REG-AOT-2223-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. D

$BD = \sqrt{12^2 + 5^2} = 13\text{ cm}$
$BH = \sqrt{5^2 + 8^2} = \sqrt{89}\text{ cm}$
$\tan\angle FBG = \dfrac{12}{8}$ and $\tan\angle DEB = \dfrac{13}{8}$
$\angle FBG \approx 56.3°$ $\angle DEB \approx 58.4°$
$\tan\angle EBH = \dfrac{12}{\sqrt{89}}$ and $\tan\angle ACB = \dfrac{12}{5}$
$\angle EBH \approx 51.8°$ $\angle ACB \approx 67.4°$

2. B

Let $VA = 2\text{ cm}$.
Note that $VA = VC$ and $\angle AVC = 60°$. $\triangle VAC$ is an equilateral triangle.
$AB = AC\cos 45°$
$= \sqrt{2}\text{ cm}$
$AB^2 = VA^2 + VB^2 - 2(VA)(VB)\cos\angle AVB$
$\angle AVB \approx 41.4°$

3. B

$AC = AD\cos\angle DAC$
$= x\cos\beta$
$BC = AC\sin\angle CAB$
$= x\sin\alpha\cos\beta$

4. B

$BC = AC\sin 60°$
$= 4\sqrt{3}\text{ m}$
$\tan\theta = \dfrac{6}{4\sqrt{3}}$
$\theta \approx 41°$

5. B

$ME = MF = \dfrac{1}{2}\sqrt{4^2 + 3^2 + 12^2}$
$= 6.5\text{ cm}$
$4^2 = ME^2 + MF^2 - 2(ME)(MF)\cos\theta$
$\theta \approx 36°$

6. D

Let the lengths of edges be 1. Let R be the midpoint of CD and Q be the midpoint of AC such that $PQ \perp QR$. Let S be the midpoint of HE such that $PS \perp HE$.

$QR = 0.5$ and $PR = \sqrt{1^2 - 0.5^2} = \sqrt{0.75}$

So, $PQ = \sqrt{(\sqrt{0.75})^2 - 0.5^2} = \sqrt{0.5}$.

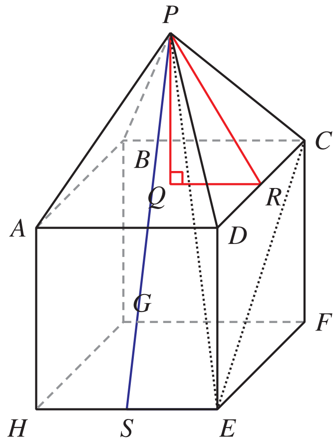
$PS = \sqrt{(\sqrt{0.5} + 1)^2 + 0.5^2} \approx 1.78$

$PE = \sqrt{PS^2 + 0.5^2} \approx 1.85$ $CE = \sqrt{1^2 + 1^2} = \sqrt{2}$

In $\triangle PCE$,

$$1^2 = PE^2 + CE^2 - 2(PE)(CE)\cos\angle PEC$$

$$\angle PEC \approx 32.4°$$



7. C

Let the length of each side be 2.

$AH = \sqrt{1^2 + 1^2} = \sqrt{2}$ and $VH = \sqrt{2^2 - (\sqrt{2})^2} = \sqrt{2}$

$\sin\angle VAH = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$

8. D

Since M and N are midpoints of OB and OC respectively, $MN = \dfrac{BC}{2} = \sqrt{2}$ cm

In $\triangle AOB$, $\angle AOB = 45°$ and $AM \perp OB$ by symmetry.

$AM = 2\sin 45° = \sqrt{2}$ cm

$AN = AM = \sqrt{2}$ cm and $\triangle AMN$ is equilateral.

Area of $\triangle AMN = \dfrac{\sqrt{3}}{2}$ cm^2

9. D

$\angle ABC = \tan^{-1}\dfrac{3}{4}$ and $CD = 4\sin\angle ABC = 2.4$ m

$\tan\theta = \dfrac{5}{2.4} = \dfrac{25}{12}$

10. E

Consider the cross section in the shape of right-angled triangle.

Hypotenuse = $1\cos\alpha = \cos\alpha$ m, length of the other two sides are $\cos\alpha\sin\beta$ m and $\cos\alpha\cos\beta$ m.

$$\text{Volume} = \frac{(\cos\alpha\cos\beta)(\cos\alpha\sin\beta)}{2} \times 1\sin\alpha$$

$$= \frac{1}{2}\sin\alpha\cos^2\alpha\sin\beta\cos\beta \text{ m}^3$$

11. A

I. ✓. VA is vertical and AB is horizontal.

II. ✗. Note that $\angle VAM = 90°$ and so $\angle VMA = 180° - 90° - \angle AVM < 90°$.

III. ✗.

12. A

I. ✓. AF is perpendicular to the plane $ABCD$, so $\angle CAF = 90°$.

II. ✓. GH is perpendicular to the plane $CDEH$, so $\angle DHG = 90°$.

III. ✗. Consider the case when all sides of the cuboid are equal (cube), then $AG = GC = AC$ and $\angle AGC = 60°$.
Thus, $\angle AGC$ is not always 90°.

13. C

A. $\tan \angle ACE = \dfrac{AE}{CE}$

B. $\tan \angle AQE = \dfrac{AE}{EQ}$

C. $\tan \angle ADE = \dfrac{AE}{DE}$

D. $\tan \angle PCR = \dfrac{PR}{CR} = \dfrac{AE}{CR}$

Since $DE < EQ = RC < CE$, we have $\tan \angle ADE$ being the greatest among all.
$\angle ADE$ is therefore the greatest angle among all.
The answer is C.

14. D

Let N be the midpoint of CD and Y be the midpoint of AC. Consider the required angle between line and plane.

A. $\angle GAB = \tan^{-1} \dfrac{10}{AB}$

B. $\angle CAH = \tan^{-1} \dfrac{10}{AC}$

C. $\angle MAN = \tan^{-1} \dfrac{10}{AN}$

D. $\angle XAY = \tan^{-1} \dfrac{10}{AY}$

Since AY is the shortest among AB, AC, AN and AY, $\angle XAY$ is the greatest angle.

15. A

Required angle is $\angle AHD$.
$DH = \sqrt{6^2 + 8^2} = 10\,\text{cm}$
$\tan \angle AHD = \dfrac{10}{10}$
$\angle AHD = 45°$

16. D

Let Q be a point on DE such that $NQ \perp DE$.
Then $\theta = \angle NPQ$.
$PQ = \frac{3}{2} = 1.5\,\text{cm}$
$\tan\theta = \frac{3}{1.5}$
$= 2$

17. A

Required angle is $\angle BDF$.
$BD = \sqrt{4^2 + 8^2} = \sqrt{80}\,\text{cm}$
$BF = \sqrt{4^2 - 3^2} = \sqrt{7}\,\text{cm}$
$\sin\angle BDF = \frac{\sqrt{7}}{\sqrt{80}}$
$\angle BDF \approx 17°$

18. B

Required angle is $\angle DBE$.
Let $AD = CE = DE = 1\,\text{cm}$.
$BE = \sqrt{1^2 + 1^2} = \sqrt{2}\,\text{cm}$
$\tan\angle DBE = \frac{1}{\sqrt{2}}$
$\angle DBE \approx 35°$

19. B

Since $VA = VB$, we have $\angle VAB = \frac{180° - 60°}{2} = 60°$ and so all lateral faces are equilateral triangles.
Required angle is $\angle VAM$, where M is the projection of V on $ABCD$.
Let $AB = 2$. Then $VA = 2$ and $AM = \frac{1}{2}\sqrt{2^2 + 2^2} = \sqrt{2}$
Required angle $= \angle VAM = \cos^{-1}\frac{\sqrt{2}}{2} = 45°$

20. B

We have $\angle VAC = 60°$.
Since $VA = VC$, we have $\angle VCA = \angle VAC = 60°$ and $\triangle VAC$ is equilateral.
$AC = \sqrt{1^2 + 1^2} = \sqrt{2}\,\text{m}$
$VB = VA = VC = AC = \sqrt{2}\,\text{m}$
In $\triangle VAB$,

$$1^2 = VA^2 + VB^2 - 2(VA)(VB)\cos\angle AVB$$

$\angle AVB \approx 41°$

21. A

$BD = \sqrt{25^2 - 15^2} = 20\,\text{m}$

$29^2 = 21^2 + 20^2 - 2(21)(20)\cos\angle BDC$

$\angle BDC = 90°$

Since $BD \perp AD$ and $BD \perp CD$, $BD \perp \triangle ACD$.

Thus, the projection of B on the plane ACD is D, and the required angle is $\angle BAD$.

$\angle BAD = \tan^{-1}\dfrac{20}{15} \approx 53°$

22. A

Let K be a point on ME such that $FK \perp ME$. [In fact, K is at the position of point M.]

Since $AF \perp EM$, we also have $AK \perp ME$. The angle required is therefore $\angle AKF$.

Since $MH = EH = 12\,\text{cm}$, $\angle EMH = 45°$ and so $\angle FEM = 45°$

$FK = FE \sin\angle FEM = 12\sqrt{2}\,\text{cm}$

Required angle $= \tan^{-1}\dfrac{AF}{FK} = \dfrac{10}{12\sqrt{2}} \approx 31°$

23. C

Let the length of cube be 2. M and N be midpoints of BD and PQ respectively.

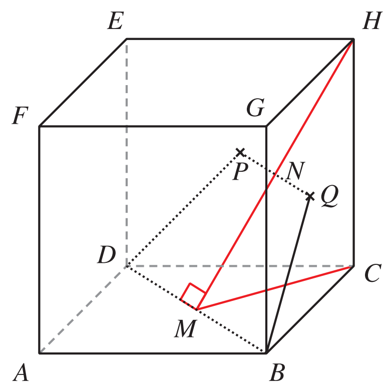
Note that since D, P, H are collinear and B, Q, H are collinear, we have M, N and H are collinear.

Note that $MH \perp BD$ and $CM \perp BD$.

The angle required is $\angle CMH$.

$CM = \dfrac{1}{2}\sqrt{2^2 + 2^2} = \sqrt{2}$.

$\angle CMH = \tan^{-1}\dfrac{2}{\sqrt{2}} \approx 55°$.



24. B

Let $BC = 1$.

$AE = BF = 1\sin 60° = \dfrac{\sqrt{3}}{2}$

$AC = \dfrac{AE}{\sin 30°} = \sqrt{3}$

$\cos\angle ACB = \dfrac{BC}{AC} = \dfrac{1}{\sqrt{3}}$

25. B

Let M and N be midpoints of BC and AD respectively.

Required angle is $\angle MVN$.

$MV = NV = \sqrt{8^2 - \left(\dfrac{4}{2}\right)^2} = \sqrt{60}\,\text{cm}$

$6^2 = MV^2 + NV^2 - 2(MV)(NV)\cos\angle MVN$

$\angle MVN \approx 46°$

26. C

Let K be a point on VB such that $AK \perp VB$. We have $CK \perp VB$ also.

Required angle is $\angle AKC$.

$AC = \sqrt{12^2 + 12^2} = 12\sqrt{2}\,\text{cm}$

$VA = VB = VC = \sqrt{8^2 + \left(\frac{12\sqrt{2}}{2}\right)^2} = \sqrt{136}\,\text{cm}$

In $\triangle VAB$,

$$\cos \angle ABV = \frac{\left(\frac{12}{2}\right)}{VB}$$

$$\angle ABV \approx 59.0^\circ$$

$AK = CK = 12 \sin \angle ABV \approx 10.3\,\text{cm}$

In $\triangle AKC$,

$$AC^2 = AK^2 + CK^2 - 2(AK)(CK)\cos \angle AKC$$

$$\angle AKC \approx 111^\circ$$

27. C

$15^2 = 9^2 + 12^2 \quad \Rightarrow \quad \angle BDC = 90^\circ$.

Since $\triangle ABD$ is vertical and CD is horizontal, $AD \perp CD$.

Therefore, $\theta = \angle ADB = \tan^{-1} \frac{12}{9}$ and so $\sin \theta = \frac{4}{5}$.

28. D

Let D be a point on BC such that $PD \perp BC$.

Required angle is $\angle ADP$.

$BC = \sqrt{4^2 + 3^2} = 5\,\text{m}$

$$\frac{(PD)(BC)}{2} = \frac{(PB)(PC)}{2}$$

$$PD = 2.4\,\text{m}$$

$$\tan \angle ADP = \frac{5}{2.4} = \frac{25}{12}$$

29. D

Refer to the figure. Let E be the midpoint of BC and length of each side be x cm.

In $\triangle AED$, $AE = DE = x \sin 60° = \dfrac{\sqrt{3}x}{2}$ cm

$$x^2 = AE^2 + DE^2 - 2(AE)(DE)\cos\angle AED$$

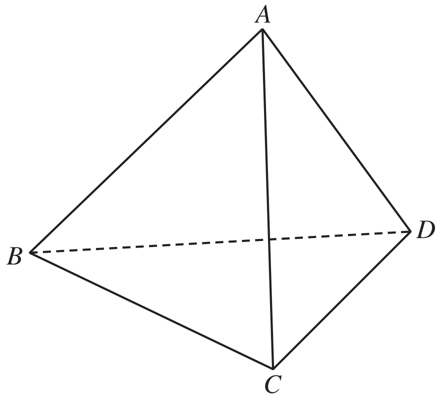
$\angle AED = \cos^{-1}\dfrac{1}{3}$

Let X be the projection of A on the plane BCD. Then X lies on DE and $AX \perp BD$.

In $\triangle AEX$,

$$AE \sin\angle AED = 4$$

$$x = 2\sqrt{6}$$



30. B

Refer to the figure. Let E be the midpoint of BC and length of each side be x cm.

In $\triangle AED$, $AE = DE = x \sin 60° = \dfrac{\sqrt{3}x}{2}$ cm

$$x^2 = AE^2 + DE^2 - 2(AE)(DE)\cos\angle AED$$

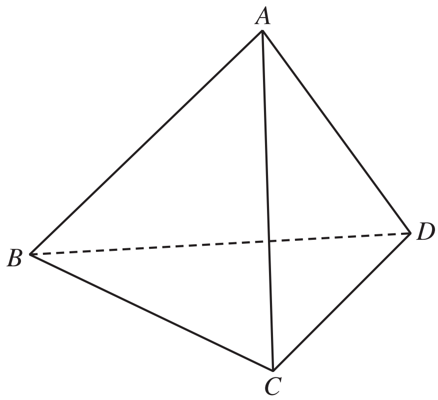
$\angle AED = \cos^{-1}\dfrac{1}{3}$

Let X be the projection of A on the plane BCD. Then X is the centroid of $\triangle BCD$ and it lies on DE.

In $\triangle AEX$, $\angle AXE = 90°$ and

$$AE \sin\angle AED = 2$$

$$x = \sqrt{6}$$



Volume $= \dfrac{1}{2}(\sqrt{6})^2 \sin 60°(2) \times \dfrac{1}{3} = \sqrt{3}\,\text{cm}^3$

Conventional Questions

31. (a) In $\triangle VAB$, $VB = VA = 24$ cm, and

$$24^2 = 12^2 + 24^2 - 2(12)(24)\cos\angle VBA \qquad \text{1M}$$

$$\angle VBA \approx 75.5^\circ \qquad \text{1A}$$

In $\triangle ABD$, $AD = 12\sin\angle VBA \approx 11.6$ cm 1A

(b) The angle between the two planes is $\angle ADC$. 1A

In $\triangle ACD$, $CD = AD$, and 1A

$$12^2 = AD^2 + CD^2 - 2(AD)(CD)\cos\angle ADC \qquad \text{1M}$$

$$\angle ADC \approx 62.2^\circ \qquad \text{1A}$$

Required angle is 62.2°.

32. (a) In $\triangle VBC$,

$$VD = 1.2\sin 60^\circ$$

$$= \frac{3\sqrt{3}}{5}\text{ m}$$

In $\triangle ABC$,

$$AD = 1.2\sin 60^\circ$$

$$= \frac{3\sqrt{3}}{5}\text{ m}$$

In $\triangle VAD$,

$$1.2^2 = \left(\frac{3\sqrt{3}}{5}\right)^2 + \left(\frac{3\sqrt{3}}{5}\right)^2 - 2\left(\frac{3\sqrt{3}}{5}\right)^2\cos\angle VDA \qquad \text{1M}$$

$$\angle VDA \approx 70.528\,779\,37^\circ$$

In $\triangle VGD$,

$$VG = \frac{3\sqrt{3}}{5}\sin\angle VDA \qquad \text{1M}$$

$$\approx 0.980\text{ m} \qquad \text{1A}$$

(b) Required angle is also equal to the angle between VBC and ABC, i.e., $\angle VDA$. 1M+1A

Required angle is 70.5°. 1A

33. (a) $CE = BE = \sqrt{12^2 + 20^2} = 4\sqrt{34}$ cm 1A

$BC^2 = 12^2 + 12^2 - 2(12)(12)\cos 40°$ 1M

$BC \approx 8.21$ cm

$BC^2 = BE^2 + CE^2 - 2(CE)(BE)\cos\angle BEC$ 1M

$\angle BEC \approx 20.3°$ 1A

(b) Let M be the midpoint of DF, such that EM is perpendicular to the plane $BCDF$.

Required angle is $\angle ECM$.

$ME = 12\cos\dfrac{40°}{2} \approx 11.3$ cm 1M

$\sin\angle ECM = \dfrac{ME}{\sqrt{544}}$ 1M

$\angle ECM \approx 28.9°$ 1A

(c) Let N be the midpoint of BC. Required angle is $\angle ENA$.

$AN = ME \approx 11.3$ cm

$\tan\angle ENA = \dfrac{20}{AN}$ 1M

$\angle ENA \approx 60.6°$ 1A

34. (a) Since $XB \perp BC$, we have $XB^2 + BC^2 = XC^2$. 1M

Since $AB \perp BC$, we have $AB^2 + BC^2 = AC^2$.

If $AX \perp XB$, then $AX^2 + XB^2 = AB^2$. 1M

$$AX^2 + XC^2 = AX^2 + (XB^2 + BC^2)$$
$$= AB^2 + BC^2$$
$$= AC^2$$

Hence, $AX \perp XC$. 1

(b) (i) In $\triangle ABF$,

$$BF^2 = 1^2 + (3\sqrt{2})^2 - 2(1)(3\sqrt{2})\cos 135°$$ 1M

$$BF = 5 \text{ m}$$

Therefore,

$$1^2 = 5^2 + (3\sqrt{2})^2 - 2(5)(3\sqrt{2})\cos\angle ABF$$ 1M

$$\cos\angle ABF = \sqrt{\frac{49}{50}}$$ 1A

In $\triangle ABX$, $XB = AB\cos\angle ABF = 3\sqrt{2} \times \sqrt{\frac{49}{50}} = \frac{21}{5}$ m 1

Remarks:

The method of using $\angle ABF \approx 8.13°$ can only gives an **approximated** value of XB. This cannot be used to show $XB = \frac{21}{5}$ m.

(ii) By (a), $AX \perp XB \Rightarrow AX \perp XC$ and $AX \perp XF \Rightarrow AX \perp XE$.

So, the required angle θ is $\angle CXE$. 1A

$$CX = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{21}{5}\right)^2} = \frac{7\sqrt{10}}{5} \text{ m}$$ 1A

$$FX = 5 - \frac{21}{5} = \frac{4}{5}$$

$$EX = \sqrt{\left(\frac{7}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \frac{\sqrt{65}}{5}$$

In $\triangle CEX$,

$$5^2 = \left(\frac{7\sqrt{10}}{5}\right)^2 + \left(\frac{\sqrt{65}}{5}\right)^2 - 2\left(\frac{7\sqrt{10}}{5}\right)\left(\frac{\sqrt{65}}{5}\right)\cos\theta$$ 1M

$$\cos\theta = \frac{1}{\sqrt{26}}$$ 1A

So, $\tan\theta = \frac{\sqrt{26 - 1^2}}{-1} = -5$. 1M+1A

 END OF PAPER