REV-LOG-2223-ASM-SET 1-MATH
建議題解

多項選擇題

1. A

$$\begin{aligned}1000^{\log(2a)} &= 10^{3\log(2a)}\\ &= 10^{\log(2a)^3}\\ &= (2a)^3\\ &= 8a^3\end{aligned}$$

2. A

$$\begin{aligned}100^{\log a} &= 10^{2\log a}\\ &= (10^{\log a})^2\\ &= a^2\end{aligned}$$

3. B

$$\begin{aligned}10^{2\log a} &= (10^{\log a})^2\\ &= a^2\end{aligned}$$

4. D

$$\begin{aligned}\log\frac{y^x}{x^y} &= \log y^x - \log x^y\\ &= x\log y - y\log x\end{aligned}$$

5. B

$$\begin{aligned}\log\sqrt[4]{x+y} &= \frac{1}{4}\log(x+y)\\ &= \frac{1}{4}\log\frac{x^2-y^2}{x-y}\\ &= \frac{1}{4}[\log(x^2-y^2) - \log(x-y)]\\ &= \frac{1}{4}(10-2)\\ &= 2\end{aligned}$$

6. A

$$\log\sqrt[3]{x} = k$$
$$\frac{1}{3}\log x = k$$
$$-2\log x = -6k$$
$$\log\frac{1}{x^2} = -6k$$

7. C

$$\frac{\log a^6 - \log \sqrt[3]{a^2}}{2\log a^4} = \frac{6\log a - \frac{2}{3}\log a}{8\log a}$$
$$= \frac{\frac{16}{3}\log a}{8\log a}$$
$$= \frac{2}{3}$$

8. A

$$\frac{\log x^3}{\log \frac{x}{2} + \log x + \log 2} = \frac{3\log x}{\log x - \log 2 + \log x + \log 2}$$
$$= \frac{3\log x}{2\log x}$$
$$= \frac{3}{2}$$

9. B

$10^c = ab$

$c = \log ab$

$= \log a + \log b$

10. D

I. ✗ ◦ $\log a + \log b = \log ab \neq \log(a + b)$

II. ✓ ◦

III. ✓ ◦ $\log \frac{1}{a} + \log \frac{1}{b} = -\log a - \log b$
$= -(\log a + \log b)$
$= -\log ab$

11. A

$$\frac{\log\left(1000x^3\right)}{\log \frac{1}{10x}} = \frac{\log 1000 + 3\log x}{-\log 10 - \log x}$$
$$= \frac{3 + 3\log x}{-1 - \log x}$$
$$= \frac{3(1 + \log x)}{-(1 + \log x)}$$
$$= -3$$

12. D

$p = \log 2$ 及 $q = \log 3$ ◦

$\log \frac{5}{3} = \log 10 - \log 2 - \log 3 = 1 - p - q$

13. A

可得 $x = \log 2$ 及 $y = \log 7$。

$$\log \frac{50}{7} = \log \frac{100}{2(7)}$$
$$= \log 100 - \log 2 - \log 7$$
$$= 2 - x - y$$

14. D

可得 $m = \log 2$ 及 $n = \log 3$。

$$\log \sqrt{15} = \frac{1}{2} \log \frac{3 \times 10}{2}$$
$$= \frac{1}{2}(\log 3 + \log 10 - \log 2)$$
$$= \frac{1}{2}(n - m + 1)$$

15. C

可得 $a = \log 3$ 及 $b = \log 4 = 2 \log 2$。

$$\log \frac{5}{6} = \log \frac{10}{2^2 \times 3}$$
$$= \log 10 - 2 \log 2 - \log 3$$
$$= 1 - a - b$$

16. D

$$100^a = 2$$
$$10^{2a} = \frac{100}{50}$$
$$2a = \log \frac{100}{50}$$
$$2a = \log 100 - \log 50$$
$$\log 50 = 2 - 2a$$

17. C

$$\log 135 = \log\left(3^3 \times 5\right)$$
$$= 3 \log 3 + \log \frac{10}{2}$$
$$= 3 \log 3 + \log 10 - \log 2$$
$$= 3y - x + 1$$

18. C

$$\log 8 = a$$
$$3\log 2 = a$$
$$\log 2 = \frac{a}{3}$$
$$\log \frac{1}{16} = \log \frac{1}{2^4}$$
$$= -4\log 2$$
$$= -\frac{4a}{3}$$

19. A

$$\log_{\frac{1}{\sqrt{5}}} 25 = \frac{\log 25}{\log \frac{1}{\sqrt{5}}}$$
$$= \frac{2\log 5}{-\frac{1}{2}\log 5}$$
$$= -4$$

20. C

可得 $\log b > \log a > 0$。
故此，$\log_a b = \dfrac{\log b}{\log a} > 1$。

21. D

$$\log_a b^3 \cdot \log_b c^3 = \frac{3\log b}{\log a} \times \frac{3\log c}{\log b}$$
$$= 9 \times \frac{\log c}{\log a}$$
$$= 9\log_a c$$

22. C

$$\log_{2\sqrt{2}} 64 = \frac{\log 64}{\log 2\sqrt{2}}$$
$$= \frac{\log 2^6}{\log 2^{1+\frac{1}{2}}}$$
$$= \frac{6\log 2}{\frac{3}{2}\log 2}$$
$$= 4$$

23. B

$$\log x^2 = 2\log 2 + \log 9$$
$$= \log(4 \times 9)$$
$$x^2 = 36$$
$$x = \pm 6$$

24. D

$$\log_2(2x+1) = \log_4(x+11)$$
$$\frac{\log(2x+1)}{\log 2} = \frac{\log(x+11)}{2\log 2}$$
$$\log(2x+1)^2 = \log(x+11)$$
$$(2x+1)^2 = x+11$$
$$4x^2 + 3x - 10 = 0$$
$$x = \frac{5}{4} \quad \text{或} \quad -2 \text{（捨去）}$$

25. A

$$\log 10a = 2\log a + 4\log b$$
$$\log 10a = \log\left(a^2b^4\right)$$
$$10a = a^2b^4$$
$$10 = ab^4$$
$$a = \frac{10}{b^4}$$

26. B

$$\log(a-2x) = 3$$
$$a - 2x = 10^3$$
$$x = \frac{1}{2}(a-1000)$$

27. D

$$\log_2 y - \log_2 x = 6.3 - 2.3$$
$$\log_2 \frac{y}{x} = 4$$
$$\frac{y}{x} = 2^4$$
$$y = 16x$$

28. A

$$\frac{1}{y} = \log_5 x$$
$$\frac{1}{y} = \frac{\log x}{\log 5}$$
$$y = \frac{\log 5}{\log x}$$
$$= \log_x 5$$

29. C

$$\log(3x+4) = \log\frac{x}{2} + 1$$
$$\log(3x+4) = \log\frac{x}{2} + \log 10$$
$$3x+4 = \frac{x}{2} \times 10$$
$$x = 2$$

30. C

繪畫直線 $x = 1$。交點為 $(1,\ b)$ 及 $(1,\ c)$。

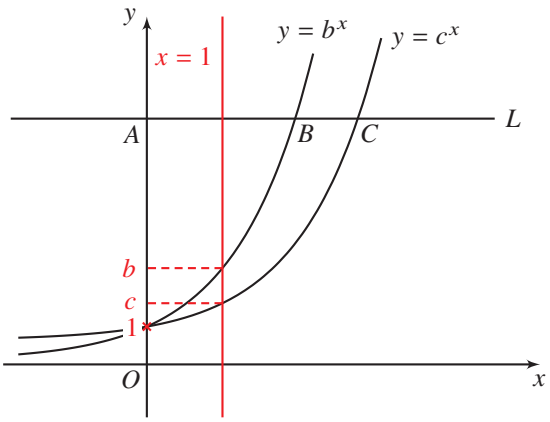
I. ✗。$b > c$

II. ✓。$b > c > 1 \quad \Rightarrow \quad bc > 1$

III. ✓。設 $L : y = k$。

$B\left(\log_b k,\ k\right)$ 及 $C\left(\log_c k,\ k\right)$

$$\frac{AB}{AC} = \frac{\log_b k}{\log_c k}$$
$$= \frac{\log k}{\log b} \div \frac{\log k}{\log c}$$
$$= \frac{\log c}{\log b}$$
$$= \log_b c$$



結構式試題

31. $\log(5x^2) - 2\log\frac{1}{\sqrt[3]{x}} + \log(2x) = \log 5 + 2\log x + \frac{2}{3}\log x + \log 2 + \log x$ 1M

$= \log(5 \times 2) + \frac{11}{3}\log x$ 1M

$= 1 + \frac{11}{3}\log x$ 1A

32. $\log_2(2x - 15) + \log_2(x - 6) = 1$

$\log_2[(2x - 15)(x - 6)] = 1$ 1M

$(2x - 15)(x - 6) = 2^1$ 1M

$2x^2 - 27x + 88 = 0$

$x = 8$ 或 $\frac{11}{2}$（捨去） 1A

33. $\log_3(x + 1) - \log_3(2x - 3) = 2$

$\log\frac{x + 1}{2x - 3} = 2$ 1M

$\frac{x + 1}{2x - 3} = 3^2$ 1M

$x + 1 = 18x - 27$

$x = \frac{28}{17}$ 1A

34. 從第一條方程，

$3^{x-1}2^{x-1} = 6^y$

$6^{x-1} = 6^y$

$x - 1 = y$ 1M

代 $y = x - 1$ 至第二條方程，

$\log_3(x - 2) - \log_3(x - 1) = -1$ 1M

$\log_3\frac{x - 2}{x - 1} = -1$

$\frac{x - 2}{x - 1} = 3^{-1}$ 1M

$3x - 6 = x - 1$

$x = \frac{5}{2}$

當 $x = \frac{5}{2}$，$y = \frac{5}{2} - 1 = \frac{3}{2}$。

所求之解為 $x = \frac{5}{2}$ 及 $y = \frac{3}{2}$。 1A