

M2REG-LIM-2223-ASM-SET 1-MATH

建議題解

$$\begin{aligned}
 1. \quad \lim_{x \rightarrow 4} \frac{x^2 - 3x - 4}{x^2 - 6x + 8} &= \lim_{x \rightarrow 4} \frac{(x - 4)(x + 1)}{(x - 4)(x - 2)} && 1M \\
 &= \lim_{x \rightarrow 4} \frac{x + 1}{x - 2} && 1M \\
 &= \frac{4 + 1}{4 - 2} \\
 &= \frac{5}{2} && 1A
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \lim_{x \rightarrow 3} \frac{x^3 - 27}{x^3 - x^2 - 5x - 3} &= \lim_{x \rightarrow 3} \frac{(x - 3)(x^2 + 6x + 9)}{(x - 3)(x^2 + 2x + 1)} && 1M \\
 &= \lim_{x \rightarrow 3} \frac{x^2 + 6x + 9}{x^2 + 2x + 1} && 1M \\
 &= \frac{3^2 + 3(3) + 9}{3^2 + 2(3) + 1} \\
 &= \frac{27}{16} && 1A
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \lim_{x \rightarrow 0} \frac{8^x - 1}{2^x - 1} &= \lim_{x \rightarrow 0} \frac{(2^x - 1)(2^{2x} + 2^x + 1)}{2^x - 1} && 1M \\
 &= \lim_{x \rightarrow 0} (2^{2x} + 2^x + 1) && 1M \\
 &= 2^0 + 2^0 + 1 \\
 &= 3 && 1A
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \lim_{h \rightarrow -2} \frac{(x - 2h^2)^2 - (x + 4h)^2}{h + 2} &= \lim_{h \rightarrow -2} \frac{[(x - 2h^2) + (x + 4h)][(x - 2h^2) - (x + 4h)]}{h + 2} && 1M \\
 &= \lim_{h \rightarrow -2} \frac{-2h(2x + 4h - 2h^2)(h + 2)}{h + 2} \\
 &= \lim_{h \rightarrow -2} -2h(2x + 4h - 2h^2) && 1M \\
 &= -2(-2)[2x + 4(-2) - 2(-2)^2] \\
 &= 8x - 64 && 1A
 \end{aligned}$$

$$5. \lim_{x \rightarrow h} \frac{(x+h)^3 + (x-3h)^3}{x-h} = \lim_{x \rightarrow h} \frac{(2x-2h)[(x+h)^2 - (x+h)(x-3h) + (x-3h)^2]}{x-h} \quad 1M$$

$$= 2 \lim_{x \rightarrow h} [(x+h)^2 - (x+h)(x-3h) + (x-3h)^2] \quad 1M$$

$$= 2(4h^2 + 4h^2 + 4h^2)$$

$$= 24h^2$$

1A

$$\lim_{x \rightarrow 2h} \frac{\frac{1}{x+h} - \frac{1}{4x-5h}}{x-2h} = \lim_{x \rightarrow 2h} \frac{(4x-5h) - (x+h)}{(x-2h)(x+h)(4x-5h)} \quad 1M$$

$$= \lim_{x \rightarrow 2h} \frac{2x-6h}{(x+h)(4x-5h)(x-2h)}$$

$$= \lim_{x \rightarrow 2h} \frac{3}{(x+h)(4x-5h)} \quad 1M$$

$$= \frac{3}{(2h+h)(8h-5h)}$$

$$= \frac{1}{3h^2}$$

1A

$$\text{因此, } \left[\lim_{x \rightarrow h} \frac{(x+h)^3 + (x-3h)^3}{x-h} \right] \cdot \left[\lim_{x \rightarrow 2h} \frac{\frac{1}{x+h} - \frac{1}{4x-5h}}{x-2h} \right] = 24h^2 \cdot \frac{1}{3h^2} = 8. \quad 1A$$

$$6. \lim_{x \rightarrow 3} \frac{\sqrt{3x+7}-4}{3-\sqrt{x+6}} = \lim_{x \rightarrow 3} \frac{(\sqrt{3x+7}-4)(\sqrt{3x+7}+4)(3+\sqrt{x+6})}{(3-\sqrt{x+6})(\sqrt{3x+7}+4)(3+\sqrt{x+6})} \quad 1M$$

$$= \lim_{x \rightarrow 3} \frac{(3x-9)(3+\sqrt{x+6})}{(3-x)(\sqrt{3x+7}+4)}$$

$$= \lim_{x \rightarrow 3} \frac{-3(3+\sqrt{x+6})}{\sqrt{3x+7}+4} \quad 1M$$

$$= \frac{-3(3+3)}{4+4}$$

$$= -\frac{9}{4}$$

1A

$$7. \quad (a) \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} = \lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x}+2)}{(\sqrt{x}-2)(\sqrt{x}+2)} \quad 1M$$

$$= \lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x}+2)}{x-4}$$

$$= \lim_{x \rightarrow 4} (\sqrt{x}+2) \quad 1M$$

$$= \sqrt{4}+2$$

$$= 4$$

1A

$$(b) \lim_{x \rightarrow 0} \frac{x}{\sqrt{x+9}-3} = \lim_{x \rightarrow 0} \frac{x(\sqrt{x+9}+3)}{(\sqrt{x+9}-3)(\sqrt{x+9}+3)} \quad 1M$$

$$= \lim_{x \rightarrow 0} \frac{x(\sqrt{x+9}+3)}{(x+9)-9}$$

$$= \lim_{x \rightarrow 0} (\sqrt{x+9}+3) \quad 1M$$

$$= \sqrt{9}+3$$

$$= 6$$

1A

$$\begin{aligned}
8. \lim_{x \rightarrow 0} \frac{\sqrt{9+x} - 3}{\sqrt{1+x} - 1} &= \lim_{x \rightarrow 0} \frac{(\sqrt{9+x} - 3)(\sqrt{9+x} + 3)(\sqrt{1+x} + 1)}{(\sqrt{1+x} - 1)(\sqrt{9+x} + 3)(\sqrt{1+x} + 1)} & 1M \\
&= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + 1)}{x(\sqrt{9+x} + 3)} \\
&= \lim_{x \rightarrow 0} \frac{\sqrt{1+x} + 1}{\sqrt{9+x} + 3} & 1M \\
&= \frac{1+1}{3+3} \\
&= \frac{1}{3} & 1A
\end{aligned}$$

$$\begin{aligned}
9. \lim_{h \rightarrow 0} \frac{(x-h)\sqrt{x+h} - x\sqrt{x}}{\sqrt{x-h} - \sqrt{x}} &= \lim_{h \rightarrow 0} \frac{[(x-h)\sqrt{x+h} - x\sqrt{x}][(\sqrt{x-h} + \sqrt{x})]}{(\sqrt{x-h} - \sqrt{x})[(x-h)\sqrt{x+h} + x\sqrt{x}](\sqrt{x-h} + \sqrt{x})} & 1M \\
&= \lim_{h \rightarrow 0} \frac{[(x-h)^2(x+h) - x^3](\sqrt{x-h} + \sqrt{x})}{-h[(x-h)\sqrt{x+h} + x\sqrt{x}]} \\
&= \lim_{h \rightarrow 0} \frac{(-hx^2 - h^2x + h^3)(\sqrt{x-h} + \sqrt{x})}{-h[(x-h)\sqrt{x+h} + x\sqrt{x}]} & 1A \\
&= \lim_{h \rightarrow 0} \frac{(x^2 + hx - h^2)(\sqrt{x-h} + \sqrt{x})}{(x-h)\sqrt{x+h} + x\sqrt{x}} & 1M \\
&= \frac{x^2(2\sqrt{x})}{2x\sqrt{x}} \\
&= x & 1A
\end{aligned}$$

$$\begin{aligned}
10. \lim_{x \rightarrow a} \frac{x^2 - 25}{\sqrt{x^2 - 9} - 4} &= \lim_{x \rightarrow a} \frac{(x^2 - 25)(\sqrt{x^2 - 9} + 4)}{(\sqrt{x^2 - 9} - 4)(\sqrt{x^2 - 9} + 4)} & 1M \\
&= \lim_{x \rightarrow a} (\sqrt{x^2 - 9} + 4) & 1M \\
&= \sqrt{a^2 - 9} + 4 & 1A \\
a^2 - 9 &= 16 \\
a &= \pm 5 & 1A
\end{aligned}$$

$$\begin{aligned}
11. \lim_{x \rightarrow -2} \frac{x^2 + 6x + 8}{2 - \sqrt{x^3 + 12}} &= \lim_{x \rightarrow -2} \frac{(x+2)(x+4)(2 + \sqrt{x^3 + 12})}{(2 - \sqrt{x^3 + 12})(2 + \sqrt{x^3 + 12})} & 1M \\
&= \lim_{x \rightarrow -2} \frac{(x+2)(x+4)(2 + \sqrt{x^3 + 12})}{-x^3 - 8} \\
&= \lim_{x \rightarrow -2} \frac{(x+2)(x+4)(2 + \sqrt{x^3 + 12})}{-(x+2)(x^2 - 2x + 4)} & 1M \\
&= \lim_{x \rightarrow -2} \frac{(x+4)(2 + \sqrt{x^3 + 12})}{-(x^2 - 2x + 4)} & 1M \\
&= \frac{(-2+4)(2 + \sqrt{-8+12})}{-(4+4+4)} \\
&= -\frac{2}{3} & 1A
\end{aligned}$$