{FE} = \frac{EG}{EB} = \frac{BG}{FB} = \tan \angle AFD$.	1M
Since $BE = DE$, we have $\frac{DE}{FE} = \frac{EG}{DE} = \tan \angle AFD$ and $EG = DE \tan \angle AFD$.	1M
$DE = (EG + FG) \tan \angle AFD$	
$= (DE \tan \angle AFD + FG) \tan \angle AFD$	
$= DE \tan^2 \angle AFD + FG \tan \angle AFD$	
$< DE \left(\frac{1}{\sqrt{3}}\right)^2 + FG \left(\frac{1}{\sqrt{3}}\right)$	1M
$DE < \frac{\sqrt{3}}{2} FG$	
The claim is agreed.	1A
15. (a) Required number $= C^8_5 C^5_2$	1M
$= 560$	1A
(b) Required number $= C^8_3 C^5_4 + C^8_2 C^5_5$	1M
$= 308$	1A

Solution	Marks
16. (a) $f(x) = \dfrac{x^2}{16} - \dfrac{x}{2} + 11$	
$= \dfrac{1}{16}(x^2 - 8x + 16) + 10$	1M
$= \dfrac{1}{16}(x-4)^2 + 10$	
The coordinates of vertex are $(4,\ 10)$.	1A
(b) $g(x) = -f(x) = -\dfrac{1}{16}(x-4)^2 - 10$	1M
$h(x) = -\dfrac{1}{16}(x-4)^2 + 6$	1M
y-intercept $= -\dfrac{1}{16}(0-4)^2 + 6 = 5$	1A
17. (a) $(x+2)(x-2) = 8(x-1)$	
$x^2 - 8x + 4 = 0$	
Thus, $p = 8$ and $q = 4$.	1A+1A
(b) Common ratio $= \dfrac{\log 8}{\log 4} = \dfrac{3\log 2}{2\log 2} = \dfrac{3}{2}$.	1M
$(\log 4)\left(\dfrac{3}{2}\right)^{\alpha} + (\log 4)\left(\dfrac{3}{2}\right)^{2\alpha} < \log 2^{2020}$	1M
$2(1.5)^{\alpha} + 2(1.5)^{2\alpha} < 2020$	
$(1.5)^{2\alpha} + 1.5^{\alpha} - 1010 < 0$	
$\dfrac{-1-\sqrt{4041}}{2} < 1.5^{\alpha} < \dfrac{-1+\sqrt{4041}}{2}$	
Since $1.5^{\alpha} > 0$,	
$0 < 1.5^{\alpha} < \dfrac{-1+\sqrt{4041}}{2}$	
$\alpha \log 1.5 < \log \dfrac{-1+\sqrt{4041}}{2}$	1M
$\alpha < 8.49$	
The greatest value of α is 8.	1A
18. (a) $\dfrac{AB}{\sin 65^\circ} = \dfrac{15}{\sin 58^\circ}$	1M
$AB \approx 16.0\text{ cm}$	1A
$AC^2 = AB^2 + 17^2 - 2(AB)(17)\cos 116^\circ$	1M
$AC \approx 28.0\text{ cm}$	1A
(b) $\angle BAD = 180^\circ - 58^\circ - 65^\circ = 57^\circ$	
$AK = AB\cos 57^\circ \approx 8.73\text{ cm}$	1M
$27^2 = AC^2 + 15^2 - 2(AC)(15)\cos\angle CAD$	
$\angle CAD \approx 70.5^\circ$	
$AC\cos\angle CAD \approx 9.36\text{ cm} \neq AK$	
So, CK is not perpendicular to AD.	1M
The claim is disagreed.	1A

Solution	Marks
19. (a) $\sqrt{(h-6)^2+(9-3)^2}=\sqrt{(h-a)^2+(11-3)^2}$	1M
$h^2-12h+72=h^2+a^2-2ah+64$	
$h=\dfrac{a^2-8}{2a-12}$	
The coordinates of G are $\left(\dfrac{a^2-8}{2a-12},\ 3\right)$.	1A
(b) (i) $\dfrac{11-3}{a-\frac{a^2-8}{2a-12}}=\dfrac{4}{3}$	1M
$24(2a-12)=4[a(2a-12)-(a^2-8)]$	
$0=4a^2-96a+320$	
$a=4 \quad \text{or} \quad 20$	
When $a=4$, $h=-2<0$; when $a=20$, $h=14>0$.	
So, $a=20$.	1A
(ii) Coordinates of G are $(14,\ 3)$. The equation of C is	
$(x-14)^2+(y-3)^2=(6-14)^2+(9-3)^2$	1M
$x^2+y^2-28x-6y+105=0$	
$x^2+(kx)^2-28x-6kx+105=0$	1M
$(1+k^2)x^2+(-28-6k)x+105=0$	
x-coordinate of $M=\dfrac{1}{2}\times\dfrac{28+6k}{1+k^2}$	
$=\dfrac{14+3k}{1+k^2}$	1

Solution	Marks
(iii) Since $\angle OMG = 90°$, $OM = 2\sqrt{41}$.	
$$\sqrt{\left(\frac{14+3k}{1+k^2}\right)^2 + \left(\frac{k(14+3k)}{1+k^2}\right)^2} = 2\sqrt{41}$$	1M
$$\frac{(14+3k)^2}{1+k^2} = 164$$ $$-155k^2 + 84k + 32 = 0$$ $$k = \frac{4}{5} \quad \text{or} \quad -\frac{8}{31} \text{ (rejected)}$$	
Coordinates of M are $(10,\ 8)$.	1M
M, G and A are collinear.	
Coordinates of B are $(4,\ 3)$.	1M
When the area of circle AUB is the least, $\angle AUB = 90°$.	1M
slope of $AM \times$ slope of $BM = \frac{8-3}{10-14} \times \frac{8-3}{10-4}$ $= -\frac{25}{24} \neq -1$	1M
So, $\angle AMB \neq 90°$ and $\angle AUB + \angle AMB \neq 180°$.	
A, M, B and U are not concyclic.	1A



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