REG-COT-2223-ASM-SET 2-MATH
Suggested solutions

Conventional Questions

1. (a) Slope of $L_2 = -\frac{1}{2}$. Slope of $L_1 = 2$. 1M

The equation of L_1 is

$$y - 4 = 2(x - 3)$$
$$y = 2x - 2$$ 1A

(b) $B\ (8,\ 0)$ and $C\ (0,\ 4)$. 1A

Since AC is parallel to the x-axis, x-coordinate of the orthocentre = 8. 1A

Let the coordinates of orthocentre be $(8,\ k)$.

$$\frac{k-4}{8-0} \times \frac{4-0}{3-8} = -1$$ 1M
$$k = 14$$

Required coordinates are $(8,\ 14)$. 1A

2. (a) Slope of $AB = \frac{4-0}{3-2} = 4$ 1M

The equation of L is $y = -\frac{x}{4}$. 1A

(b) Orthocentre lies on the altitude through B, i.e., $x = 3$.

Put $x = 3$ into equation of L, $y = -\frac{3}{4}$. 1M

The coordinates of orthocentre are $\left(3,\ -\frac{3}{4}\right)$. 1A

3. (a) $CE \perp AB$ *(property of orthocentre)*

$BD \perp AC$ *(property of orthocentre)*

$\angle BEC = \angle BDC = 90°$

Thus, $BCDE$ is a cyclic quadrilateral. *(converse of ∠s in the same segment)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) Coordinates of centre $= \left(\dfrac{-6+14}{2},\ \dfrac{-6-6}{2}\right) = (4,\ -6)$ 1A

The equation of the circle is

$$(x-4)^2 + (y+6)^2 = (0-4)^2 + (8+6)^2$$ 1M

$$(x-4)^2 + (y+6)^2 = 100$$ 1A

(ii) Distance between A and centre $= \sqrt{4^2 + (-6-8)^2} = \sqrt{212}$

Radius of circle = 10

Angle between two tangents $= 2 \times \sin^{-1} \dfrac{10}{\sqrt{212}} \approx 86.8° \neq 90°$ 1M+1A

The claim is not agreed. 1A

4. (a) $$\frac{y-2}{x-1} \times \frac{y-8}{x-9} = -1$$ 1M+1A

$(y-2)(y-8) + (x-1)(x-9) = 0$

$x^2 + y^2 - 10x - 10y + 25 = 0$ 1A

(b) (i) $8^2 + 1^2 - 10(8) - 10(1) + 25 = 0$

Thus, C lies on S. 1

(ii) $(5,\ 5)$ 1A

(iii) Since AB is a diameter of the circle, $\angle ACB = 90°$.

So, the orthocentre H is at point $C\ (8,\ 1)$. 1M

Circumcentre J is the midpoint of AB. The line joining J and H is a median of $\triangle ABC$.

Since centroid G lies on median of $\triangle ABC$, G, J and H are collinear.

The claim is agreed. 1A

5. (a) Let $x = \angle BAI$ and $y = \angle ABI$.

$\angle PAC = \angle BAI = x$ *(property of incentre)*

$PB = PC$ *(equal ∠s, equal chords)*

$\angle PIB = \angle ABI + \angle BAI$ *(ext. ∠ of △)*

$= x + y$

$\angle IBC = \angle ABI = y$ *(property of incentre)*

$\angle PBC = \angle PAC = x$ *(∠s in the same segment)*

$\angle IBP = x + y$

$= \angle PIB$

$PB = PI$ *(sides opp. equal ∠s)*

Thus, $PB = PI = PC$.

Marking Scheme		
Case 1 Any correct proof with correct reasons.		3
Case 2 Any correct proof without reasons.		2
Case 3 Incomplete proof with any one correct step with reason.		1

(b) $\angle IAY = \angle PSC$ *(∠s in the same segment)*

$\angle AYI = 90°$ *(given)*

$\angle SCP = 90°$ *(∠ in semicircle)*

$= \angle AYI$

$\triangle IAY \sim \triangle PSC$ *(AA)*

Marking Scheme		
Case 1 Any correct proof with correct reasons.		3
Case 2 Any correct proof without reasons.		2
Case 3 Incomplete proof with any one correct step with reason.		1

(c) $\dfrac{IY}{PC} = \dfrac{IA}{PS}$ 1M

$\dfrac{r}{IP} = \dfrac{AI}{2R}$

$AI \cdot IP = 2Rr$

The claim is agreed. 1A

(d) Let the equation of circle BPC be $x^2 + y^2 + Dx + Ey + F = 0$, where D, E and F are constants.

$$\begin{cases} (-16)^2 - 16D + F = 0 \\ (-8)^2 - 8E + F = 0 \\ 16^2 + 16D + F = 0 \end{cases}$$ 1M

Solving, we have $D = 0$, $E = -24$ and $F = -256$.

Radius of the circumcircle $= \sqrt{0^2 + 12^2 + 256} = 20$ 1A

$BP = \sqrt{16^2 + 8^2} = 8\sqrt{5}$ 1A

By (c), $AI = \dfrac{2Rr}{IP} = \dfrac{2(20)(9)}{8\sqrt{5}} = 9\sqrt{5}$ 1A

6. (a) $\angle OMQ = 90° = \angle ONQ$ *(given)*

$AB = CD$ *(given)*

$OM = ON$ *(equal chords, equidistant from centre)*

$OQ = OQ$ *(common side)*

$\triangle QNO \cong \triangle QMO$ *(RHS)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $OT = ON = 130$

$OQ = \sqrt{312^2 + 130^2} = 338$ and $OT : OQ = 130 : 338 = 5 : 13$

x-coordinate of $T = 312 \times \left(-\frac{5}{13}\right) = -120$ 1M

y-coordinate of $T = -130 \times \left(-\frac{5}{13}\right) = 50$

The coordinates of T are $(-120,\ 50)$. 1A

Let the coordinates of R be $(h,\ -130)$ such that $QR \perp ON$.

$$\frac{-130 - 50}{h + 120} \times \frac{50 - 0}{-120 - 0} = -1$$ 1M

$$h = -195$$

Let the coordinates of P be $(a,\ b)$. Note that T is the midpoint of PR.

$$\frac{a + (-195)}{2} = -120 \quad \text{and} \quad \frac{b + (-130)}{2} = 50$$ 1M

$$a = -45 \qquad\qquad b = 230$$

The coordinates of P are $(-45,\ 230)$. 1A

(ii) $OP = \sqrt{45^2 + 230^2} = \sqrt{54\,925} \neq OQ$ 1M

Thus, O is not the circumcentre of $\triangle PQR$.

The claim is disagreed. 1A

END OF PAPER