**REG-FG-2223-ASM-SET 2-MATH**
**Suggested solutions**

**Multiple Choice Questions**

1. A

The graph opens downwards when the coefficient of $x^2$ is negative.
The answer is A.

2. A

$y = -2x^2 - 8x + 10 \quad \rightarrow \quad a = -2$, $b = -8$ and $c = 10$

| Sign | Graph | Conclusion |
|---|---|---|
| $a < 0$ | open downwards | B ✗ and D ✗ |
| $b < 0$ | slope at $y$-intercept is negative | C ✗ |

The answer is A.

3. B

| Graph | Conclusion |
|---|---|
| Open upwards | $a > 0$ |
| $y$-intercept is positive | $b > 0$ |

The answer is B.

4. D

| Graph | Conclusion |
|---|---|
| Slope at $y$-intercept is negative | $p < 0$ |
| $y$-intercept is negative | $q > 0$ |

The answer is D.

5. A

Consider the graph of $y = ax^2 + bx + c$.

| Graph | Conclusion |
|---|---|
| Open upwards | $a > 0$ |
| Slope at $y$-intercept is negative | $b < 0$ |
| $y$-intercept is positive | $c > 0$ |

Consider the graph of $y = bx^2 + cx + a$.

| Sign | Graph | Conclusion |
|---|---|---|
| $b < 0$ | open downwards | C ✗ and D ✗ |
| $c > 0$ | slope at $y$-intercept is positive | B ✗ |

The answer is A.

6. C

$y = x^2 - 6x + 6 \quad \rightarrow \quad a = 1, b = -6$ and $c = 6$

| Sign | Graph | Conclusion |
|---|---|---|
| $a > 0$ | open upwards | |
| $b < 0$ | slope at $y$-intercept is negative | B ✗ |
| $c > 0$ | $y$-intercept is positive | |

When $y = 0$,

$0 = x^2 - 6x + 6$

$x \approx 4.73$ or $1.27$

There are two $x$-intercepts.

The answer is C.

7. C

| Graph | Conclusion |
|---|---|
| Open downwards | $m < 0$ |
| $y$-intercept $> 0$ | $n > 0$ |

The answer is C.

8. B

The graph opens downwards. We have $a < 0$.

The coordinates of the vertex of the graph are $(-b,\ c)$.

We have $-b < 0$ $(b > 0)$ and $c > 0$.

Thus, $ab < 0$ and $c > 0$.

9. C

$y = x^2 - 2x - 3 \quad \rightarrow \quad a = 1, b = -2$ and $c = -3$

| Sign | Graph | Conclusion |
|---|---|---|
| $a > 0$ | open upwards | A ✗ and B ✗ |
| $b < 0$ | slope at $y$-intercept is negative | D ✗ |
| $c < 0$ | $y$-intercept is negative | |

The answer is C.

10. A

Slope at $y$-intercept = $b < 0$

When $y = 0$, the equation $-3x^2 + bx + c$ has no real roots.

$$\Delta = b^2 - 4(-3)(c) < 0$$
$$b^2 + 12c < 0$$

11. C

| Graph | Conclusion |
|---|---|
| Open upwards | $m > 0$ |
| $y$-intercept $< 0$ | $n < 0$ |

The answer is C.

12. A

| Graph | Conclusion |
|---|---|
| Open downwards | $a < 0$ |
| $y$-intercept is positive | $c > 0$ |
| Two $x$-intercepts | $\Delta = b^2 - 4ac > 0$ |

The answer is A.

13. D

The equation $x^2 - 16x + c$ has double real roots.

$$\Delta = 16^2 - 4(1)(c) = 0$$
$$c = 64$$

14. C

Coordinates of vertex are $(1,\ 1)$.
Vertex of the graph lies in the first quadrant.
The answer is C.

15. B

Coordinates of vertex are $(-h,\ k)$.
$-h < 0$ and $k < 0$

$h > 0$
The answer is B.

16. C

The coordinates of the vertex are $(-b,\ c)$.
We have $b = 0$ and $c < 0$.

17. B

I. ✗. Put $x = 0$, $y = 2(0 + a)^2 + b = 2a^2 + b$.
$y$-intercept $= 2a^2 + b$

II. ✓. The coordinates of the vertex are $(-a,\ b)$.
We have $-a > 0$ $(a < 0)$ and $b < 0$.
Thus, $ab > 0$.

III. ✗.

18. B

$y = (3x - 1)^2 - 9$

$= 9\left(x - \frac{1}{3}\right)^2 - 9$

The coordinates of vertex are $\left(\frac{1}{3},\ -9\right)$.

19. D

Coordinates of vertex are $(-1,\ 1)$.
Vertex of the graph lies in the second quadrant.
$y$-intercept $= -2(0 + 1)^2 + 1$

$= -1 < 0$
The answer is D.

20. A

$y$-intercept $= -(0-1)^2 + 9$

$= 8$

21. D

Vertex $(-b,\ 0) \quad \Rightarrow \quad -b < 0 \quad \Rightarrow \quad b > 0$

The graph opens upwards $\quad \Rightarrow \quad a > 0$

22. D

$y = (ax+1)^2 + a$

$= a^2\left(x + \dfrac{1}{a}\right)^2 + a$

Coordinates of vertex are $\left(-\dfrac{1}{a},\ a\right)$.

Since $-1 < a < 0$, vertex $(\oplus,\ \ominus)$ lies in quadrant IV.

When $x = 0$, $y = 1 + a > 0 \quad \Rightarrow \quad y$-intercept is positive.

23. D

$y = -(4x-5)^2 + 8$

$= -16\left(x - \dfrac{5}{4}\right)^2 + 8$

The axis of symmetry is $x = \dfrac{5}{4}$.

24. B

When $x = 0$, $y = (0+2)^2 - 8 = -4$.

The coordinates of $C$ are $(0,\ -4)$.

$y = (3x+2)^2 - 8$

$= 9\left(x + \dfrac{2}{3}\right)^2 - 8$

The coordinates of $A$ and $B$ are $\left(-\dfrac{2}{3},\ 0\right)$ and $\left(-\dfrac{2}{3},\ -8\right)$ respectively.

Required area $= \dfrac{(4+8)\left(\frac{2}{3}\right)}{2}$

$= 4$

25. B

The coordinates of $A$ are $(0,\ 7)$.

The coordinates of $B$ are $(12,\ 7)$.

Consider the axis of symmetry.

$$h = \frac{0+12}{2}$$
$$= 6$$

Put $(0,\ 7)$ into $y = \frac{1}{2}(x-6)^2 + k$.

$$7 = \frac{1}{2}(0-6)^2 + k$$
$$k = -11$$

26. B

We have $c = 5$.

Let the other $x$-intercept be $\beta$.

Then 1 and $\beta$ are roots of $x^2 + bx + 5 = 0$.

Product of roots $= 1\beta = \frac{5}{1}$

$$\beta = 5$$

Equation of axis of symmetry is

$$x = \frac{1+5}{2}$$
$$x = 3$$

27. D

Using the $x$-intercepts, we have $f(x) = a(x+2)(x-3)$, where $a$ is a constant.

Put $(0,\ 2)$ into $y = a(x+2)(x-3)$.

$$2 = a(0+2)(0-3)$$
$$a = -\frac{1}{3}$$

Thus, $f(x) = -\frac{1}{3}(x+2)(x-3)$.

28. C

The coordinates of the vertex are $(-2,\ 6)$.

The equation is in the form $y = a(x+2)^2 + 6$, where $a$ is a constant.

The answer is C.

29. A

I. ✓. Coefficient of $x^2 = 1 > 0$.

II. ✗. $y$-intercept $= m^2 + n$.
Take $m = 0$ and $n = -1$, then the $y$-intercept is not positive.

III. ✗. When $x = n$, $y = (m - n)^2 + n \geq n$.
Take $m = -1$ and $n = 1$, then $y \neq m$ obviously.

30. C

$(3, -4)$ lies on $y = f(x) \quad \Rightarrow \quad f(3) = -4$

$$f(x) + 4 = 0$$
$$f(x) = -4$$
$$x = 3 \qquad \text{(vertex} \rightarrow 3 \text{ is a repeated root)}$$

The roots of the equation $f(x) + 4 = 0$ are real numbers.

**Conventional Questions**

31. (a) $0 = -x^2 + 7x - 6$

$x = 1 \quad \text{or} \quad 6$

Coordinates of $A$ and $B$ are $(1,\ 0)$ and $(6,\ 0)$ respectively. 1A+1A

Coordinates of $C$ are $(0,\ -6)$. 1A

(b) $y = -x^2 + 7x - 6$

$$= -\left[x^2 - 2\left(\frac{7}{2}\right)x + \left(\frac{7}{2}\right)^2\right] + \frac{25}{4}$$ 1M

$$= -\left(x - \frac{7}{2}\right)^2 + \frac{25}{4}$$

Thus, coordinates of $P$ are $\left(\frac{7}{2},\ \frac{25}{4}\right)$. 1A

Equation of $L$ is $y = \frac{25}{4}$. 1A

32. (a) Substitute $x = 8$ into $2x - y - 1 = 0$, we have $y = 6$.

The coordinates of $C$ are $(8,\ 6)$ and therefore $k = 6$. 1A

Substitute $y = 0$ into $2x - y - 10 = 0$, we have $x = 5$.

The coordinates of $A$ are $(5,\ 0)$. 1A

Substitute $(5,\ 0)$ into $y = a(x - 8)^2 + 6$,

$$0 = a(5 - 8)^2 + 6$$ 1M

$$a = -\frac{2}{3}$$ 1A

(b) When $y = 0$,

$$0 = -\frac{2}{3}(x - 8)^2 + 6$$ 1M

$x = 11 \quad \text{or} \quad 5$

The coordinates of $B$ are $(11,\ 0)$. 1A

Slope of $BC = \frac{6 - 0}{8 - 11} = -2$.

The equation of $BC$ is

$$y - 6 = -2(x - 8)$$ 1M

$$y = -2x + 22$$ 1A

33. (a) Since the coordinates of the vertex of the graph are $(3,\ 8)$ and the graph intersects the $x$-axis at two points,

$(3,\ 8)$ is the maximum point of the graph of $y = f(x)$. 1M

Thus, the graph of $y = f(x)$ opens downwards. 1A

(b) By (a), $f(x) \leq 8$.

Thus, $k > 8$. 1A

(c) The axis of symmetry is $x = 3$. 1A

One of the $x$-intercept is 1. 1A

Let the other $x$-intercept be $\beta$.

$$\frac{1+\beta}{2} = 3 \quad \text{1M}$$

$$\beta = 5$$

The two $x$-intercepts are 1 and 5. 1A

34. (a) $3 = -3^2 + 8(3) + k$ 1M

$k = -12$ 1A

(b) $0 = -x^2 + 8x - 12$

$x = 2$ or $6$

The coordinates of $A$ and $B$ are $(2,\ 0)$ and $(6,\ 0)$ respectively. 1A+1A

(c) (i) $x$-coordinate of midpoint of $AB = \dfrac{2+6}{2} = 4$.

The axis of symmetry is $x = 4$. 1A

(ii) $CP : PB = (4 - 3) : (6 - 4)$ 1M

$= 1 : 2$ 1A

 **END OF PAPER**