M2REG-VEC-2223-ASM-SET 3-MATH

建議題解

1. (a) $\overrightarrow{AR} = \frac{1}{4}\overrightarrow{AB} + \frac{3}{4}\overrightarrow{AQ}$ 1M

$= \frac{3}{4}\mathbf{u} + \frac{1}{4}\mathbf{v}$ 1A

(b) $\overrightarrow{AP} = \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC}$

$= \frac{1}{2}\overrightarrow{AB} + \frac{3}{2}\overrightarrow{AQ}$

$= \frac{3}{2}\mathbf{u} + \frac{1}{2}\mathbf{v}$ 1A

(c) $\overrightarrow{AR} = \frac{3}{4}\mathbf{u} + \frac{1}{4}\mathbf{v}$

$= \frac{1}{2}\left(\frac{3}{2}\mathbf{u} + \frac{1}{2}\mathbf{v}\right)$ 1M

$= \frac{1}{2}\overrightarrow{AP}$

可得 $AR // AP$。

因此，A、R 和 P 共線。 1

2. (a) $\overrightarrow{OC} = \frac{3}{5}\overrightarrow{OA} + \frac{2}{5}\overrightarrow{OB}$ 1M

$= \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ 1A

(b) $\overrightarrow{OM} = \frac{1}{k+1}\overrightarrow{OA} + \frac{k}{k+1}\overrightarrow{OD}$

$= \frac{1}{k+1}\mathbf{a} + \frac{k}{k+1}\left(\frac{2}{7}\mathbf{b}\right)$ 1A

$= \frac{1}{k+1}\mathbf{a} + \frac{2k}{7(k+1)}\mathbf{b}$ 1A

(c) 可得 $OM // OC$。

$$\frac{1}{k+1} \div \frac{3}{5} = \frac{2k}{7(k+1)} \div \frac{2}{5}$$ 1M+1A

$$\frac{5}{3} = \frac{5k}{7}$$

$$k = \frac{7}{3}$$ 1A

(d) $\overrightarrow{OM} = \frac{1}{\frac{7}{3}+1}\mathbf{a} + \frac{2\left(\frac{7}{3}\right)}{7\left(\frac{7}{3}+1\right)}\mathbf{b}$

$= \frac{3}{10}\mathbf{a} + \frac{1}{5}\mathbf{b}$

$= \frac{1}{2}\left(\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}\right)$

$= \frac{1}{2}\overrightarrow{OC}$ 1A

因此，M 為 OC 的中點。 1

3. (a) $\overrightarrow{OE} = \overrightarrow{OA} + (h+1)\overrightarrow{AB}$ 1M

$= \mathbf{a} + (h+1)(\mathbf{b} - \mathbf{a})$

$= -h\mathbf{a} + (h+1)\mathbf{b}$ 1A

(b) $\overrightarrow{OC} = \overrightarrow{OE} + \overrightarrow{EC}$

$= \overrightarrow{OE} + \dfrac{1+k}{k}\overrightarrow{OA}$ 1A

$= [-h\mathbf{a} + (h+1)\mathbf{b}] + \dfrac{1+k}{k}\mathbf{a}$

$= \dfrac{1+k-hk}{1+k}\mathbf{a} + (h+1)\mathbf{b}$ 1A

(c) $\overrightarrow{OC} = 4\overrightarrow{OB} = 4\mathbf{b}$

可得 $\dfrac{1+k-hk}{1+k} = 0$ 及 $h+1=4$。. 1M

求解後，可得 $h = 3$ 及 $k = \dfrac{1}{2}$。

$ED : DC = 1 : \dfrac{1}{2} = 2 : 1$ 1A

4. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= -\mathbf{a} + 3\mathbf{b}$ 1A

(b) $\overrightarrow{AD} = \dfrac{h}{1+h}\overrightarrow{AO} + \dfrac{1}{1+h}\overrightarrow{AC}$ 1M

$= \dfrac{h}{1+h}(-\mathbf{a}) + \dfrac{1}{1+h}\left(\dfrac{2}{3}(3\mathbf{b})\right)$ 1A

$= -\dfrac{h}{1+h}\mathbf{a} + \dfrac{2}{1+h}\mathbf{b}$ 1A

(c) $\dfrac{-h}{1+h} \div (-1) = \dfrac{2}{1+h} \div 3$ 1M+1A

$h = \dfrac{2}{3}$ 1A

(d) $\overrightarrow{OD} = \overrightarrow{AD} + \overrightarrow{OA}$

$= -\dfrac{\left(\frac{2}{3}\right)}{1+\frac{2}{3}}\mathbf{a} + \dfrac{2}{1+\frac{2}{3}}\mathbf{b} + \mathbf{a}$

$= \dfrac{3}{5}\mathbf{a} + \dfrac{6}{5}\mathbf{b}$

$\overrightarrow{ED} = \overrightarrow{OD} - \overrightarrow{OE}$

$= \left(\dfrac{3}{5}\mathbf{a} + \dfrac{6}{5}\mathbf{b}\right) - \dfrac{2}{5}(3\mathbf{b})$

$= \dfrac{3}{5}\mathbf{a}$ 1A

$\overrightarrow{DF} = \overrightarrow{AF} - \overrightarrow{AD}$

$= \dfrac{3}{5}\overrightarrow{AC} - \left(-\dfrac{2}{5}\mathbf{a} + \dfrac{6}{5}\mathbf{b}\right)$

$= \dfrac{3}{5}(2\mathbf{b}) + \dfrac{2}{5}\mathbf{a} - \dfrac{6}{5}\mathbf{b}$

$= \dfrac{2}{5}\mathbf{a}$

$= \dfrac{2}{3}\left(\dfrac{3}{5}\mathbf{a}\right)$ 1M

$= \dfrac{2}{3}\overrightarrow{ED}$

可得 $ED // DF$。

因此，E、D 和 F 共線。 1A

5. (a) $\overrightarrow{BS} = \overrightarrow{OS} - \overrightarrow{OB}$

$= 3\mathbf{a} - \mathbf{b}$ 1A

(b) $\overrightarrow{BM} = \dfrac{1-k}{1-k+k}\overrightarrow{BT} + \dfrac{k}{1-k+k}\overrightarrow{BA}$ 1M

$= (1-k)(4\mathbf{b}) + k(\mathbf{a} - \mathbf{b})$ 1A

$= k\mathbf{a} + (4-5k)\mathbf{b}$ 1A

(c) $\dfrac{k}{3} = \dfrac{4-5k}{-1}$ 1M+1A

$k = \dfrac{6}{7}$ 1A

$\overrightarrow{BM} = \dfrac{6}{7}\mathbf{a} - \dfrac{2}{7}\mathbf{b}$

$= \dfrac{2}{7}\overrightarrow{BS}$ 1M

$= \dfrac{2}{7-2}\overrightarrow{MS}$

$BM : MS = 2 : 5$ 1A

6. (a) $\overrightarrow{AQ} = \overrightarrow{OQ} - \overrightarrow{OA} = \frac{3}{2}\mathbf{b} - \mathbf{a}$ 1A

$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \mathbf{b} - \frac{4}{3}\mathbf{a}$ 1A

(b) (i) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \frac{3}{2}\mathbf{b} - \frac{4}{3}\mathbf{a}$

$\overrightarrow{PA} = -\frac{1}{3}\overrightarrow{OA} = -\frac{1}{3}\mathbf{a}$

$\overrightarrow{PT} = \frac{m}{1+m}\overrightarrow{PA} + \frac{1}{m+1}\overrightarrow{PQ}$ 1M

$= \frac{m}{1+m}\left(-\frac{1}{3}\mathbf{a}\right) + \frac{1}{m+1}\left(\frac{3}{2}\mathbf{b} - \frac{4}{3}\mathbf{a}\right)$

$= \frac{-m-4}{3(1+m)}\mathbf{a} + \frac{3}{2(1+m)}\mathbf{b}$ 1A

(ii) $\frac{-m-4}{3(1+m)} \div \left(\frac{-4}{3}\right) = \frac{3}{2(1+m)} \div 1$ 1M

$\frac{2}{m+4} = \frac{1}{3}$

$m = 2$ 1A

(c) $\overrightarrow{OU} = \frac{k}{1+k}\overrightarrow{OP} + \frac{1}{1+k}\overrightarrow{OQ}$

$= \frac{4k}{3(1+k)}\mathbf{a} + \frac{3}{2(1+k)}\mathbf{b}$ 1M

$\overrightarrow{PT} = \frac{-2-4}{3(1+2)}\mathbf{a} + \frac{3}{2(1+2)}\mathbf{b} = -\frac{2}{3}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$\overrightarrow{OT} = \overrightarrow{OP} + \overrightarrow{PT}$

$= \frac{4}{3}\mathbf{a} + \left(-\frac{2}{3}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$

$= \frac{2}{3}\mathbf{a} + \frac{1}{2}\mathbf{b}$ 1M

假定 O、T 和 U 共線。

$\frac{4k}{3(1+k)} \div \frac{2}{3} = \frac{3}{2(1+k)} \div \frac{1}{2}$ 1M

$k = \frac{3}{2}$ 1A

7. (a) $\overrightarrow{OC} = \dfrac{b}{a+b}\overrightarrow{OA} + \dfrac{a}{a+b}\overrightarrow{OB}$ 1M

可得 $r = \dfrac{b}{a+b}$ 及 $s = \dfrac{a}{a+b}$。 1M

$r + s = \dfrac{b}{a+b} + \dfrac{a}{a+b} = 1$ 1

(b) (i) $\overrightarrow{OG} = \dfrac{1}{1+2}\overrightarrow{OQ} + \dfrac{2}{1+2}\overrightarrow{OX}$

$= \dfrac{1}{3}\overrightarrow{OQ} + \dfrac{2}{3}\left(\dfrac{1}{2}\mathbf{p}\right)$

$= \dfrac{1}{3}\mathbf{p} + \dfrac{1}{3}\mathbf{q}$ 1A

(ii) $\overrightarrow{OY} = k\overrightarrow{OG} = \dfrac{k}{3}\mathbf{p} + \dfrac{k}{3}\mathbf{q}$

$\dfrac{k}{3} + \dfrac{k}{3} = 1$ 1M

$k = \dfrac{3}{2}$ 1A

$\overrightarrow{OY} = \dfrac{1}{2}\mathbf{p} + \dfrac{1}{2}\mathbf{q}$ 1A

(iii) (1) $\overrightarrow{OG} = \dfrac{1}{3}\mathbf{p} + \dfrac{1}{3}\mathbf{q} = \dfrac{1}{3}\mathbf{p} + \dfrac{1}{3h}\overrightarrow{OZ}$

$\dfrac{1}{3} + \dfrac{1}{3h} = 1$ 1M

$h = \dfrac{1}{2}$ 1A

(2) $\overrightarrow{ZY} = \overrightarrow{OY} - \overrightarrow{OZ} = \dfrac{1}{2}\mathbf{p} + \dfrac{1}{2}\mathbf{q} - \dfrac{1}{2}\mathbf{q} = \dfrac{1}{2}\mathbf{p} = \dfrac{1}{2}\overrightarrow{OP}$ 1M

因此，$ZY \,//\, OP$。 1A