REG-AOT-2223-ASM-SET 3-MATH
Suggested solutions

Multiple Choice Questions

1. B

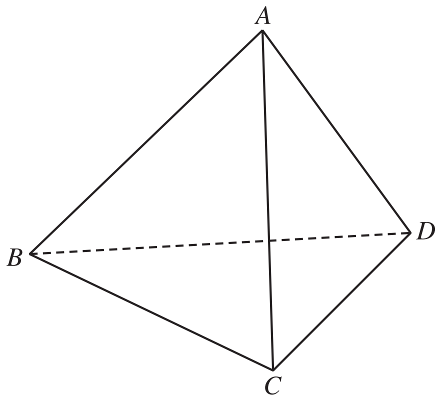
$PE = \frac{1}{2}\sqrt{4^2 + 4^2}$
$= \sqrt{8}$ cm
$PD = \sqrt{PE^2 + 4^2}$
$= 2\sqrt{6}$ cm

2. B

Refer to the figure. Let E be the midpoint of BC and length of each side be 12 cm.

In $\triangle AED$, $AE = DE = 12\sin 60° = 6\sqrt{3}$ cm
$12^2 = AE^2 + DE^2 - 2(AE)(DE)\cos\angle AED$

$\angle AED = \cos^{-1}\frac{1}{3}$



Let X be the projection of A on the plane BCD. Then X is the centroid of $\triangle BCD$ and it lies on DE.
In $\triangle AEX$, $\angle AXE = 90°$ and
height $= AE\sin\angle AED$
$= \sqrt{96}$ cm
Volume $= \frac{1}{2}(12)^2\sin 60°(\sqrt{96}) \times \frac{1}{3}$
$= 144\sqrt{2}\text{ cm}^3$

3. A

Required angle is $\angle PHF$.
$PF = 12 \times \frac{2}{1+2} = 8$ cm and $FH = \sqrt{8^2 + 6^2} = 10$ cm
$\tan\angle PHF = \frac{8}{10}$
$\angle PHF \approx 39°$

4. C

Let N be the midpoint of GH.
Required angle is MFN.
$MN = 24$ cm
$FN = \sqrt{5^2 + 8^2}$
$= \sqrt{89}$ cm
$\tan\angle MFN = \frac{24}{\sqrt{89}}$
$\angle MFN \approx 69°$

5. B

Required angle is $\angle YXH$.
Let $CH = 2$ cm.
We have $YH = \frac{2}{2} = 1$ cm and $XH = \sqrt{1^2 + 2^2} = \sqrt{5}$ cm.

$\tan \angle YXH = \frac{1}{\sqrt{5}}$

$\angle YXH \approx 24^\circ$

6. C

Note that $\theta = \angle BEG$.

$$EG = \sqrt{6^2 + 8^2} = 10\text{ cm}$$

$$\tan\theta = \frac{BG}{EG} = \frac{2}{5}$$

7. A

A. Angle between AC and $BCHG$ is $\angle ACB$.

B. Angle between DH and $BCHG$ is $\angle DHC$.

C. Angle between DG and $BCHG$ is $\angle DGC$.

D. Let Y be the midpoint of GH.
Angle between XB and $BCHG$ is $\angle XBY$.

By simple observation, we have $\angle ACB$ being the greatest angle among all.
The answer is A.

8. B

A. Angle between DH and $EFGH$ is $\angle DHE$.

B. Angle between CF and $EFGH$ is $\angle CFH$.

C. Note that the projection of M on $EFGH$ is the midpoint of HE.
Angle between MH and $EFGH$ is $\angle MHE$.

D. Note that the projection of K on $EFGH$ is the midpoint of EG.
Angle between KG and $EFGH$ is $\angle KGE$.

Comparing, we have $\angle CFH$ is the smallest.
The answer is B.

9. B

A. Required angle is $\angle EBF$.

B. Required angle is $\angle ENF$.

C. Let Q be a point on $ABGF$ such that PQ is perpendicular to $ABGF$.
Required angle is $\angle PFQ$, which is equal to $\angle FPE$.

D. Let K be the midpoint of BG.
Required angle is $\angle MNK$, which is equal to $\angle CAB$.

By simple observation, we have $\angle ENF$ being the greatest angle among all.
The answer is B.

10. B

Note that $GH = CF$ and $BF < AH < BH$.
We have $\dfrac{CF}{BF} > \dfrac{GH}{AH} > \dfrac{GH}{BH}$.
We have $\tan a > \tan c > \tan b$.
Thus, $a > c > b$.

11. D

Let E be a point on AC such that $VE \perp AC$.
Required angle is $\angle VED$.
Since the pyramid is symmetric about the plane VAC,
we have $\angle VEB = \angle VED = \dfrac{180°}{2} = 90°$.

12. D

Let K be a point on EG such that $AK \perp EG$.
Required angle is $\angle AKF$.
Consider the area of $\triangle EFG$.

$$\frac{1}{2}(8)(6) = \frac{1}{2}(EG)(FK)$$
$$24 = \frac{1}{2}\sqrt{8^2+6^2}(EK)$$
$$EK = 4.8\,\text{cm}$$

$$\tan\theta = \frac{12}{4.8}$$
$$\theta \approx 68°$$

13. D

Let E be a point on CD such that $BE \perp CD$. Then $AE \perp CD$ and $\theta = \angle AEB$.
$\angle BCD = \tan^{-1}\dfrac{15}{8}$, and $BE = BC\sin\angle BCD = \dfrac{120}{17}$ m
$\tan\theta = \dfrac{8}{BE} = \dfrac{17}{15}$

14. D

Let E be the midpoint of BC. Required angle is $\angle AED$.

Let length of side of each edge be 2.

Then $AE = \sqrt{2^2 - 1^2} = \sqrt{3}$ and $DE = AE = \sqrt{3}$.

In $\triangle AED$,

$$2^2 = (\sqrt{3})^2 + (\sqrt{3})^2 - 2(\sqrt{3})^2 \cos\angle AED$$
$$\angle AED \approx 71°$$

15. B

Let F be a point on AC such that $BF \perp AC$.

Required angle is $\angle BFD$.

Let $BC = 1$ cm.

Note that $\triangle ABC$ and $\triangle ACD$ are equilateral.

$DF = BF = 1\sin 60° = \dfrac{\sqrt{3}}{2}$ cm

$BD = \sqrt{1^2 + 1^2} = \sqrt{2}$ cm

$$BD^2 = DF^2 + BF^2 - 2(DF)(BF)\cos\angle BFD$$
$$\angle BFD \approx 109°$$

16. C

Let M and N be midpoint of AB and CD respectively.

Let X be the midpoint of MN such that VX is perpendicular to the plane $ABCD$.

We have $\theta = \angle MVN$ and $\dfrac{\theta}{2} = \angle VNX$.

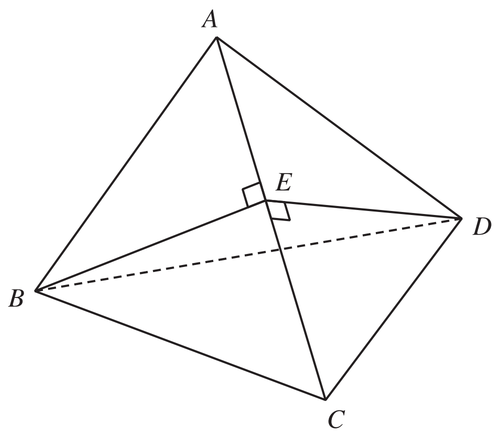
$DX = \dfrac{1}{2}\sqrt{4^2 + 2^2} = \sqrt{5}$ cm

$VX = \sqrt{7^2 - DX^2} = 2\sqrt{11}$ cm

$$\tan\frac{\theta}{2} = \frac{MX}{VX} = \frac{2}{2\sqrt{11}} = \frac{\sqrt{11}}{11}$$

17. D

Since $BE \perp AC$ and $DE \perp AC$, required angle is $\angle BED$.



18. C

Let K be a point on AM such that $DK \perp AM$.
Then $\alpha = \angle EMD$ and $\beta = \angle EKD$.

I. ✓. Since $\angle DKM = 90°$, we have $DK < DM$ and $\alpha < \beta$.

II. ✗. Since $\alpha < \beta$, we have $\cos\alpha > \cos\beta$.

III. ✓. $\angle DAM = \angle BMA = \tan^{-1}\dfrac{5}{2.5} \approx 63.4°$
$DK = AD\sin\angle DAM \approx 4.47\text{ cm}$
$\tan\beta = \dfrac{DE}{DK} = \dfrac{4}{\sqrt{5}}$

Conventional Questions

19. (a) $BC^2 = 30^2 + 30^2 - 2(30)(30)\cos 40°$ 1M

$BC \approx 20.5 \text{ cm}$ 1A

(b) Since $\triangle ABC$ is equilateral, the circumcentre of $\triangle ABC$ coincides with centroid of $\triangle ABC$.

$r = \frac{2}{3} \times BC \sin 60°$ 1M

$\approx 11.8 \text{ cm}$ 1A

(c) Required angle $= \cos^{-1} \frac{r}{30}$ 1M

$\approx 66.7°$ 1A

20. (a) (i) $17^2 = 23^2 + 10^2 - 2(23)(10)\cos \angle CAB$ 1M

$\angle CAB \approx 42.3°$ 1A

(ii) $\frac{AF}{\sin(180° - 58° - \angle CAB)} = \frac{10}{\sin 58°}$ 1M

$AF \approx 11.6 \text{ cm}$ 1A

(b) (i) $BF = 23 - AF \approx 11.4 \text{ cm}$

$AB^2 = AF^2 + BF^2 - 2(AF)(BF)\cos 75°$ 1M

$AB \approx 14.0 \text{ cm}$ 1A

(ii) $AP = AF \cos \angle FAC \approx 8.57 \text{ cm}$ 1M

$17^2 = 10^2 + (AB)^2 - 2(10)(AB)\cos \angle BAC$

$\angle BAC \approx 88.6°$

$BP^2 = AP^2 + AB^2 - 2(AP)(AB)\cos \angle BAC$

$BP \approx 16.2 \text{ cm}$ 1M

$AB^2 = AP^2 + BP^2 - 2(AP)(BP)\cos \angle APB$

$\angle APB \approx 59.6° \neq 90°$

The claim is disagreed. 1A

21. (a) $\angle BDC = \dfrac{360° - 150°}{2} = 105°$ 1A

$50^2 = 32^2 + BD^2 - 2(32)(BD)\cos 105°$ 1M

$0 = BD^2 - 64(BD)\cos 105° - 1476$

$BD \approx 31.0$ cm or -47.6 cm (rejected) 1A

(b) (i) $AC^2 = 32^2 + 32^2 - 2(32)^2 \cos 40°$ 1M

$AC \approx 21.9$ cm

Required distance is 21.9 cm. 1A

(ii) Let E be the midpoint of AC.

Required angle is $\angle BED$. 1A

$DE = 32\cos\dfrac{40°}{2} \approx 30.1$ cm 1M

$BE = \sqrt{50^2 - \left(\dfrac{AC}{2}\right)^2} \approx 48.8$ cm

$BD^2 = DE^2 + BE^2 - 2(DE)(BE)\cos\angle BED$

$\angle BED \approx 37.7°$ 1A

(iii) Area of $\triangle ACD = \dfrac{1}{2}(32)(32)\sin 40°$ 1M

$= 512 \sin 40°$ cm^2

Volume $= \dfrac{1}{3}(512\sin 40°)(BE \sin\angle BED)$ 1M

≈ 3270 cm^3 1A

Alternative method:

Volume $= \dfrac{1}{3}\left(\dfrac{1}{2}(AC)(BE)\right)(DE\sin\angle BED)$ 1M+1M

≈ 3270 cm^3 1A

(c) Let D' and P' be the projections of D and P on plane ABC respectively.

Since $\triangle BDD' \sim \triangle BPP'$, we have $\dfrac{PP'}{DD'} = \dfrac{BP}{BD} = \dfrac{1}{2}$. 1M

$$\frac{\text{volume of } ABCP}{\text{volume of } ABCD} = \frac{\frac{1}{3}(\text{area of } \triangle ABC)(PP')}{\frac{1}{3}(\text{area of } \triangle ABC)(DD')}$$

$$= \frac{PP'}{DD'} = \frac{1}{2}$$

The claim is agreed. 1A

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