

REG-ST-2223-ASM-SET 2-MATH

Suggested solutions

Multiple Choice Questions

1. ☐ D

$\angle QPR$ is the largest angle.

$$8^2 = 6^2 + 7^2 - 2(6)(7) \cos \angle QPR$$

$$\angle QPR \approx 76^\circ$$

Required angle is 76° .

2. ☐ D

Let E be a point on CD such that $AE \parallel BC$.

Then $ABCE$ is a parallelogram.

$$\frac{DE}{AD} = \frac{100 - 60}{40} = \frac{40}{40}, \angle ADE = 180^\circ - 120^\circ = 60^\circ \text{ and } \angle AED = \angle BCD = 180^\circ - 110^\circ = 70^\circ$$

$$\frac{\sin 70^\circ}{\sin 180^\circ - 60^\circ - 70^\circ} = \frac{40}{\sin(180^\circ - 60^\circ - 70^\circ)}$$

$$AD \approx 49 \text{ cm}$$

3. ☐ A

A. **X**. We have $A : B : C = 1 : 2 : 3$. Take $A = 30^\circ$, then $B = 60^\circ$ and $C = 90^\circ$.

We have $\sin A : \sin B : \sin C \neq 1 : 2 : 3$.

B. **✓**.

C. **X**. Take $A = 30^\circ$, then $B = 60^\circ$ and $C = 90^\circ$.

$$\text{We have } \cos A : \cos B : \cos C = \frac{\sqrt{3}}{2} : \frac{1}{2} : 0 \neq \sqrt{3} : 1 : 1.$$

4. ☐ D

$$\angle AOB = 360^\circ - 240^\circ = 120^\circ \text{ and } \angle ACB = \frac{1}{2}(120^\circ) = 60^\circ$$

$$AB^2 = 4^2 + 3^2 - 2(3)(4) \cos 60^\circ$$

$$AB = \sqrt{13} \text{ cm}$$

5. ☐ D

Let $ED = 3$ cm. Then $DE = 2$ cm and $AB = 5$ cm.

$$BE = \sqrt{5^2 + 3^2} = \sqrt{34} \text{ cm and } CE = \sqrt{5^2 + 2^2} = \sqrt{29} \text{ cm.}$$

$$5^2 = (\sqrt{34})^2 + (\sqrt{29})^2 - 2(\sqrt{34})(\sqrt{29}) \cos \theta$$

$$\cos \theta \approx 0.605$$

6. A

$$\begin{aligned}\text{Required area} &= 4^2\pi \times \frac{60^\circ}{360^\circ} - \frac{1}{2}(4)(4) \sin 60^\circ \\ &\approx 1.45 \text{ cm}^2\end{aligned}$$

7. A

$$\begin{aligned}\frac{\sin \theta}{3} &= \frac{\sin 150^\circ}{5} \\ \sin \theta &= \frac{3}{10} \\ \tan \theta &= \frac{3}{\sqrt{10^2 - 3^2}} \\ &= \frac{3}{\sqrt{91}}\end{aligned}$$

8. B

$$\begin{aligned}AC &= r \sin \alpha \\ \angle CAB &= 180^\circ - \beta - \gamma \\ \frac{BC}{\sin[180^\circ - (\beta + \gamma)]} &= \frac{r \sin \alpha}{\sin \beta} \\ \frac{BC}{\sin(\beta + \gamma)} &= \frac{r \sin \alpha}{\sin \beta} \\ BC &= \frac{r \sin \alpha \sin(\beta + \gamma)}{\sin \beta}\end{aligned}$$

9. C

$$\begin{aligned}\frac{1}{2}(16)(AB) \sin 30^\circ &= 24 \\ AB &= 6 \text{ cm}\end{aligned}$$

10. C

$$\begin{aligned}10^2 &= x^2 + (3x)^2 - 2(x)(3x) \cos 50^\circ \\ 100 &= x^2(10 - 6 \cos 50^\circ) \\ x &= \sqrt{\frac{100}{10 - 6 \cos 50^\circ}} \quad \text{or} \quad -\sqrt{\frac{100}{10 - 6 \cos 50^\circ}} \text{ (rejected)} \\ &\approx 4.03\end{aligned}$$

11. C

$$\begin{aligned}\frac{1}{2}(8)(AC) \sin 30^\circ &= 28 \\ AC &= 14 \text{ cm}\end{aligned}$$

12. D

$$\begin{aligned}\frac{\sin \angle ACB}{5} &= \frac{\sin 50^\circ}{8} \\ \angle ACB &\approx 28.6^\circ \quad \text{or} \quad 151^\circ \text{ (rejected)}\end{aligned}$$

13. B

$$\frac{\sin x^\circ}{8} = \frac{\sin 78^\circ}{12}$$

$$x \approx 41$$

14. C

Since $5^2 + 12^2 = 13^2$, $\angle PRQ = 90^\circ$.

$$\cos P : \cos Q = \frac{5}{13} : \frac{12}{13}$$

$$= 5 : 12$$

15. C

$$\frac{\sin \angle TPQ}{5} = \frac{\sin 120^\circ}{7}$$

$$\angle TPQ \approx 38.2^\circ \quad \text{or} \quad 140^\circ \text{ (rejected)}$$

Let X be a point on the circle such that XQ is a diameter of the circle.

$$\angle PXQ = \angle TPQ \text{ and radius} = \frac{XQ}{2} = \frac{1}{2} \times \frac{PQ}{\sin \angle PXQ} \approx 5.66 \text{ cm}$$

16. B

$$2(5)(\pi) \times \frac{\angle AOB}{360^\circ} = \pi$$

$$\angle AOB = 36^\circ$$

$$\text{Required area} = \frac{1}{2}(5)(5) \sin 36^\circ$$

$$\approx 7.35 \text{ cm}^2$$

17. A

$$b^2 = a^2 + (2\sqrt{2})^2 - 2(a)(2\sqrt{2}) \cos 45^\circ$$

$$(6 - a)^2 = a^2 + 8 - 4a$$

$$8a = 28$$

$$a = 3.5$$

18. D

$$\text{Let } s = \frac{6 + (x + 5) + (x + 3)}{2} = 7 + x.$$

$$\sqrt{(7 + x)(1 + x)(2)(4)} = \sqrt{728}$$

$$x^2 + 8x - 84 = 0$$

$$x = 6 \quad \text{or} \quad -14 \text{ (rejected)}$$

$$\text{Distance} = \frac{\sqrt{728} \times 2}{11} \approx 4.9$$

19. B

Let O be the centre. Radius = $\frac{AD}{2} = \frac{\sqrt{10^2 + 24^2}}{2} = 13$ cm

$$AC^2 = OA^2 + OC^2 - 2(OA)(OC) \cos \angle AOC$$

$$\angle AOC \approx 135^\circ$$

$$\text{Area} = 13^2 \pi \times \frac{\angle AOC}{360^\circ} - \frac{1}{2}(13)^2 \sin \angle AOC \approx 139 \text{ cm}^2$$

20. B

Join AC as shown.

In $\triangle ABC$,

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC) \cos 60^\circ$$

$$AC = \sqrt{21}$$

And

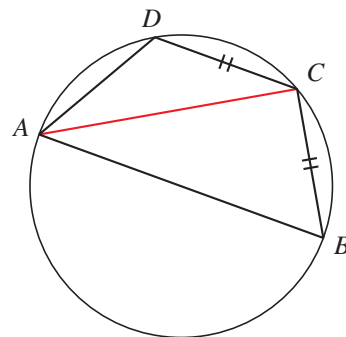
$$BC^2 = AC^2 + AB^2 - 2(AC)(AB) \cos \angle BAC$$

$$\angle BAC \approx 49.1^\circ$$

Since $CD = BC$, $\angle CAD = \angle BAC \approx 49.1^\circ$.

In $\triangle ACD$, $\angle ADC = 120^\circ$ and $\angle ACD = 180^\circ - 120^\circ - \angle BAC \approx 10.9^\circ$.

Area of quadrilateral = $\frac{1}{2}(CD)(AC) \sin \angle ACD + \frac{1}{2}(AB)(BC) \sin 60^\circ \approx 10.4$ and it refers to $6\sqrt{3}$.



21. B

$$\text{Required ratio} = \frac{1}{2}(BD)(BE) \sin \angle ABC : \frac{1}{2}(AB)(BC) \sin \angle ABC$$

$$= (BD)(BE) : (AB)(BC)$$

$$= (BD)(BE) : \left(\frac{5BD}{3}\right) \left(\frac{9BE}{4}\right)$$

$$= 4 : 15$$

22. B

$$\text{Required area} = 6^2 \pi \times \frac{40^\circ}{360^\circ} - \frac{1}{2}(6)(3) \sin 40^\circ$$

$$\approx 6.78 \text{ cm}^2$$

23. C

$$\text{Let } s = \frac{5+6+7}{2} = 9 \text{ cm.}$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \sqrt{9(9-5)(9-6)(9-7)} \\ &= \sqrt{216} \text{ cm}^2 \end{aligned}$$

Let r cm be the radius of the inscribed circle.

Consider the area of $\triangle ABC$.

$$\begin{aligned} \sqrt{216} &= \frac{5r}{2} + \frac{6r}{2} + \frac{7r}{2} \\ r &= \frac{\sqrt{216}}{9} \end{aligned}$$

$$\text{Required area} = r^2 \pi$$

$$= \frac{8\pi}{3} \text{ cm}^2$$

24. B

Put $AB = 5$ cm. Then $BC = 7$ cm and $CA = 9$ cm.

$$9^2 = 5^2 + 7^2 - 2(5)(7) \cos \angle B$$

$$\angle B \approx 95.7^\circ$$

25. D

$$AB^2 + BC^2 = CA^2 \text{ and so } \angle ABC = 90^\circ.$$

Let $AC = 5k$. Then $AB = 3k$ and $BC = 4k$.

$$\frac{(3k)(4k)}{2} = 150$$

$$k = 5 \quad \text{or} \quad -5 \text{ (rejected)}$$

$$AC = 5(5) = 25$$

26. B

$$\angle POQ = 230^\circ - 110^\circ = 120^\circ$$

$$PQ = 2 \times \sqrt{2} \sin \frac{120^\circ}{2} = \sqrt{6}$$

27. A

Let $BP = 1$. So, $PQ = QC = 1$ and $AB = AC = 3$.

In $\triangle ABP$, $AP = \sqrt{3^2 + 1^2 - 2(1)(3) \cos 60^\circ} \approx 2.65$. Similarly, $AQ = AP \approx 2.65$

$$\text{In } \triangle APQ, \angle PAQ = \cos^{-1} \frac{AP^2 + AQ^2 - PQ^2}{2(AP)(AQ)} \approx 22^\circ$$

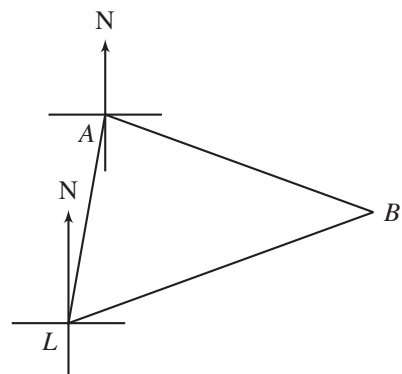
28. C

Refer to the notations in the figure.

$$\angle LAB = 10^\circ + 70^\circ = 80^\circ \text{ and } \angle ALB = 70^\circ - 10^\circ = 60^\circ$$

$$\frac{400}{\sin 60^\circ} = \frac{BL}{\sin 80^\circ}$$

$$BL = \frac{400 \sin 80^\circ}{\sin 60^\circ}$$



29. B

$$\angle ABC = 30^\circ + 50^\circ = 80^\circ$$

$$AC^2 = 8^2 + 6^2 - 2(8)(6) \cos 80^\circ$$

$$AC \approx 9.13 \text{ km}$$

Required distance is 9.13 km.

30. C

$$\angle CAB = 90^\circ - 65^\circ = 25^\circ \text{ and } \angle ABC = 90^\circ + 12^\circ = 102^\circ.$$

$$\angle ACB = 180^\circ - 25^\circ - 102^\circ = 53^\circ$$

$$\frac{BC}{\sin 25^\circ} = \frac{4.5}{\sin 53^\circ}$$

$$BC \approx 2.4 \text{ m}$$

Conventional Questions

31. Let C be a point due south of A .

$$\angle HAC = 45^\circ$$

$$BH^2 = 50^2 + 100^2 - 2(50)(100) \cos 75^\circ$$

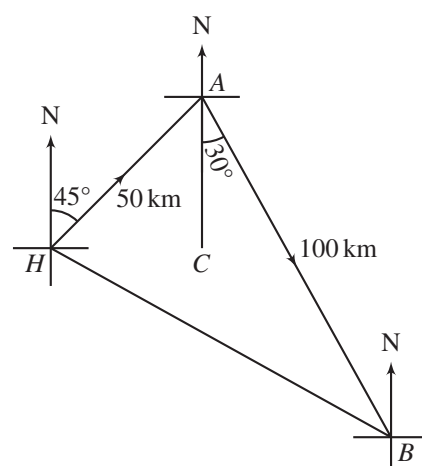
$$BH \approx 99.6 \text{ km}$$

$$50^2 = 100^2 + HB^2 - 2(100)(HB) \cos \angle ABH$$

$$\angle ABH \approx 29.0^\circ$$

$$\angle ABH + 30^\circ \approx 59.0^\circ$$

Thus, the compass bearing of H from B is N59.0°W.



32. (a) $\angle ACD = 85^\circ - 46^\circ = 39^\circ$.

$$\frac{AD}{\sin 39^\circ} = \frac{6}{\sin 46^\circ}$$

$$AD \approx 5.25 \text{ cm}$$

- (b) $\angle BCD = 180^\circ - 85^\circ - 32^\circ = 63^\circ$

$$\frac{DB}{\sin 63^\circ} = \frac{6}{\sin 32^\circ}$$

$$DB \approx 10.1 \text{ cm}$$

- (c) Area of $\triangle DBC = \frac{1}{2}(6)(DB) \sin 85^\circ$

$$\approx 30.2 \text{ cm}^2$$

33. (a) $AC^2 = 4.3^2 + 6^2 - 2(4.3)(6) \cos 130^\circ$

$$AC \approx 9.36 \text{ cm}$$

$$\angle CED = 180^\circ - 34^\circ - 121^\circ = 25^\circ$$

$$\frac{CE}{\sin 121^\circ} = \frac{4.9}{\sin 25^\circ}$$

$$CE \approx 9.94 \text{ cm}$$

Let $AE = x \text{ cm}$.

$$7.6^2 = 5.5^2 + x^2 - 2(5.5)(x) \cos 37^\circ$$

$$0 = x^2 - (11 \cos 37^\circ)x - 27.51$$

$$x = 11.2 \quad \text{or} \quad -2.45 \text{ (rejected)}$$

Thus, $AE \approx 11.2 \text{ cm}$.

- (b) Let $s = \frac{AC + CE + AE}{2} \approx 15.3 \text{ cm}$.

$$\text{Required area} = \sqrt{s(s - AC)(s - CE)(s - AE)}$$

$$\approx 44 \text{ cm}^2$$