

M2REG-VEC-2223-ASM-SET 1-MATH

建議題解

1. (a) $\overrightarrow{SR} = \overrightarrow{SD} + \overrightarrow{DR}$
 $= \frac{2}{3}\mathbf{p} + \frac{3}{4}\mathbf{q}$ 1A
 - (b) $\overrightarrow{QR} = \overrightarrow{QC} + \overrightarrow{CR}$
 $= \frac{2}{3}\mathbf{p} - \frac{1}{4}\mathbf{q}$ 1A
 - (c) $\overrightarrow{PS} = \overrightarrow{PA} + \overrightarrow{AS}$
 $= \frac{1}{3}\mathbf{p} - \frac{1}{2}\mathbf{q}$ 1A
 - (d) $\overrightarrow{QP} = \overrightarrow{QB} + \overrightarrow{BP}$
 $= -\frac{1}{3}\mathbf{p} - \frac{1}{2}\mathbf{q}$ 1A
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2. (a) $\overrightarrow{AF} = \frac{2}{3}(\overrightarrow{AB} + \overrightarrow{BC})$
 $= \frac{2}{3}\mathbf{p} + \frac{2}{3}\mathbf{q}$ 1A
 - (b) $\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE}$
 $= \frac{3}{4}\mathbf{p} + \frac{2}{5}\mathbf{q}$ 1A
 - (c) $\overrightarrow{DF} = \overrightarrow{AF} - \overrightarrow{AD}$
 $= \overrightarrow{AF} - \frac{1}{4}\overrightarrow{AB}$
 $= \left(\frac{2}{3}\mathbf{p} + \frac{2}{3}\mathbf{q}\right) - \frac{1}{4}\mathbf{p}$
 $= \frac{5}{12}\mathbf{p} + \frac{2}{3}\mathbf{q}$ 1A
 - (d) $\overrightarrow{FE} = \overrightarrow{DE} - \overrightarrow{DF}$
 $= \frac{1}{3}\mathbf{p} - \frac{4}{15}\mathbf{q}$ 1A
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3. (a) $\overrightarrow{BE} = \overrightarrow{BA} + \frac{1}{2}\overrightarrow{AC} = -\mathbf{p} + \frac{1}{2}\mathbf{q}$ 1A
 $\overrightarrow{CD} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB} = -\mathbf{q} + \frac{1}{2}\mathbf{p}$ 1A
 - (b) $2\overrightarrow{BE} + \overrightarrow{CD} = -2\mathbf{p} + \frac{1}{2}\mathbf{p}$ 1M
 $\mathbf{p} = -\frac{4}{3}\overrightarrow{BE} - \frac{2}{3}\overrightarrow{CD}$ 1A
 $\overrightarrow{BE} + 2\overrightarrow{CD} = \frac{1}{2}\mathbf{q} - 2\mathbf{q}$
 $\mathbf{q} = -\frac{2}{3}\overrightarrow{BE} - \frac{4}{3}\overrightarrow{CD}$ 1A

4. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -5\mathbf{a} + 5\mathbf{b}$ 1A
- $$\overrightarrow{OP} = \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AB} = 5\mathbf{a} + \frac{3}{4}(-5\mathbf{a} + 5\mathbf{b}) = \frac{5}{4}\mathbf{a} + \frac{15}{4}\mathbf{b}$$
- 1A
- $$\overrightarrow{MN} = \frac{1}{2}\overrightarrow{AP}$$
- 1M
- $$= \frac{1}{2}\left(\frac{3}{4}\overrightarrow{AB}\right)$$
- $$= -\frac{15}{8}\mathbf{a} + \frac{15}{8}\mathbf{b}$$
- 1A
- (b) $\overrightarrow{MC} = -\frac{5}{2}\mathbf{a} + k\mathbf{b}$
- $$\frac{\left(-\frac{15}{8}\right)}{\left(-\frac{5}{2}\right)} = \frac{\left(\frac{15}{8}\right)}{k}$$
- 1M
- $$k = \frac{5}{2}$$
- 1A
5. (a) $\overrightarrow{EF} = \frac{1}{5}\overrightarrow{AC} - \frac{1}{4}\overrightarrow{AD} = \frac{1}{5}(\mathbf{a} + \mathbf{b}) - \frac{1}{4}\mathbf{b} = \frac{1}{5}\mathbf{a} - \frac{1}{20}\mathbf{b}$ 1A
- $$\overrightarrow{FB} = \overrightarrow{AB} - \overrightarrow{AF} = \mathbf{a} - \frac{1}{5}(\mathbf{a} + \mathbf{b}) = \frac{4}{5}\mathbf{a} - \frac{1}{5}\mathbf{b}$$
- 1A
- (b) $\overrightarrow{FB} = \frac{4}{5}\mathbf{a} - \frac{1}{5}\mathbf{b} = 4\left(\frac{1}{5}\mathbf{a} - \frac{1}{20}\mathbf{b}\right) = 4\overrightarrow{EF}$ 1M
- 故此， $FB \parallel EF$ 及 E 、 F 和 B 共線，且 $EF : FB = 1 : 4$ 。 1+1A
6. (a) $\overrightarrow{SP} = \overrightarrow{SA} + \overrightarrow{AP}$
- $$= \frac{1}{2}\overrightarrow{DA} + \frac{1}{2}\overrightarrow{AB}$$
- 1M
- $$= \frac{1}{2}(\overrightarrow{DA} + \overrightarrow{AB})$$
- $$2\overrightarrow{SP} = \overrightarrow{DB}$$
- 1
- (b) $\overrightarrow{RQ} = \overrightarrow{RC} + \overrightarrow{CQ}$
- $$= \frac{1}{2}\overrightarrow{DC} + \frac{1}{2}\overrightarrow{CB}$$
- 1M
- $$= \frac{1}{2}(\overrightarrow{DC} + \overrightarrow{CB})$$
- $$\overrightarrow{RQ} = \frac{1}{2}\overrightarrow{DB}$$
- $$= \overrightarrow{SP}$$
- 可得 $RQ \parallel SP$ 及 $RQ = SP$ 。
- 因此， $PQRS$ 為平行四邊形。 1

7. (a) $\overrightarrow{CB} = \overrightarrow{AB} - \overrightarrow{AC}$
 $= 4\mathbf{a} - 4\mathbf{b}$ 1A
 $\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE}$
 $= \overrightarrow{AB} - \frac{1}{2}\overrightarrow{CB}$
 $= 4\mathbf{a} - \frac{1}{2}(4\mathbf{a} - 4\mathbf{b})$
 $= 2\mathbf{a} + 2\mathbf{b}$ 1A
 $\overrightarrow{FD} = \overrightarrow{AD} - \overrightarrow{AF}$
 $= \frac{1}{4}\overrightarrow{AB} - \frac{1}{4}\overrightarrow{AC}$
 $= \mathbf{a} - \mathbf{b}$ 1A
- (b) $\overrightarrow{AG} = \overrightarrow{AD} + \frac{1}{2}\overrightarrow{DF}$
 $= \frac{1}{4}(4\mathbf{a}) - \frac{1}{2}(\mathbf{a} - \mathbf{b})$
 $= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ 1M
 $\overrightarrow{AG} = k\overrightarrow{AE}$
 $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} = 2k\mathbf{a} + 2k\mathbf{b}$ 1M
 $k = \frac{1}{4}$ 1A
8. (a) $3\mathbf{x} = \mathbf{y}$
 $6\mathbf{a} + 3n\mathbf{b} = 2k\mathbf{a} + 3\mathbf{b}$
 故此, $\begin{cases} 6 = 2k \\ 3n = 3 \end{cases}$ 1M
 因此, $k = 3$ 及 $n = 1$ 1A
- (b) $3\mathbf{x} = 5\mathbf{y}$
 $3k\mathbf{a} + 3n\mathbf{b} = 15\mathbf{a} + \frac{30}{k}\mathbf{b}$
 故此, $\begin{cases} 3k = 15 \\ 3n = \frac{30}{k} \end{cases}$ 1M
 因此, $k = 5$ 及 $n = 2$ 1A
9. $\mathbf{x} = 4\mathbf{y}$
 $k^2\mathbf{a} + n^2\mathbf{b} = (4k - 4)\mathbf{a} - (8n + 16)\mathbf{b}$
 $k^2 = 4k - 4$ 及 $n^2 = -(8n + 16)$ 1M
 $k^2 - 4k + 4 = 0$ $n^2 + 8n + 16 = 0$
 $k = 2$ $n = -4$ 1A+1A

$$10. \quad (a) \quad \overrightarrow{AQ} = \frac{1}{5}\overrightarrow{AC} = \frac{1}{5}(\overrightarrow{AB} + \overrightarrow{AD}) = \frac{1}{5}\mathbf{a} + \frac{1}{5}\mathbf{b} \quad 1A$$

$$\overrightarrow{DQ} = \overrightarrow{AQ} - \overrightarrow{AD} = \frac{1}{5}\mathbf{a} - \frac{4}{5}\mathbf{b} \quad 1A$$

$$\overrightarrow{DP} = \overrightarrow{DA} + \frac{1}{4}\overrightarrow{AB} = \frac{1}{4}\mathbf{a} - \mathbf{b} \quad 1A$$

$$(b) \quad \overrightarrow{DQ} = r\overrightarrow{DP}$$

$$\frac{1}{5}\mathbf{a} - \frac{4}{5}\mathbf{b} = \frac{r}{4}\mathbf{a} - r\mathbf{b}$$

$$\text{故此, } \begin{cases} \frac{1}{5} = \frac{r}{4} \\ -\frac{4}{5} = -r \end{cases} \quad 1M$$

$$\text{因此, } r = \frac{4}{5} \quad 1A$$

$$11. \quad (a) \quad \overrightarrow{OP} = h\left(\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = h\mathbf{a} + \frac{h}{2}\mathbf{b} \quad 1A$$

$$(b) \quad (i) \quad \overrightarrow{MP} = k\overrightarrow{MA} = k\left(\frac{1}{2}\mathbf{a} - \mathbf{b}\right) = \frac{k}{2}\mathbf{a} - k\mathbf{b} \quad 1A$$

$$(ii) \quad \overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = \frac{1}{2}\mathbf{a} + \mathbf{b} + \frac{k}{2}\mathbf{a} - k\mathbf{b} = \frac{1+k}{2}\mathbf{a} + (1-k)\mathbf{b} \quad 1A$$

$$(c) \quad \text{利用 (a) 及 (b), } \begin{cases} h = \frac{1+k}{2} \\ \frac{h}{2} = 1-k \end{cases} \quad 1M$$

$$\text{求解後, 可得 } h = \frac{4}{5} \text{ 及 } k = \frac{3}{5} \quad 1A$$

$$\text{因此, } h:k = 4:3 \quad 1A$$