M2REG-SOLE-2223-ASM-SET 1-MATH
Suggested solutions

1. $\begin{vmatrix} k & -3 & 3 \\ k+1 & 1 & 1 \\ k+2 & 0 & 7 \end{vmatrix} = k\begin{vmatrix} 1 & 1 \\ 0 & 7 \end{vmatrix} - (k+1)\begin{vmatrix} -2 & 3 \\ 0 & 7 \end{vmatrix} + (k+2)\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix}$

   $= 16k + 4$ 1A

   The system has a unique solution if $16k + 4 \neq 0$. 1M

   Thus, $k < -\frac{1}{4}$ or $k > \frac{1}{4}$. 1A

2. $\begin{vmatrix} k & k-1 & 2 \\ 1 & k & -1 \\ 5 & 3 & 1 \end{vmatrix} = k^2 - 13k + 12$ 1A

   $= (k-1)(k-12)$

   The system has a unique solution if $(k-1)(k-12) \neq 0$. 1M

   Thus, $k < 1$ or $1 < k < 12$ or $k > 12$. 1A

3. $\Delta = \begin{vmatrix} 2 & -2 & 1 \\ 1 & -1 & 2 \\ 4 & -1 & 2 \end{vmatrix} = -9$ 1A

   $x = \dfrac{\begin{vmatrix} 3 & -2 & 1 \\ 6 & -1 & 2 \\ 9 & -1 & 2 \end{vmatrix}}{-9}$ 1M

   $= \dfrac{-9}{-9}$

   $= 1$

   $y = \dfrac{\begin{vmatrix} 2 & 3 & 1 \\ 1 & 6 & 2 \\ 4 & 9 & 2 \end{vmatrix}}{-9}$

   $= \dfrac{-9}{-9}$

   $= 1$

   $z = \dfrac{\begin{vmatrix} 2 & -2 & 3 \\ 1 & -1 & 6 \\ 4 & -1 & 9 \end{vmatrix}}{-9}$

   $= \dfrac{-27}{-9}$

   $= 3$

   Thus, $x = 1$, $y = 1$ and $z = 3$. 1A

4. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 1 & -4 \end{vmatrix} = -3$ 1A

$x = \dfrac{\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ -5 & 1 & -4 \end{vmatrix}}{-3}$ 1M

$= \dfrac{-1}{-3}$

$= \dfrac{1}{3}$

$y = \dfrac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 4 \\ 3 & -5 & -4 \end{vmatrix}}{-3}$

$= \dfrac{2}{-3}$

$= -\dfrac{2}{3}$

$z = \dfrac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 1 & -5 \end{vmatrix}}{-3}$

$= \dfrac{-4}{-3}$

$= \dfrac{4}{3}$

Thus, $x = \dfrac{1}{3}$, $y = -\dfrac{2}{3}$ and $z = \dfrac{4}{3}$. 1A

5. (a) $\begin{vmatrix} a & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 3 \end{vmatrix} = 6a + 9 - 4 - 24 - a + 9$

$= 5(a - 2)$

The system has a unique solution, so $5(a-2) \neq 0$. 1M

We have $a \neq 2$.

Thus, $a < 2$ or $a > 2$. 1A

(b) $x = \dfrac{\begin{vmatrix} 2 & -3 & 4 \\ 7 & 2 & -1 \\ -1 & -1 & 3 \end{vmatrix}}{5(a-2)}$ 1M

$= \dfrac{10}{a-2}$ 1A

$y = \dfrac{\begin{vmatrix} a & 2 & 4 \\ 1 & 7 & -1 \\ 3 & -1 & 3 \end{vmatrix}}{5(a-2)}$

$= \dfrac{4(a-5)}{a-2}$

$z = \dfrac{\begin{vmatrix} a & -3 & 2 \\ 1 & 2 & 7 \\ 3 & -1 & -1 \end{vmatrix}}{5(a-2)}$

$= \dfrac{a-16}{a-2}$ 1A

6. (a) $\Delta = \begin{vmatrix} k & 0 & -1 \\ k+2 & -1 & 2 \\ 1 & k & -1 \end{vmatrix}$

$= -3k^2 - k - 1$ 1A

$= -3\left(k + \frac{1}{6}\right)^2 - \frac{11}{12}$

$\leq -\frac{11}{12}$

Thus, $\Delta \neq 0$. 1M

The system (E) has a unique solution for all values of k. 1

(b) $x = \dfrac{\begin{vmatrix} 1 & 0 & -1 \\ -1 & -1 & 2 \\ 1 & k & -1 \end{vmatrix}}{-3k^2 - k - 1}$ 1M

$= \dfrac{-k}{-3k^2 - k - 1}$

$= \dfrac{k}{3k^2 + k + 1}$ 1A

$y = \dfrac{\begin{vmatrix} k & 1 & -1 \\ k+2 & -1 & 2 \\ 1 & 1 & -1 \end{vmatrix}}{-3k^2 - k - 1}$

$= \dfrac{1-k}{-3k^2 - k - 1}$

$= \dfrac{k-1}{3k^2 + k + 1}$

$z = \dfrac{\begin{vmatrix} k & 0 & 1 \\ k+2 & -1 & -1 \\ 1 & k & 1 \end{vmatrix}}{-3k^2 - k - 1}$

$= \dfrac{2k^2 + k + 1}{-3k^2 - k - 1}$

$= -\dfrac{2k^2 + k + 1}{3k^2 + k + 1}$ 1A

(c) $\left(\dfrac{k-1}{3k^2 + k + 1}\right)^2 = 2\left(\dfrac{k}{3k^2 + k + 1}\right)\left(-\dfrac{2k^2 + k + 1}{3k^2 + k + 1}\right)$ 1M

$(k-1)^2 = -2k(2k^2 + k + 1)$

$4k^3 + 3k^2 + 1 = 0$

$(k+1)(4k^2 - k + 1) = 0$

$k = -1$ or $4k^2 - k + 1 = 0$ (rejected) 1A

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