M2REG-VEC-2223-ASM-SET 2-MATH
Suggested solutions

1. (a) $\mathbf{p} = 6\mathbf{i} + 3\mathbf{j}$ 1A

$\mathbf{q} = 3\mathbf{i} + 6\mathbf{j}$ 1A

(b) $\mathbf{p} - 2\mathbf{q} = (6-6)\mathbf{i} + (3-12)\mathbf{j}$ 1M

$= -9\mathbf{j}$

$\overrightarrow{AB} = 6\mathbf{j} = -\frac{2}{3}(\mathbf{p} - 2\mathbf{q}) = -\frac{2}{3}\mathbf{p} + \frac{4}{3}\mathbf{q}$ 1A

$\overrightarrow{BC} = \overrightarrow{AP} + \overrightarrow{PO}$

$= \frac{1}{2}\overrightarrow{AB} - \mathbf{p}$ 1M

$= -\frac{4}{3}\mathbf{p} + \frac{2}{3}\mathbf{q}$ 1A

2. (a) $\overrightarrow{OA} = 4\cos 120°\mathbf{i} + 4\sin 120°\mathbf{j} = -2\mathbf{i} + 2\sqrt{3}\mathbf{j}$ 1A

(b) $\overrightarrow{OB} = (-2\mathbf{i} + 2\sqrt{3}\mathbf{j}) + (-4\mathbf{i}) = -6\mathbf{i} + 2\sqrt{3}\mathbf{j}$ 1A

$\overrightarrow{AC} = (-4\mathbf{i}) - (-2\mathbf{i} + 2\sqrt{3}\mathbf{j}) = -2\mathbf{i} - 2\sqrt{3}\mathbf{j}$ 1A

3. (a) $\mathbf{p} = 6\mathbf{i} + 3\mathbf{j}$ 1A

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4. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= 6\mathbf{i} - 8\mathbf{j}$ 1A

(b) (i) $\overrightarrow{AC} = \left(\frac{3}{AB}\right)\overrightarrow{AB}$

$= \frac{3}{\sqrt{6^2 + 8^2}}(6\mathbf{i} - 8\mathbf{j})$ 1M

$= \frac{9}{5}\mathbf{i} - \frac{12}{5}\mathbf{j}$ 1A

(ii) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$= -\frac{11}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$ 1A

5. (a) $\overrightarrow{AP} = 2\overrightarrow{AB}$

$(-3-m)\mathbf{i} + (4-n)\mathbf{j} = 2[(2-m)\mathbf{i} + (-1-n)\mathbf{j}]$

$-(m+3)\mathbf{i} + (4-n)\mathbf{j} = 2(2-m)\mathbf{i} - 2(n+1)\mathbf{j}$

We have $-(m+3) = 2(2-m)$ and $4-n = -2(n+1)$. 1M

Solving, we have $m = 7$ and $n = -6$. 1A+1A

(b) $\overrightarrow{AB} = -5\mathbf{i} + 5\mathbf{j}$

Required vector $= \dfrac{1}{AB}\overrightarrow{AB}$

$= \dfrac{1}{\sqrt{5^2+5^2}}(-5\mathbf{i} + 5\mathbf{j})$ 1M

$= -\dfrac{1}{\sqrt{2}}\mathbf{i} + \dfrac{1}{\sqrt{2}}\mathbf{j}$ 1A

(c) Required vector $= -\dfrac{\sqrt{8}}{AB}\overrightarrow{AB}$

$= -\sqrt{8}\left(-\dfrac{1}{\sqrt{2}}\mathbf{i} + \dfrac{1}{\sqrt{2}}\mathbf{j}\right)$ 1M

$= 2\mathbf{i} - 2\mathbf{j}$ 1A

6. (a) $2\mathbf{u} + \mathbf{v} = (12-1)\mathbf{i} + (4-4)\mathbf{j}$ 1M

$\mathbf{i} = \dfrac{2}{11}\mathbf{u} + \dfrac{1}{11}\mathbf{v}$ 1A

$\mathbf{v} = -\left(\dfrac{2}{11}\mathbf{u} + \dfrac{1}{11}\mathbf{v}\right) - 4\mathbf{j}$

$\mathbf{j} = -\dfrac{1}{22}\mathbf{u} - \dfrac{3}{11}\mathbf{v}$ 1A

(b) $4\mathbf{i} - 5\mathbf{j} = a\mathbf{u} + b\mathbf{v}$

$\left(4\times\dfrac{2}{11} - 5\times\left(-\dfrac{1}{22}\right)\right)\mathbf{u} + \left(4\times\dfrac{1}{11} - 5\times\left(-\dfrac{3}{11}\right)\right)\mathbf{v} = a\mathbf{u} + b\mathbf{v}$ 1M

$\dfrac{21}{22}\mathbf{u} + \dfrac{19}{11}\mathbf{v} = a\mathbf{u} + b\mathbf{v}$

Therefore, $a = \dfrac{21}{22}$ and $b = \dfrac{19}{11}$. 1A

7. (a) $\overrightarrow{AB} = -\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ 1A

(b) $\overrightarrow{BC} = r\mathbf{i} + (s-2)\mathbf{j} + (-r-6)\mathbf{k}$ 1A

(c) If A, B and C are collinear, then $\dfrac{r}{-1} = \dfrac{-r-6}{3} = \dfrac{s-2}{1}$. 1M

Solving, we have $r = 3$ and $s = -1$. 1A

The coordinates of C are $(3,\ -1,\ -4)$. 1A

8. $\overrightarrow{AB} = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$

$\overrightarrow{CD} = 6\mathbf{i} - \lambda\mathbf{j} + (2\mu+2)\mathbf{k}$ 1A

If $\overrightarrow{AB} // \overrightarrow{CD}$, then $\dfrac{-2}{6} = \dfrac{1}{-\lambda} = \dfrac{-4}{2\mu+2}$. 1M

Solving, we have $\lambda = 3$ and $\mu = 5$. 1A

9. $\overrightarrow{CD} = \overrightarrow{BA} = 22\mathbf{i} - \mathbf{j}$ 1A

$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD} = 25\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ 1M

The coordinates of D are $(25,\ -2,\ 1)$. 1A

10. $\overrightarrow{AB} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\left|\overrightarrow{AB}\right| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$ 1M

$\overrightarrow{DC} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} = \overrightarrow{AB}$ and $\left|\overrightarrow{DC}\right| = \sqrt{14}$

$\overrightarrow{AD} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\left|\overrightarrow{AD}\right| = \sqrt{2^2 + 3^1 + 1^2} = \sqrt{14}$

$\overrightarrow{BC} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} = \overrightarrow{AD}$ and $\left|\overrightarrow{BC}\right| = \sqrt{14}$ 1A

Since the lengths of four sides are equal, $ABCD$ is a rhombus. 1

11. (a) $\overrightarrow{PQ} = -5\mathbf{i} + \mathbf{j} + 6\mathbf{k}$ 1A

$\overrightarrow{PR} = -\frac{15}{4}\mathbf{i} + \frac{3}{4}\mathbf{j} + \frac{9}{2}\mathbf{k}$ 1A

(b) $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$

$= (2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}) + \left(-\frac{15}{4}\mathbf{i} + \frac{3}{4}\mathbf{j} + \frac{9}{2}\mathbf{k}\right)$ 1M

$= -\frac{7}{4}\mathbf{i} + \frac{15}{4}\mathbf{j} - \frac{1}{2}\mathbf{k}$ 1A

(c) When $\overrightarrow{PS} = 3\overrightarrow{PQ}$,

$\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{PS}$

$= (2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}) + (-15\mathbf{i} + 3\mathbf{j} + 18\mathbf{k})$

$= -13\mathbf{i} + 6\mathbf{j} + 13\mathbf{k}$ 1M

When $\overrightarrow{PS} = -3\overrightarrow{PQ}$,

$\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{PS}$

$= (2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}) + (15\mathbf{i} - 3\mathbf{j} - 18\mathbf{k})$

$= 17\mathbf{i} - 23\mathbf{k}$

Required coordinates are $(-13,\ 6,\ 13)$ or $(17,\ 0,\ -23)$. 1A

12. Let $OQ : QB = t : 1 - t$.

$\overrightarrow{CQ} = t\overrightarrow{CB} + (1 - t)\overrightarrow{CO}$ 1M

$\overrightarrow{CP} = \overrightarrow{CO} + \frac{1}{2}\overrightarrow{OA} = \overrightarrow{CO} + \frac{1}{2}\overrightarrow{CB}$ 1A

Since C, Q and P are collinear,

$$\frac{t}{\left(\frac{1}{2}\right)} = \frac{1 - t}{1}$$ 1M

$$t = \frac{1}{3}$$

Therefore, $OQ : QB = \frac{1}{3} : \frac{2}{3} = 1 : 2$. 1A

13. (a) $\overrightarrow{AE} = k\overrightarrow{AC} = 3k\overrightarrow{AB} + 2k\overrightarrow{AD} = 3k\mathbf{u} + 2k\mathbf{v}$ 1A

(b) $\overrightarrow{AE} = \frac{1}{r+1}\overrightarrow{AB} + \frac{r}{r+1}\overrightarrow{AD}$ 1M

$= \frac{1}{r+1}\mathbf{u} + \frac{r}{r+1}\mathbf{v}$ 1A

(c) By (a) and (b), $3k = \frac{1}{r+1}$ and $2k = \frac{r}{r+1}$. 1M

$\frac{2k}{3k} = \frac{r}{r+1} \div \frac{1}{r+1}$ 1M

$r = \frac{2}{3}$ 1A

When $r = \frac{2}{3}$, $k = \frac{1}{3} \times \frac{1}{r+1} = \frac{1}{5}$. 1A

(d) $\overrightarrow{BE} = \overrightarrow{AE} - \overrightarrow{AB} = \frac{3}{5}\mathbf{u} + \frac{2}{5}\mathbf{v} - \mathbf{u} = -\frac{2}{5}\mathbf{u} + \frac{2}{5}\mathbf{v}$ 1A

14. (a) $\overrightarrow{AR} = \frac{1}{4}\overrightarrow{AB} + \frac{3}{4}\overrightarrow{AQ}$ 1M

$= \frac{3}{4}\mathbf{u} + \frac{1}{4}\mathbf{v}$ 1A

(b) $\overrightarrow{AP} = \frac{1}{2}(3\overrightarrow{AQ}) + \frac{1}{2}\overrightarrow{AB} = \frac{3}{2}\mathbf{u} + \frac{1}{2}\mathbf{v}$ 1A

(c) $\overrightarrow{AP} = \frac{3}{2}\mathbf{u} + \frac{1}{2}\mathbf{v} = 2\overrightarrow{AR}$ 1M

Thus, A, R and P are collinear. 1

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