REG-COT-2223-ASM-SET 3-MATH
Suggested solutions

Conventional Questions

1. (a) $a = \dfrac{(PQ)r}{2} + \dfrac{(QR)r}{2} + \dfrac{(PR)r}{2}$ 1M

$= \dfrac{r}{2}(PQ + QR + PR)$

$= \dfrac{pr}{2}$ 1

(b) $AB = \sqrt{(9-2)^2 + (18+6)^2} = 25$, $BC = \sqrt{32^2 + 24^2} = 40$ and $AC = 39$.

Let the radius of the inscribed circle of $\triangle ABC$ be r.

$\dfrac{(41-2)(18+6)}{2} = \dfrac{(25+40+39)r}{2}$ 1M

$r = 9$ 1A

Required y-coordinate $= -6 + 9 = 3$ 1A

2. (a) $f(x) = x^2 - 6kx + 12k^2 + 6$

$= (x^2 - 6x + 9k^2) + 3k^2 + 6$ 1M

$= (x - 3k)^2 + 3k^2 + 6$

The coordinates of vertex are $\left(3k,\ 3k^2 + 6\right)$. 1A

(b) $P\left(3k - 3,\ 3k^2 + 6\right)$ and $Q\left(3k + 3,\ -3k^2 - 6\right)$. 1A

Consider the the distances from point $(0,\ 6)$ to P and to Q.

$\sqrt{(3k-3)^2 + (3k^2)^2} = \sqrt{(3k+3)^2 + (-3k^2 - 6 - 6)^2}$ 1M

$-72k^2 - 36k - 144 = 0$

$\Delta = 36^2 - 4(-72)(-144) = -40\,176 < 0$ 1M

The equation has no real roots.

Thus, such point R does not exist. 1A

3. (a) $PB = PD$ and $\angle PBD = \dfrac{180^\circ - x}{2} = 90^\circ - \dfrac{x}{2}$ 1M

$\angle BAD = \angle PBD = 90^\circ - \dfrac{x}{2}$ 1A

$\angle ABC = 90^\circ$

$\angle AQB = 180^\circ - 90^\circ - \left(90^\circ - \dfrac{x}{2}\right) = \dfrac{x}{2}$ 1A

(b) (i) Since $PB = PD = PR$, we have $\angle BRD = \angle PDR$. 1M

$$\angle BRD + \angle PDR = x$$

$$\angle BRD = \frac{x}{2}$$ 1A

(ii) Note that $\angle BRD = \angle BQD = \dfrac{x}{2}$.

B, D, Q, R are concyclic. 1M

Since $PB = PD = PR$, P is the centre of the circle BDR,

and hence is the centre of the circle $BDQR$.

Thus, P is the centre of the circumcircle of $\triangle BDQ$.

The claim is agreed. 1A

4. (a) $\angle BPA = 90°$ *(given)*

$\angle GMA = 90°$ *(property of circumcentre)*

$= \angle BPA$

$BC // GM$ *(corr. ∠s equal)*

$\angle BDC = \angle MDG$ *(vert. opp. ∠s)*

$\angle CBD = \angle GMD$ *(alt. ∠s, BC//GM)*

$\triangle BCD \sim \triangle MGD$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(b) (i) $GM = \sqrt{r^2 - \left(\frac{a}{2}\right)^2} = \frac{1}{2}\sqrt{4r^2 - a^2}$ 1A

The coordinates of G are $\left(\frac{a}{2}, \frac{\sqrt{4r^2 - a^2}}{2}\right)$.

$$BG = r$$

$$\left(\frac{a}{2} - b\right)^2 + \left(\frac{\sqrt{4r^2 - a^2}}{2} - h\right)^2 = r^2$$ 1M

$$\left(\frac{\sqrt{4r^2 - a^2}}{2} - h\right)^2 = r^2 - \frac{(a - 2b)^2}{4}$$

Since $h > GM > 0$, $\frac{\sqrt{4r^2 - a^2}}{2} - h < 0$.

$$\frac{\sqrt{4r^2 - a^2}}{2} - h = -\sqrt{r^2 - \frac{(a - 2b)^2}{4}}$$

$$h = \frac{\sqrt{4r^2 - a^2}}{2} + \frac{\sqrt{4r^2 - (a - 2b)^2}}{\sqrt{4}}$$

$$= \frac{\sqrt{4r^2 - a^2 + 4ab - 4b^2} + \sqrt{4r^2 - a^2}}{2}$$ 1

(ii) $BD : DM = 2 : 1$ 1A

$BC : GM = BD : DM = 2 : 1$ and $BC = 2GM = \sqrt{4r^2 - a^2}$ 1M

$CP = h - \sqrt{4r^2 - a^2} = \frac{\sqrt{4r^2 - a^2 + 4ab - 4b^2} - \sqrt{4r^2 - a^2}}{2}$

(Slope of AB) $\times$ (slope of OC)

$= \dfrac{h-0}{b-a} \times \dfrac{CP}{b}$ 1M

$= \dfrac{1}{4b(b-a)}\left[(\sqrt{4r^2-a^2+4ab-4b^2})^2-(\sqrt{4r^2-a^2})^2\right]$

$= \dfrac{4b(a-b)}{4b(b-a)}$

$= -1$

Since $BC \perp OA$ and $OC \perp AB$, C is the orthocentre of $\triangle OAB$. 1

(iii) $h = \dfrac{\sqrt{4r^2-a^2+4ab-4b^2}+\sqrt{4r^2-a^2}}{2} = 49$

$CP = h - \sqrt{4r^2-a^2} = 31$

x-coordinate of $D = \dfrac{2\left(\frac{80}{2}\right)+31}{1+2} = 37$ 1M

Required area $= \dfrac{(40-31)(49)}{2} - \dfrac{(49-31)(37-31)}{2}$

$= \dfrac{333}{2}$ 1A

 END OF PAPER