REG-ST-2223-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. $\boxed{\text{C}}$

$$\begin{aligned} 12\sqrt{3} &= 2 \times \frac{1}{2}(AB)(AD)\sin 120^\circ \\ &= AB(6) \cdot \frac{\sqrt{3}}{2} \\ AB &= 4 \end{aligned}$$

2. $\boxed{\text{A}}$

$$\begin{aligned} \frac{1}{2}(10)(8)\sin\theta &= 20 \\ \sin\theta &= \frac{1}{2} \\ \theta &= 30^\circ \quad \text{or} \quad 150^\circ \text{ (rejected)} \end{aligned}$$

3. $\boxed{\text{B}}$

$$\begin{aligned} \text{Required area} &= \frac{1}{2}(4)(6)\sin 60^\circ \\ &= 12\left(\frac{\sqrt{3}}{2}\right) \\ &= 6\sqrt{3}\,\text{cm}^2 \end{aligned}$$

4. $\boxed{\text{B}}$

$\angle AOB = \angle BOC = \angle COA = 120^\circ$

$$\begin{aligned} \text{Required area} &= 6^2\pi - 3 \times \frac{1}{2}(6)(6)\sin 120^\circ \\ &\approx 66.3\,\text{cm}^2 \end{aligned}$$

5. $\boxed{\text{B}}$

Let $OA = r$ cm.

$$\begin{aligned} r^2\pi \times \frac{60^\circ}{360^\circ} - \frac{1}{2}r^2\sin 60^\circ &= 10 \\ r &\approx 10.5 \end{aligned}$$

6. $\boxed{\text{B}}$

$$\begin{aligned} \frac{1}{2}(AB)(BC)\sin 80^\circ &= 80 \\ \frac{1}{2}(AB)\left(\frac{3AB}{2}\right)\sin 80^\circ &= 80 \\ AB &\approx 10.4\,\text{cm} \end{aligned}$$

7. $\boxed{\text{A}}$

Note that $\sin 70° = \sin 110°$.

$$\begin{aligned}
\text{Area of } \triangle ABD &= \frac{1}{2}(EA)(EB)\sin 110° + \frac{1}{2}(EA)(ED)\sin 70° \\
&= \frac{1}{2}(EA)(EB)\sin 70° + \frac{1}{2}(EA)(ED)\sin 70° \\
&= \frac{1}{2}(EA)(EB+ED)\sin 70° \\
&= \frac{1}{2}(EA)(BD)\sin 70°
\end{aligned}$$

$$\begin{aligned}
\text{Area of } \triangle BCD &= \frac{1}{2}(EB)(EC)\sin 70° + \frac{1}{2}(EC)(ED)\sin 110° \\
&= \frac{1}{2}(EC)(EB+ED)\sin 70° \\
&= \frac{1}{2}(EC)(BD)\sin 70°
\end{aligned}$$

$$\begin{aligned}
\text{Required area} &= \frac{1}{2}(EA)(BD)\sin 70° + \frac{1}{2}(EC)(BD)\sin 70° \\
&= \frac{1}{2}(EA+EC)(BD)\sin 70° \\
&= \frac{1}{2}(AC)(BD)\sin 70° \\
&\approx 19.7\text{ cm}^2
\end{aligned}$$

8. $\boxed{\text{B}}$

Note that $EA = EC = 3\text{ cm}$.

$$\begin{aligned}
\text{Required area} &= \frac{1}{2}(5)(3)\sin 120° \\
&= \frac{15}{2}\left(\frac{\sqrt{3}}{2}\right) \\
&= \frac{15\sqrt{3}}{4}\text{ cm}^2
\end{aligned}$$

9. $\boxed{\text{B}}$

$\angle BAC = 180° - 90° - 60° = 30°$

$AE = 6\cos 30° = 3\sqrt{3}\text{ cm}$

$AD = BC = \dfrac{6}{\tan 60°} = 2\sqrt{3}\text{ cm}$

$\angle DAE = \angle ACB = 60°$

$$\begin{aligned}
\text{Required area} &= \frac{1}{2}(AD)(AE)\sin 60° \\
&= \frac{9\sqrt{3}}{2}\text{ cm}^2
\end{aligned}$$

10. $\boxed{\text{B}}$

$BC = 36 - 8 - 16$

$= 12\text{ cm}$

Let $s = \dfrac{8 + 12 + 16}{2} = 18\text{ cm}.$

Required area $= \sqrt{18(18-8)(18-12)(18-16)}$

$= 12\sqrt{15}\text{ cm}^2$

11. $\boxed{\text{D}}$

Let $s = \dfrac{5 + 6 + 7}{2} = 9\text{ cm}.$

Base area $= \sqrt{9(9-5)(9-6)(9-7)}$

$= \sqrt{216}$

$= 6\sqrt{6}\text{ cm}^2$

Required area $= 2(6\sqrt{6}) + (5)(8) + (6)(8) + (7)(8)$

$= (144 + 12\sqrt{6})\text{ cm}^2$

12. $\boxed{\text{C}}$

$AC = \sqrt{10^2 + 24^2}$

$= 26\text{ cm}$

Let $s = \dfrac{11 + 25 + 26}{2} = 31\text{ cm}.$

Required area $= \sqrt{31(31-25)(31-11)(31-26)}$

$\approx 136.4\text{ cm}^2$

13. $\boxed{\text{A}}$

Let $s = \dfrac{9 + 9 + 12}{2} = 15\text{ cm}.$

Required area $= \sqrt{15(15-9)^2(15-12)}$

$= 18\sqrt{5}\text{ cm}^2$

14. $\boxed{\text{D}}$

$a = 36 \times \dfrac{3}{3 + 4 + 5} = 9\text{ cm}$

$b = 9 \times \dfrac{4}{3} = 12\text{ cm}$ and $c = 9 \times \dfrac{5}{3} = 15\text{ cm}$

Let $s = \dfrac{a + b + c}{2} = 18\text{ cm}.$

Area $= \sqrt{18(18-9)(18-12)(18-15)}$

$= 54\text{ cm}^2$

15. $\boxed{\text{C}}$

$\dfrac{\sin \angle BAC}{14} = \dfrac{\sin 35^\circ}{10}$

$\angle BAC \approx 53.4^\circ$ or 127°

2 triangles can be constructed.

16. $\boxed{\text{C}}$

$$\frac{BP}{\sin x} = \frac{AB}{\sin \angle APB} \quad \text{and} \quad \frac{PC}{\sin y} = \frac{AC}{\sin \angle APC}$$

$$BP = \frac{AB \sin x}{\sin \angle APB} \qquad PC = \frac{AC \sin y}{\sin \angle APC}$$

Note that $\sin \angle APB = \sin \angle APC$.

$$\frac{BP}{PC} = \frac{AB \sin x}{\sin \angle APB} \div \frac{AC \sin y}{\sin \angle APC}$$

$$= \frac{\sin x}{\sin y}$$

$BP : PC = \sin x : \sin y$

17. $\boxed{\text{B}}$

Note that $AD // BC$ and $\angle APB = \beta$. In $\triangle ABP$,

$$\frac{AP}{\sin \alpha} = \frac{AB}{\sin \beta}$$

$$\frac{AP}{\sin \alpha} = \frac{2AP}{\sin \beta}$$

$$\frac{\sin \beta}{\sin \alpha} = \frac{2AP}{AP}$$

$\sin \alpha : \sin \beta = 1 : 2$

18. $\boxed{\text{D}}$

When $b = 20$, $c = 14$, $\angle C = 50°$,

$$\frac{\sin B}{20} = \frac{\sin 50°}{14}$$

$\sin B \approx 1.09 > 1$

The equation has no solution.

Thus, the triangle in D cannot exist.

19. $\boxed{\text{A}}$

$\angle BAC = 180° - 62° - 74° = 44°$

$$\frac{BC}{\sin 44°} = \frac{4}{\sin 62°}$$

$$BC = \frac{4 \sin 44°}{\sin 62°}$$

20. $\boxed{\text{D}}$

$a^2 = b^2 + c^2 - 2bc \cos 120°$

$a^2 = b^2 + c^2 - 2bc\left(-\frac{1}{2}\right)$

$a^2 = b^2 + c^2 + bc$

21. $\boxed{\text{B}}$

$$\cos B = \sqrt{\cos^2 B} = \sqrt{1 - sin^2 B} = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

$$AC^2 = 16^2 + 15^2 - 2(16)(15)\cos B$$
$$AC = \sqrt{97}\text{ cm}$$

22. $\boxed{\text{C}}$

Let O be the midpoint of BD.

Note that $\triangle ABO$, $\triangle AEO$ and $\triangle DEO$ are equilateral, we have $BD = 16$ cm.

$$16^2 = 10^2 + 10^2 - 2(10)(10)\cos \angle BCD$$
$$\angle BCD \approx 106^\circ$$

23. $\boxed{\text{D}}$

Let r cm be the radius of the circle.

$$8^2 = r^2 + r^2 - 2r^2 \cos 120^\circ$$
$$64 = 3r^2$$
$$r \approx 4.62$$

$$\text{Required area} = r^2\pi - \frac{1}{2}(8)(8)\sin 60^\circ$$
$$\approx 39.3\text{ cm}^2$$

24. $\boxed{\text{B}}$

$$10^2 = 11^2 + 3^2 - 2(11)(3)\cos \angle CAB$$
$$\angle CAB \approx 63.0^\circ$$
$$CD = 11 \sin \angle CAB$$
$$\approx 9.80\text{ cm}$$

25. $\boxed{\text{C}}$

$$10^2 = 9^2 + 12^2 - 2(9)(12)\cos\theta$$
$$\theta \approx 55^\circ$$

26. $\boxed{\text{B}}$

Consider $\triangle BCD$.

$$10^2 = 5^2 + 6^2 - 2(5)(6)\cos \angle BCD$$
$$\angle BCD \approx 131^\circ$$

$$\angle ABC = 180^\circ - \angle BCD \approx 49^\circ$$

27. $\boxed{\text{D}}$

$3^2 = 5^2 + 7^2 - 2(5)(7)\cos A$ and $5^2 = 7^2 + 3^2 - 2(7)(3)\cos B$

$\cos A = \dfrac{13}{14}$ $\cos B = \dfrac{11}{14}$

$\cos A : \cos B = \dfrac{13}{14} : \dfrac{11}{14} = 13 : 11$

28. $\boxed{\text{A}}$

$x^2 = y^2 + z^2 - 2yz\cos 135°$

$x^2 = y^2 + z^2 - 2zy\left(-\dfrac{\sqrt{2}}{2}\right)$

$x^2 = y^2 + z^2 + \sqrt{2}yz$

29. $\boxed{\text{D}}$

$AC^2 = 3^2 + 4^2 - 2(3)(4)\cos 120°$

$AC \approx 6.08$

30. $\boxed{\text{B}}$

$4^2 = 6^2 + 4^2 - 2(6)(4)\cos \angle ACB$

$\angle ACB \approx 41.4°$

$AD^2 = 4^2 + 4^2 - 2(4)(4)\cos \angle ACB$

$AD \approx 2.83\,\text{cm}$

Conventional Questions

31. (a) Let $s = \dfrac{12 + 5 + 15}{2} = 16\,\text{cm}$.

Area of $\triangle ACD = \sqrt{16(16-15)(16-12)(16-5)}$ 1M

$\approx 26.5\,\text{cm}^2$ 1A

(b) Area of $\triangle BDC$ = area of $\triangle ADC$

$= \dfrac{1}{2}(5)(17)\sin\angle BDC$ 1M+1M

$\angle BDC \approx 38.6°$ or $141°$ (rejected) 1A

32. (a) $\dfrac{AC}{\sin 40°} = \dfrac{AB}{\sin 110°}$ 1M

$AB \approx 8.77\,\text{cm}$ 1A

(b) $\angle A = 180° - 40° - 110° = 30°$

Required area $= \dfrac{1}{2} \times 6 \times AB \times \sin 30°$ 1M

$\approx 13.2\,\text{cm}^2$ 1A

33. Let the length of XZ be x cm.

$$\frac{x}{\sin(180° - 23° - 61°)} = \frac{YZ}{\sin 23°}$$ 1M

$$YZ = \frac{x \sin 23°}{\sin 96°}$$ 1A

$$\frac{1}{2}(YZ)(XZ)\sin 61° = 40$$ 1M

$$\frac{x^2 \sin 23° \sin 61°}{2\sin 96°} = 40$$ 1M

$$x \approx 15.3$$ 2A

Length of XZ is 15.3 cm.

34. (a) In $\triangle ABD$,

$$\angle BAD = 180° - \theta$$

$$BD^2 = 5^2 + 8^2 - 2(5)(8)\cos(180° - \theta) \qquad \text{1M}$$

$$= 89 + 80\cos\theta$$

In $\triangle BCD$,

$$BD^2 = 6^2 + 7^2 - 2(6)(7)\cos\theta \qquad \text{1M}$$

$$= 85 - 84\cos\theta$$

Therefore,

$$89 + 80\cos\theta = 85 - 84\cos\theta \qquad \text{1M}$$

$$(80 + 84)\cos\theta = (85 - 89)$$

$$\cos\theta = -\frac{1}{41} \qquad \text{1}$$

(b) $BD^2 = 89 + 80\cos\theta$

$$BD^2 = 89 + 80\left(-\frac{1}{41}\right) \qquad \text{1M}$$

$$BD \approx 9.33 \qquad \text{1A}$$

END OF PAPER