REV-EOSL-2223-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. B

   Slope = -1 and y-intercept = 5
   The answer is B.

2. D

   Slope = -1 and y-intercept = -5
   The answer is D.

3. C

   Slope = $m < 0$ and y-intercept = $c < 0$
   The answer is C.

4. B

   $y = 3(0) + 6$ and $0 = 3x + 6$

   $y = 6$ $\qquad$ $x = -2$
   x-intercept = -2 and y-intercept = 6

5. C

   Slope = $m = \dfrac{3-0}{0+6} = \dfrac{1}{2}$
   y-intercept = $c = 3$

6. C

   Slope = $a \dfrac{-2-0}{0-4} = \dfrac{1}{2}$
   y-intercept = $b = -2$

7. B

   Slope of $L = -\dfrac{1}{4} \div \dfrac{-1}{6} = \dfrac{3}{2}$.

   I. ✓. Slope of $2x + 3y - 4 = 0$ is $-\dfrac{2}{3}$. Product of slopes = -1.

   II. ✓. Slope of $3x - 2y + 1 = 0$ is $\dfrac{3}{2}$, equal to slope of L.

   III. ✗. $\dfrac{0}{4} - \dfrac{y}{6} = 1$

   $y = -6 \neq 6$

8. $\boxed{\text{C}}$

$\frac{x}{3} - \frac{y}{4} = 1$

$y = \frac{4x}{3} - 4$

Slope $= \frac{4}{3}$

Only option C is a straight line with slope $\frac{4}{3}$.

9. $\boxed{\text{C}}$

Slope of $L_1 = -1$

Slope of $L_2 = -1$

Slope of $L_3 = 1$

I. ✓.

II. ✗.

III. ✓.

10. $\boxed{\text{B}}$

$$(\text{slope of } L_1)(\text{slope of } L_2) = -1$$

$$(3)\left(\frac{a}{9}\right) = -1$$

$$a = -3$$

11. $\boxed{\text{D}}$

Equation of straight line perpendicular to L_2 is in the form $\frac{x}{2} + \frac{y}{5} + k = 0$, where k is a constant.

$$\frac{6}{2} + \frac{-2}{5} + k = 0$$

$$k = -\frac{13}{5}$$

Required equation is

$$\frac{x}{2} + \frac{y}{5} - \frac{13}{5} = 0$$

$$5x + 2y - 26 = 0$$

12. D

Consider the x-intercept of two lines,

$$-\frac{15}{h} = \frac{5}{4}$$

$$h = -12$$

Two lines are perpendicular to each other,

$$\frac{12}{k} \times -\frac{4}{3} = -1$$

$$k = 16$$

13. B

Let the inclination of L be θ_1.

$$\tan \theta_1 = \frac{9-0}{12-0}$$

$$\theta_1 \approx 36.9°$$

Let the angle between the line $x + 3y - 14 = 0$ and the x-axis be θ_2.

$$-\tan \theta_2 = \frac{-1}{3}$$

$$\theta_2 \approx 18.4°$$

$$\theta = \theta_1 + \theta_2 \approx 55°$$

14. B

Slope $= -\frac{a}{b} > 0$

y-intercept $= -\frac{c}{b} < 0$

The answer is B.

15. B

I. ✓. Consider the x-intercepts of two lines.

$$\frac{\ell}{h} = \frac{c}{a}$$

$$a\ell = ch$$

II. ✗. Consider the y-intercepts of two lines.

$$\frac{\ell}{k} > 0 > \frac{c}{b}$$

$$\frac{\ell}{k} - \frac{c}{b} > 0$$

$$\frac{b\ell - ck}{bk} > 0$$

We have $b\ell - ck \neq 0$.

Thus, $ck \neq b\ell$.

III. ✓. Consider the x-intercept of the line $ax + by + c$.

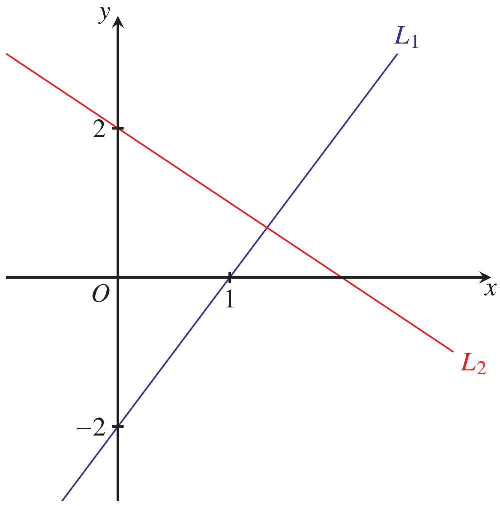
We have $\frac{c}{a} < 0$.

END OF PAPER

16. A

Assign reasonable values to the intercepts. We need only one intercept for L_2 since there is only one variable to be determined.

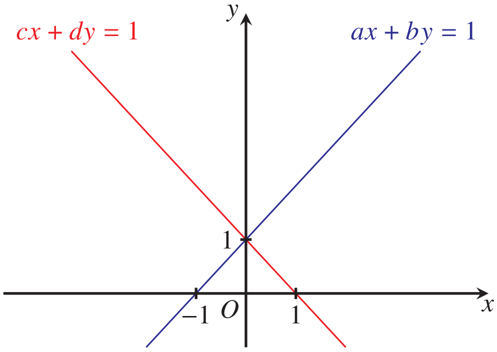
L_1:
$(1,\ 0) \rightarrow b = -1$
$(0,\ -2) \rightarrow a = \dfrac{1}{2}$
L_2:
$(0,\ 2) \rightarrow c = 2$
The result follows.



17. D

Assign reasonable values to the intercepts.

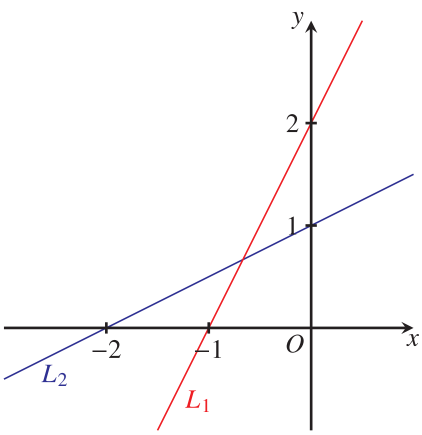
Sub $(-1,\ 0)$ and $(0,\ 1)$ into $ax + by = 1$,
we have $a = -1$ and $b = 1$.
Sub $(1,\ 0)$ and $(0,\ 1)$ into $cx + dy = 1$,
we have $c = d = 1$.
Statements I, II and III are true.



18. D

Assign reasonable values to the intercepts.

$(-1,\ 0)$ and $(0,\ 2)$ in L_1
$\rightarrow \quad a = -\dfrac{3}{2}$ and $b = -3$
$(0,\ 1)$ and $(-2,\ 0)$ in L_2
$\rightarrow \quad c = -\dfrac{1}{2}$ and $d = 1$
Only statements I and III are true.



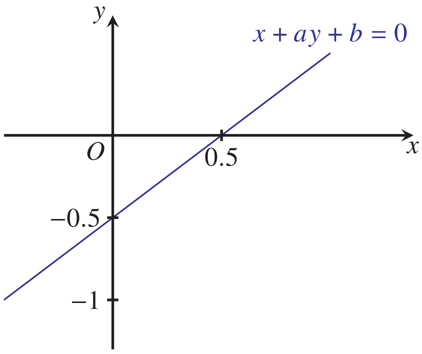
19. D

Assign reasonable values to the intercepts.

$(0.5,\ 0) \rightarrow b = -0.5$

$(0,\ -0.5) \rightarrow a = 1$

Statements I, II and III are true.



20. B

Assign reasonable values to the intercepts.

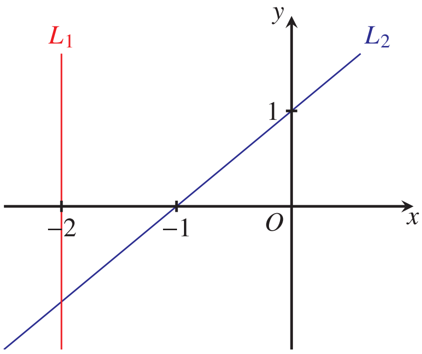
Sub $(-2,\ 0)$ into L_1,

we have $a = -\frac{1}{2}$.

Sub $(-1,\ 0)$ and $(0,\ 1)$ into L_2,

we have $b = -1$ and $c = 1$.

Only statements I and III are true.



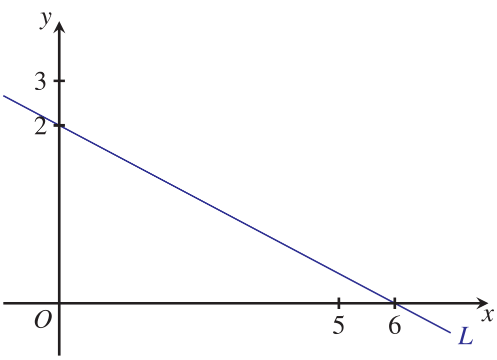
21. A

Assign reasonable values to the intercepts.

$(6,\ 0) \quad \rightarrow \quad a = -2.5$

$(0,\ 2) \quad \rightarrow \quad b = -7.5$

Only statements I and II are true.



22. C

Solve $\begin{cases} -2x + 3y - 10 = 0 \\ 3x - 2y - 5 = 0 \end{cases}$, we have $x = 7$ and $y = 8$.

Slope of $AB = \dfrac{8 - 1}{7 + 1} = \dfrac{7}{8}$

Required equation is

$$y - 8 = \frac{7}{8}(x - 7)$$

$$7x - 8y + 15 = 0$$

23. A

Solve $\begin{cases} 2x + 3y - 7 = 0 \\ x - 3y - 5 = 0 \end{cases}$, we have $x = 4$ and $y = -\dfrac{1}{3}$.

L_2 and L_3 intersect at $\left(4,\ -\dfrac{1}{3}\right)$.

If three lines intersect at a point, then L_1 passes through $\left(4,\ -\dfrac{1}{3}\right)$.

$$7(4) + k\left(-\frac{1}{3}\right) - 31 = 0$$

$$k = -9$$

24. B

y-intercept of L_1 is 2.

$5(0) + 3(2) + k = 0$

$$k = -6$$

25. D

Equation of OA is $y = \dfrac{4x}{3}$.

Solve $\begin{cases} y = \dfrac{4x}{3} \\ y = 8 \end{cases}$, we have $x = 6$ and $y = 8$.

The coordinates of A are $(6,\ 8)$.

$AB = OA = \sqrt{6^2 + 8^2} = 10$

The coordinates of B are $(16,\ 8)$.

26. A

Slope of $AC = \frac{-6+2}{8+8} = -\frac{1}{4}$

Equation of AC is

$$y + 6 = -\frac{1}{4}(x - 8)$$

$$x + 4y + 16 = 0$$

Solve $\begin{cases} x + 4y + 16 = 0 \\ 3x - 4y - 12 = 0 \end{cases}$, the coordinates of C are $\left(-1,\ -\frac{15}{4}\right)$.

Consider the x-intercepts of two lines.

The coordinates of A and B are $(-16,\ 0)$ and $(4,\ 0)$ respectively.

$$\text{Required area} = \frac{(4+16)\left(\frac{15}{4}\right)}{2}$$

$$= 37.5$$

27. B

$2x + 3(2) + 6 = 0$ and $2(0) + 3y + 6 = 0$

$x = -6$ $y = -2$

The coordinates of A and C are $(-6,\ 2)$ and $(0,\ -2)$ respectively.

$$\text{Required area} = \frac{(0+6)(2+2)}{2}$$

$$= 12$$

28. B

$$\frac{-k}{4} \times \frac{2}{3} = -1$$

$$k = 6$$

L: $6x + 4y - 12 = 0$ intersects the x-axis and y-axis at $(2,\ 0)$ and $(0,\ 3)$ respectively.

$$\text{Required area} = \frac{(2)(3)}{2}$$

$$= 3$$

29. D

Two straight lines are parallel. They have equal slopes.

$$\frac{4}{a} = \frac{a}{1}$$

$$a^2 = 4$$

$$a = \pm 2$$

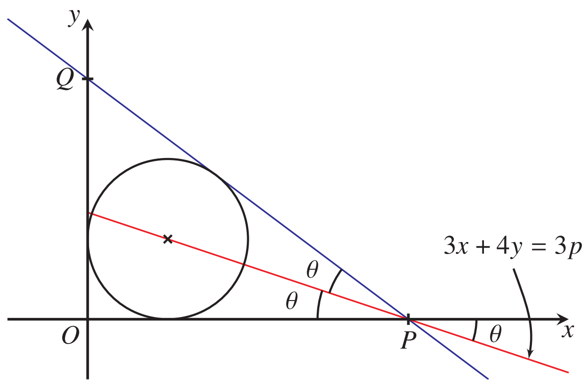
30. D

Note that $(p,\ 0)$ satisfies the equation $3x + 4y = 3p$.

The straight line $3x + 4y = 3p$ passes through $P\ (p,\ 0)$ and the in-centre of $\triangle OPQ$.

Therefore, it is the angle bisector of $\angle OPQ$.

Let the acute angle between the straight line and the x-axis be θ.



Slope of the line $= -\dfrac{3}{4} = -\tan\theta$ and $\dfrac{OQ}{OP} = \tan 2\theta$

$\theta = \tan^{-1}\dfrac{3}{4}$ $\qquad \dfrac{q}{p} = \dfrac{24}{7}$

$p : q = 7 : 24$