M2REG-LIM-2223-ASM-SET 4-MATH

建議題解

1. $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sqrt{x}) = \lim\limits_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$ 1M

$= \lim\limits_{h\to 0}\dfrac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})}$ 1M

$= \lim\limits_{h\to 0}\dfrac{1}{\sqrt{x+h}+\sqrt{x}}$ 1M

$= \dfrac{1}{2\sqrt{x}}$ 1A

2. $g(1+h)-g(1) = \dfrac{1}{\sqrt{5(1+h)+4}} - \dfrac{1}{\sqrt{5+4}}$

$= \dfrac{3-\sqrt{5h+9}}{3\sqrt{5h+9}} \times \dfrac{3+\sqrt{5h+9}}{3+\sqrt{5h+9}}$ 1M

$= \dfrac{-5h}{3\sqrt{5h+9}(3+\sqrt{5h+9})}$ 1

$g'(1) = \lim\limits_{h\to 0}\dfrac{g(1+h)-g(1)}{h}$

$= \lim\limits_{h\to 0}\dfrac{-5}{3\sqrt{5h+9}(3+\sqrt{5h+9})}$ 1M

$= \dfrac{-5}{3\sqrt{9}(3+\sqrt{9})}$

$= -\dfrac{5}{54}$ 1A

3. $\dfrac{\mathrm{d}}{\mathrm{d}x}(\cos 3x) = \lim\limits_{h\to 0}\dfrac{\cos[3(x+h)]-\cos 3x}{h}$ 1M

$= \lim\limits_{h\to 0}\dfrac{-2\sin\left(3x+\frac{3h}{2}\right)\sin\frac{3h}{2}}{h}$ 1M

$= \lim\limits_{h\to 0}\left[-\dfrac{\sin\frac{3h}{2}}{\frac{3h}{2}}\cdot 3\sin\left(3x+\dfrac{3h}{2}\right)\right]$ 1M

$= -3\sin 3x$ 1A

4. $\dfrac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \lim\limits_{h\to 0}\dfrac{\tan(x+h)-\tan x}{h}$ 1M

$= \lim\limits_{h\to 0}\dfrac{\tan[(x+h)-x][1+\tan x\tan(x+h)]}{h}$ 1M

$= \lim\limits_{h\to 0}\left[\dfrac{\sin h}{h}\cdot\dfrac{1+\tan x\tan(x+h)}{\cos h}\right]$ 1M

$= 1\cdot(1+\tan^2 x)$

$= \sec^2 x$ 1A

5. $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sec 4x) = \lim\limits_{h\to 0}\dfrac{\sec[4(x+h)]-\sec 4x}{h}$ 1M

$= \lim\limits_{h\to 0}\dfrac{\cos 4x-\cos(4x+4h)}{h\cos 4x\cos(4x+4h)}$

$= \lim\limits_{h\to 0}\dfrac{2\sin(4x+2h)\sin 2h}{h\cos 4x\cos(4x+4h)}$ 1M

$= \lim\limits_{h\to 0}\left[\dfrac{\sin 2h}{2h}\cdot\dfrac{4\sin(4x+2h)}{\cos 4x\cos(4x+4h)}\right]$ 1M

$= \dfrac{4\sin 4x}{\cos^2 4x}$

$= 4\sec 4x\tan 4x$ 1A

6. $f'(0) = \lim\limits_{h\to 0}\dfrac{e^{2(0+h)}-e^{2(0)}}{h}$ 1M

$= \lim\limits_{h\to 0}\dfrac{e^{2h}-1}{2h}\cdot 2$ 1M

$= 2$ 1A

7. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(e^{x^2}\right) = \lim\limits_{h\to 0}\dfrac{e^{(x+h)^2}-e^{x^2}}{h}$ 1M

$= \lim\limits_{h\to 0}\dfrac{e^{x^2}(e^{2hx+h^2}-1)}{h}$ 1M

$= e^{x^2}\lim\limits_{h\to 0}\left[\dfrac{e^{h(2x+h)}-1}{h(2x+h)}\cdot(2x+h)\right]$ 1M

$= e^{x^2}\cdot 1\cdot 2x$

$= 2xe^{x^2}$ 1A

8. 設 $u = x-\dfrac{\pi}{8}$。 1M

$\lim\limits_{x\to\frac{\pi}{8}}\dfrac{x-\frac{\pi}{8}}{\cos\left(x+\frac{11\pi}{8}\right)} = \lim\limits_{u\to 0}\dfrac{u}{\cos\left(\frac{3\pi}{2}+u\right)}$

$= \lim\limits_{u\to 0}\dfrac{u}{\sin u}$ 1M

$= 1$ 1A

9. $\lim\limits_{x\to 0}\dfrac{4x^2}{e^{2x^3}-e^{3x^2}} = \lim\limits_{x\to 0}\dfrac{4x^2}{e^{3x^2}(e^{2x^3-3x^2}-1)}$

$= \lim\limits_{x\to 0}\dfrac{2x^3-3x^2}{e^{2x^3-3x^2}-1}\cdot\dfrac{4}{e^{3x^2}(2x-3)}$ 1M

$= 1\cdot\dfrac{4}{0-3}$

$= -\dfrac{4}{3}$ 1A

10. $\lim\limits_{x\to 0}\dfrac{e^{3x}+e^{-3x}-2}{x^2} = \lim\limits_{x\to 0}\dfrac{e^{6x}+1-2e^{3x}}{x^2e^{3x}}$

$= \lim\limits_{x\to 0}\dfrac{(e^{3x}-1)^2}{x^2e^{3x}}$ 1M

$= \lim\limits_{x\to 0}\left[\left(\dfrac{e^{3x}-1}{3x}\right)^2\cdot\dfrac{9}{e^{3x}}\right]$ 1M

$= 1^2\cdot 9$

$= 9$ 1A

11. (a) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{e^{x+1}-e} = \lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{e(e^x-1)}\cdot\dfrac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$ 1M

$= \lim\limits_{x\to 0}\dfrac{x}{e^x-1}\cdot\dfrac{1}{e(\sqrt{1+x}+1)}$ 1M

$= 1\cdot\dfrac{1}{e(1+1)}$

$= \dfrac{1}{2e}$ 1A

(b) $\lim\limits_{x\to 0}\dfrac{e^{5x}-\sin 3x-1}{e^{-2x}-\cos 4x} = \lim\limits_{x\to 0}\dfrac{e^{5x}-\sin 3x-1}{e^{-2x}-(1-2\sin^2 2x)}$ 1M

$= \lim\limits_{x\to 0}\dfrac{\frac{e^{5x}-1}{x}-\frac{\sin 3x}{x}}{\frac{e^{-2x}-1}{x}+\frac{2\sin^2 2x}{x}}$ 1M

$= \lim\limits_{x\to 0}\dfrac{\frac{e^{5x}-1}{5x}\cdot 5-\frac{\sin 3x}{3x}\cdot 3}{\frac{e^{-2x}-1}{-2x}\cdot(-2)+\frac{\sin 2x}{2x}\cdot 4\sin 2x}$ 1M

$= \dfrac{1\cdot 5-1\cdot 3}{1\cdot(-2)+1\cdot 0}$

$= -1$ 1A

12. (a) $\lim\limits_{x\to 0}\dfrac{e^{2x}+e^{-2x}-2}{\cos^2 2x-\cos 2x}=\lim\limits_{x\to 0}\dfrac{(e^x-e^{-x})^2}{\cos 2x(\cos 2x-1)}$ 1M

$=\lim\limits_{x\to 0}\dfrac{e^{-2x}(e^{2x}-1)^2}{\cos 2x(-2\sin^2 x)}$ 1M

$=\lim\limits_{x\to 0}\left(\dfrac{e^{2x}-1}{2x}\right)^2\left(\dfrac{x}{\sin x}\right)^2\cdot\dfrac{2e^{-2x}}{-\cos 2x}$ 1M

$=1^2\cdot 1^2\cdot(-2)$

$=-2$ 1A

(b) $\lim\limits_{x\to 0}\dfrac{\sqrt{\cos 2x}-1}{2^{2x}-2^{x+1}+1}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{\cos 2x}-1}{(2^x-1)^2}\cdot\dfrac{\sqrt{\cos 2x}+1}{\sqrt{\cos 2x}+1}$ 1M

$=\lim\limits_{x\to 0}\dfrac{\cos 2x-1}{(2^x-1)^2(\sqrt{\cos 2x}+1)}$

$=\lim\limits_{x\to 0}\dfrac{-2\sin^2 x}{(e^{x\ln 2}-1)^2(\sqrt{\cos 2x}+1)}$ 1M

$=\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^2\left(\dfrac{x\ln 2}{e^{x\ln 2}-1}\right)^2\cdot\dfrac{-2}{(\ln 2)^2(\sqrt{\cos 2x}+1)}$ 1M

$=1^2\cdot 1^2\cdot\dfrac{-2}{(\ln 2)^2(1+1)}$

$=-\dfrac{1}{(\ln 2)^2}$ 1A

13. $\lim\limits_{x\to 0}\dfrac{(e^x-e^{-x})^2}{1-\cos 2x}=\lim\limits_{x\to 0}\dfrac{(e^{2x}-1)^2}{e^{2x}[1-(1-2\sin^2 x)]}$ 1M

$=\lim\limits_{x\to 0}\left[\left(\dfrac{e^{2x}-1}{2x}\right)^2\cdot\left(\dfrac{x}{\sin x}\right)^2\cdot\dfrac{2}{e^{2x}}\right]$ 1M

$=1^2\cdot 1^2\cdot 2$

$=2$ 1A