

REV-FG-2223-ASM-SET 2-MATH**Suggested solutions****Conventional Questions**

1. (a) $3 = -3^2 + 8(3) + k$ 1M
 $k = -12$ 1A
- (b) $0 = -x^2 + 8x - 12$
 $x = 2$ or 6
The coordinates of A and B are $(2, 0)$ and $(6, 0)$ respectively. 1A+1A
- (c) (i) x -coordinate of midpoint of $AB = \frac{2+6}{2} = 4$.
The axis of symmetry is $x = 4$. 1A
- (ii) $CP : PB = (4 - 3) : (6 - 4)$ 1M
 $= 1 : 2$ 1A
2. (a) $CD = BE = \sqrt{(3x)^2 + (4x)^2}$ 1M
 $= 5x \text{ cm}$ 1A
- (b) (i) $BC = \frac{96 - 3x - 4x - 5x}{2}$ 1M
 $= (48 - 6x) \text{ cm}$
Area of rectangle $= (48 - 6x)(5x)$
 $= 30x(8 - x) \text{ cm}^2$ 1A
- (ii) Area $= -30x^2 + 240x + 6x^2$
 $= -24[x^2 - 2(x)(5) + 5^2] + 600$ 1M
 $= -24(x - 5)^2 + 600$ 1A
The maximum area of the figure is 600 cm^2 . 1A

3. (a) We have

$$\begin{cases} 400 = 150 + A(180)(5) + 25B \\ 438 = 150 + A(200)(8) + 64B \end{cases} \quad 1M$$

Solving, we have $A = \frac{1}{2}$ and $B = -8$. 1M+1

Thus, $P = 150 + \frac{\theta t}{2} - 8t^2$.

(b) (i) $P = 150 + \frac{220(10)}{2} - 8(10)^2 = 450$ 1A

(ii) When $\theta = 160$ and $P = 450$,

$$450 = 150 + \frac{160t}{2} - 8t^2$$

$$2t^2 - 20t + 75 = 0$$

$$\Delta = (-20)^2 - 4(2)(75) = -200 < 0 \quad 1M$$

There is no real solution for t .

Thus, it is not possible to achieve the same value of P in (i) by varying t . 1A

(c) $P = 150 + \frac{160}{2}t - 8t^2$ 1M

$$= -8[t^2 - 2(5)(t) + 5^2] + 350$$

$$= -8(t - 5)^2 + 350$$

Thus, P is maximum when $t = 5$.

Required times is 5 minutes. 1A

4. (a) $0 = 3x^2 - 9x - 12$

$$x = 4 \quad \text{or} \quad -1$$

The coordinates of A and B are $(-1, 0)$ and $(4, 0)$ respectively. 1A+1A

The coordinates of C are $(0, -12)$. 1A

(b) $y = 3x^2 - 9x - 12$

$$= 3 \left[x^2 - 2 \left(\frac{3}{2} \right) x + \left(\frac{3}{2} \right)^2 \right] - \frac{75}{4} \quad 1M$$

$$= 3 \left(x - \frac{3}{2} \right)^2 - \frac{75}{4} \quad 1M$$

Coordinates of Q are $\left(\frac{3}{2}, -\frac{75}{4} \right)$. 1A

(c) Area of $ABQC = \frac{1}{2}(1)(12) + \frac{1}{2} \left(\frac{3}{2} \right) \left(12 + \frac{75}{4} \right) + \frac{1}{2} \left(4 - \frac{3}{2} \right) \left(\frac{75}{4} \right)$ 1M

$$= \frac{105}{2} \quad 1A$$

5. (a) (i) $\alpha + \beta = 6$ and $\alpha\beta = p$ 1A
- $$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$
- $$= (6)^2 - 2p$$
- $$= 36 - 2p$$
- 1M
-
- $$= 36 - 2p$$
- 1A
-
- (ii)
- $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
- 1M
- $$= (6)^2 - 4p$$
- $$= 36 - 4p$$
- 1A
-
- (b)
- $\Delta = (-6)^2 - 4(1)(p) > 0$
- 1M+1A
- $$p < 9$$
- 1A
-
- (c) (i)
- $y = x^2 - 6x + 2$
- $$= [x^2 - 2(3)(x) + 3^2] - 7$$
- 1M
- $$= (x - 3)^2 - 7$$
-
- The coordinates of
- V
- are
- $(3, -7)$
- .
- 1A
-
- (ii) Let the coordinates of
- A
- and
- B
- be
- $(\alpha, 0)$
- and
- $(\beta, 0)$
- respectively.
- $$OB - OA = \beta - \alpha$$
- $$= \sqrt{(\beta - \alpha)^2}$$
- 1M
-
- Using the result of (a)(ii), putting
- $p = 2$
- ,
- $$OB - OA = \sqrt{36 - 4(2)}$$
- $$= 2\sqrt{7}$$
- 1A
-
- (iii) Area of
- $\triangle ABV = \frac{(2\sqrt{7})(7)}{2}$
- 1M
- $$= 7\sqrt{7}$$

6. (a) $b = 6$

Since α and β are roots of the equation $x^2 + ax + b = 0$.

$$\alpha\beta = b = 6.$$

1A

(b) (i) $y = x^2 + ax + 6$

$$= \left[x + ax + \left(\frac{a}{2} \right)^2 \right] + 6 - \frac{a^2}{4}$$

1M

$$= \left(x + \frac{a}{2} \right)^2 + 6 - \frac{a^2}{4}$$

1A

If the area of $\triangle ABC$ is twice the area of $\triangle ABP$,

$$2 \left[\frac{a^2}{4} - 6 \right] = 6$$

1M+1M

$$\frac{a^2}{2} - 9 = 0$$

$$a = 6 \quad \text{or} \quad -6 \text{ (rejected)}$$

1A

(ii) By (b)(i), coordinates of P are $\left(\frac{-6}{2}, 6 - \frac{6^2}{4} \right) = (-3, -3)$.

1A

(c) $x^2 + 6x + 6 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(6)}}{2(1)}$$

1M

$$= -3 \pm \sqrt{3}$$

Therefore, $\alpha = -3 + \sqrt{3}$ and $\beta = -3 - \sqrt{3}$.

$$OA = 3 - \sqrt{3} \text{ and } AB = 2\sqrt{3} \neq 2OA.$$

1A

The claim is disagreed.

1A

7. (a) Vertex of the graph of $y = -\frac{1}{2}(x - h)^2 + 8$ is $(h, 8)$. 1M

The two graphs have the same vertex. Since $f(2) = 8$, we have $h = 2$. 1A

(b) (i) Required area = $\frac{(AB + d)k}{2} = \frac{(12 - k)k}{2}$ 1A

(ii) Area of $OABD = -\frac{1}{2}k^2 + 6k$
 $= -\frac{1}{2}[k^2 - 2(6)(k) + 6^2] + 18$ 1M
 $= -\frac{1}{2}(k - 6)^2 + 18$

Area of $OABD$ is maximum when $k = 6$. 1A

When $y = 6$,

$$6 = -\frac{1}{2}(x - 2)^2 + 8$$
 1M

$$(x - 2)^2 = 4$$

$$x = 0 \quad \text{or} \quad 4$$

So, $AB = 4$ 1A

$$4 + d + 6 = 12$$

$$d = 2$$

Thus, $D = (2, 0)$. 1A

8. (a) $y = x^2 - 8x + 12$
 $= (x^2 - 8x + 16) - 4$ 1M
 $= (x - 4)^2 - 4$
Coordinates of V are $(4, -4)$. 1A
- (b) $C(0, 12)$. 1A
 $0 = x^2 - 8x + 12$
 $x = 2$ or 6
So, $A(2, 0)$ and $B(6, 0)$. 1A
Area of $AVBC = \frac{(6-2)(12)}{2} + \frac{(6-2)(4)}{2}$
 $= 32$ 1A
- (c) Slope of $AV = \frac{0+4}{2-4} = -2$, slope of $AC = \frac{12-0}{0-2} = -6$ 1M
Slope of $BV = \frac{0+4}{6-4} = 2$, slope of $BC = \frac{12-0}{0-6} = -2$
 $AV \parallel BC$ while AC is not parallel to BV .
So, $AVBC$ is a trapezium. 1
- (d) Let the required height be h .
 $32 = \frac{(\sqrt{12^2 + 6^2} + \sqrt{4^2 + (2-4)^2})h}{2}$ 1M+1M
 $h \approx 3.58$ 1A
Required height is 3.58.

9. (a) Slope of $L_1 = \frac{4-1}{2+1} = 1$. Equation of L_1 is

$$y - 4 = 1(x - 2) \quad 1M$$

$$y = x + 2 \quad 1A$$

- (b) (i) Slope of $L_2 = -1$. Equation of L_2 is

$$y - a^2 = -(x - a) \quad 1M$$

$$y = -x + a + a^2 \quad 1A$$

(ii) Solve $\begin{cases} y = x + 2 \\ y = -x + a + a^2 \end{cases}$.

$$x + 2 = -x + a + a^2 \quad 1M$$

$$x = \frac{a^2 + a - 2}{2}$$

When $x = \frac{a^2 + a - 2}{2}$, $y = \frac{a^2 + a - 2}{2} + 2 = \frac{a^2 + a + 2}{2}$.

So, $D \left(\frac{a^2 + a - 2}{2}, \frac{a^2 + a + 2}{2} \right)$. 1A

$$DP^2 = \left(\frac{a^2 + a - 2}{2} - a \right)^2 + \left(\frac{a^2 + a + 2}{2} - a^2 \right)^2 \quad 1M$$

$$= \frac{(a^2 - a - 2)^2}{4} + \frac{(-a^2 + a + 2)^2}{4}$$

$$= \frac{2(a^2 - a - 2)^2}{4}$$

$$DP = \frac{1}{\sqrt{2}} \sqrt{(a^2 - a - 2)^2}$$

Since $a^2 - a - 2 = (a - 2)(a + 1) \leq 0$ for $-1 \leq a \leq 2$, 1M

$$DP = \frac{-1}{\sqrt{2}}(a^2 - a - 2) \quad 1$$

(iii) $AB = \sqrt{(4-1)^2 + (2+1)^2} = 3\sqrt{2}$

$$\text{Area of } \triangle ABP = \frac{1}{2}(3\sqrt{2}) \left(\frac{-1}{\sqrt{2}}(a^2 - a - 2) \right) \quad 1M$$

$$= -\frac{3}{2} \left[\left(a^2 - a + \frac{1}{4} \right) - \frac{9}{4} \right] \quad 1M$$

$$= -\frac{3}{2} \left(a - \frac{1}{2} \right)^2 + \frac{27}{8}$$

Maximum area of $\triangle ABP$ is $\frac{27}{8}$ when $P \left(\frac{1}{2}, \frac{1}{4} \right)$. 1A+1A