

REV-LOG-2223-ASM-SET 1-MATH**Suggested solutions****Multiple Choice Questions**1. A

$$\begin{aligned}1000^{\log(2a)} &= 10^{3\log(2a)} \\&= 10^{\log(2a)^3} \\&= (2a)^3 \\&= 8a^3\end{aligned}$$

2. A

$$\begin{aligned}100^{\log a} &= 10^{2\log a} \\&= (10^{\log a})^2 \\&= a^2\end{aligned}$$

3. B

$$\begin{aligned}10^{2\log a} &= (10^{\log a})^2 \\&= a^2\end{aligned}$$

4. D

$$\begin{aligned}\log \frac{y^x}{x^y} &= \log y^x - \log x^y \\&= x \log y - y \log x\end{aligned}$$

5. B

$$\begin{aligned}\log \sqrt[4]{x+y} &= \frac{1}{4} \log(x+y) \\&= \frac{1}{4} \log \frac{x^2 - y^2}{x - y} \\&= \frac{1}{4} [\log(x^2 - y^2) - \log(x - y)] \\&= \frac{1}{4} (10 - 2) \\&= 2\end{aligned}$$

6. A

$$\begin{aligned}\log \sqrt[3]{x} &= k \\ \frac{1}{3} \log x &= k \\ -2 \log x &= -6k \\ \log \frac{1}{x^2} &= -6k\end{aligned}$$

7. C

$$\begin{aligned}\frac{\log a^6 - \log \sqrt[3]{a^2}}{2 \log a^4} &= \frac{6 \log a - \frac{2}{3} \log a}{8 \log a} \\ &= \frac{\frac{16}{3} \log a}{8 \log a} \\ &= \frac{2}{3}\end{aligned}$$

8. A

$$\begin{aligned}\frac{\log x^3}{\log \frac{x}{2} + \log x + \log 2} &= \frac{3 \log x}{\log x - \log 2 + \log x + \log 2} \\ &= \frac{3 \log x}{2 \log x} \\ &= \frac{3}{2}\end{aligned}$$

9. B

$$\begin{aligned}10^c &= ab \\ c &= \log ab \\ &= \log a + \log b\end{aligned}$$

10. D

$$\begin{aligned}\text{I. } \times. \log a + \log b &= \log ab \neq \log(a + b) \\ \text{II. } \checkmark. \\ \text{III. } \checkmark. \log \frac{1}{a} + \log \frac{1}{b} &= -\log a - \log b \\ &= -(\log a + \log b) \\ &= -\log ab\end{aligned}$$

11. A

$$\begin{aligned}\frac{\log(1000x^3)}{\log \frac{1}{10x}} &= \frac{\log 1000 + 3 \log x}{-\log 10 - \log x} \\ &= \frac{3 + 3 \log x}{-1 - \log x} \\ &= \frac{3(1 + \log x)}{-(1 + \log x)} \\ &= -3\end{aligned}$$

12. D

$$\begin{aligned}p &= \log 2 \text{ and } q = \log 3. \\ \log \frac{5}{3} &= \log 10 - \log 2 - \log 3 = 1 - p - q\end{aligned}$$

13. A

We have $x = \log 2$ and $y = \log 7$.

$$\begin{aligned}\log \frac{50}{7} &= \log \frac{100}{2(7)} \\ &= \log 100 - \log 2 - \log 7 \\ &= 2 - x - y\end{aligned}$$

14. D

We have $m = \log 2$ and $n = \log 3$.

$$\begin{aligned}\log \sqrt{15} &= \frac{1}{2} \log \frac{3 \times 10}{2} \\ &= \frac{1}{2} (\log 3 + \log 10 - \log 2) \\ &= \frac{1}{2} (n - m + 1)\end{aligned}$$

15. C

We have $a = \log 3$ and $b = \log 4 = 2 \log 2$.

$$\begin{aligned}\log \frac{5}{6} &= \log \frac{10}{2^2 \times 3} \\ &= \log 10 - 2 \log 2 - \log 3 \\ &= 1 - a - b\end{aligned}$$

16. D

$$100^a = 2$$

$$10^{2a} = \frac{100}{50}$$

$$2a = \log \frac{100}{50}$$

$$2a = \log 100 - \log 50$$

$$\log 50 = 2 - 2a$$

17. C

$$\begin{aligned}\log 135 &= \log (3^3 \times 5) \\ &= 3 \log 3 + \log \frac{10}{2} \\ &= 3 \log 3 + \log 10 - \log 2 \\ &= 3y - x + 1\end{aligned}$$

18. C

$$\log 8 = a$$

$$3 \log 2 = a$$

$$\log 2 = \frac{a}{3}$$

$$\begin{aligned} \log \frac{1}{16} &= \log \frac{1}{2^4} \\ &= -4 \log 2 \\ &= -\frac{4a}{3} \end{aligned}$$

19. A

$$\begin{aligned} \log_{\frac{1}{\sqrt{5}}} 25 &= \frac{\log 25}{\log \frac{1}{\sqrt{5}}} \\ &= \frac{2 \log 5}{-\frac{1}{2} \log 5} \\ &= -4 \end{aligned}$$

20. C

We have $\log b > \log a > 0$.

Therefore, $\log_a b = \frac{\log b}{\log a} > 1$.

21. D

$$\begin{aligned} \log_a b^3 \cdot \log_b c^3 &= \frac{3 \log b}{\log a} \times \frac{3 \log c}{\log b} \\ &= 9 \times \frac{\log c}{\log a} \\ &= 9 \log_a c \end{aligned}$$

22. C

$$\begin{aligned} \log_{2\sqrt{2}} 64 &= \frac{\log 64}{\log 2\sqrt{2}} \\ &= \frac{\log 2^6}{\log 2^{1+\frac{1}{2}}} \\ &= \frac{6 \log 2}{\frac{3}{2} \log 2} \\ &= 4 \end{aligned}$$

23. B

$$\begin{aligned} \log x^2 &= 2 \log 2 + \log 9 \\ &= \log(4 \times 9) \\ x^2 &= 36 \\ x &= \pm 6 \end{aligned}$$

24. D

$$\log_2(2x + 1) = \log_4(x + 11)$$

$$\frac{\log(2x + 1)}{\log 2} = \frac{\log(x + 11)}{2 \log 2}$$

$$\log(2x + 1)^2 = \log(x + 11)$$

$$(2x + 1)^2 = x + 11$$

$$4x^2 + 3x - 10 = 0$$

$$x = \frac{5}{4} \quad \text{or} \quad -2 \text{ (rejected)}$$

25. A

$$\log 10a = 2 \log a + 4 \log b$$

$$\log 10a = \log(a^2 b^4)$$

$$10a = a^2 b^4$$

$$10 = ab^4$$

$$a = \frac{10}{b^4}$$

26. B

$$\log(a - 2x) = 3$$

$$a - 2x = 10^3$$

$$x = \frac{1}{2}(a - 1000)$$

27. D

$$\log_2 y - \log_2 x = 6.3 - 2.3$$

$$\log_2 \frac{y}{x} = 4$$

$$\frac{y}{x} = 2^4$$

$$y = 16x$$

28. A

$$\frac{1}{y} = \log_5 x$$

$$\frac{1}{y} = \frac{\log x}{\log 5}$$

$$y = \frac{\log 5}{\log x}$$

$$= \log_x 5$$

29. C

$$\log(3x + 4) = \log \frac{x}{2} + 1$$

$$\log(3x + 4) = \log \frac{x}{2} + \log 10$$

$$3x + 4 = \frac{x}{2} \times 10$$

$$x = 2$$

30. C

Draw the line $x = 1$. The intersections are $(1, b)$ and $(1, c)$.

I. ✗. $b > c$

II. ✓. $b > c > 1 \Rightarrow bc > 1$

III. ✓. Let $L: y = k$.

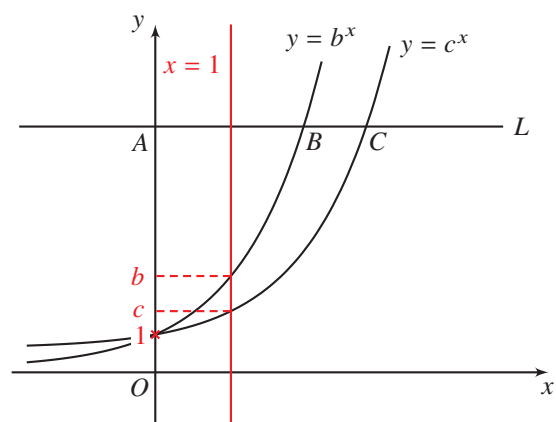
$B(\log_b k, k)$ and $C(\log_c k, k)$

$$\frac{AB}{AC} = \frac{\log_b k}{\log_c k}$$

$$= \frac{\log k}{\log b} \div \frac{\log k}{\log c}$$

$$= \frac{\log c}{\log b}$$

$$= \log_b c$$



Conventional Questions

$$31. \log(5x^2) - 2 \log \frac{1}{\sqrt[3]{x}} + \log(2x) = \log 5 + 2 \log x + \frac{2}{3} \log x + \log 2 + \log x \quad 1M$$

$$= \log(5 \times 2) + \frac{11}{3} \log x \quad 1M$$

$$= 1 + \frac{11}{3} \log x \quad 1A$$

$$32. \log_2(2x - 15) + \log_2(x - 6) = 1$$

$$\log_2[(2x - 15)(x - 6)] = 1 \quad 1M$$

$$(2x - 15)(x - 6) = 2^1 \quad 1M$$

$$2x^2 - 27x + 88 = 0$$

$$x = 8 \quad \text{or} \quad \frac{11}{2} \text{ (rejected)} \quad 1A$$

$$33. \log_3(x + 1) - \log_3(2x - 3) = 2$$

$$\log \frac{x + 1}{2x - 3} = 2 \quad 1M$$

$$\frac{x + 1}{2x - 3} = 3^2 \quad 1M$$

$$x + 1 = 18x - 27$$

$$x = \frac{28}{17} \quad 1A$$

34. From the first equation,

$$3^{x-1} 2^{x-1} = 6^y$$

$$6^{x-1} = 6^y$$

$$x - 1 = y \quad 1M$$

Substitute $y = x - 1$ into the second equation,

$$\log_3(x - 2) - \log_3(x - 1) = -1 \quad 1M$$

$$\log_3 \frac{x - 2}{x - 1} = -1$$

$$\frac{x - 2}{x - 1} = 3^{-1} \quad 1M$$

$$3x - 6 = x - 1$$

$$x = \frac{5}{2}$$

$$\text{When } x = \frac{5}{2}, y = \frac{5}{2} - 1 = \frac{3}{2}. \text{ The solution is } x = \frac{5}{2} \text{ and } y = \frac{3}{2}. \quad 1A$$