Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Extended Part

S5 – S6 M2 Integration Assignment $\boxed{\text{Set 6}}$

Name: ______________________ Centre: ____________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S5 – S6 M2 Integration
Phase 2 – Lesson 2

1. (a) $\displaystyle\int \sin(\ln x)\,\mathrm{d}x = x\sin(\ln x) - \int x\cos(\ln x)\left(\frac{1}{x}\right)\mathrm{d}x$ 1M

$\displaystyle = x\sin(\ln x) - x\cos(\ln x) + \int x[-\sin(\ln x)]\left(\frac{1}{x}\right)\mathrm{d}x$ 1M

$\displaystyle 2\int \sin(\ln x)\,\mathrm{d}x = x\sin(\ln x) - x\cos(\ln x) + \text{constant}$

$\displaystyle \int \sin(\ln x)\,\mathrm{d}x = \frac{x\sin(\ln x)}{2} - \frac{x\cos(\ln x)}{2} + \text{constant}$ 1A

(b) Let $u = x+1$. Then $\mathrm{d}u = \mathrm{d}x$.

$\displaystyle \int_0^{e^\pi - 1} \sin[\ln(x+1)]\,\mathrm{d}x = \int_1^{e^\pi} \sin(\ln x)\,\mathrm{d}x$

$\displaystyle = \left[\frac{x\sin(\ln x)}{2} - \frac{x\cos(\ln x)}{2}\right]_1^{e^\pi}$ 1M

$\displaystyle = \frac{e^\pi + 1}{2}$ 1A

2. (a) $\displaystyle \frac{x}{x^2+4} - \frac{1}{x+4} = \frac{x(x+4)-(x^2+4)}{(x^2+4)(x+4)}$ 1M

$\displaystyle = \frac{4x-4}{(x^2+4)(x+4)}$

$\displaystyle = \frac{4(x-1)}{(x^2+4)(x+4)}$ 1

(b) Let $t = 2\tan\theta$. Then $\mathrm{d}t = 2\sec^2\theta\,\mathrm{d}\theta$.

$\displaystyle \int_0^{\frac{\pi}{4}} \frac{2\tan\theta - 1}{\tan\theta + 2}\,\mathrm{d}\theta = \int_0^{\frac{\pi}{4}} \frac{2\tan\theta - 1}{2\sec^2\theta(\tan\theta+2)}\cdot 2\sec^2\theta\,\mathrm{d}\theta$

$\displaystyle = \int_0^2 \frac{t-1}{2\left(1+\frac{t^2}{4}\right)\left(\frac{t}{2}+2\right)}\,\mathrm{d}t$ 1A

$\displaystyle = \int_0^2 \frac{4(t-1)}{(t^2+4)(t+4)}\,\mathrm{d}t$

$\displaystyle = \int_0^2 \left(\frac{t}{t^2+4} - \frac{1}{t+4}\right)\mathrm{d}t$ 1M

Let $u = t^2+4$. Then $\mathrm{d}u = 2t\,\mathrm{d}t$. 1M

$\displaystyle \int_0^{\frac{\pi}{4}} \frac{2\tan\theta - 1}{\tan\theta + 2}\,\mathrm{d}\theta = \frac{1}{2}\int_4^8 \frac{\mathrm{d}u}{u} - \int_0^2 \frac{\mathrm{d}t}{t+4}$

$\displaystyle = \frac{1}{2}\Big[\ln|u|\Big]_4^8 - \Big[\ln|t+4|\Big]_0^2$ 1A

$\displaystyle = \frac{1}{2}(\ln 8 - \ln 4) - (\ln 6 - \ln 2)$

$\displaystyle = \frac{5}{2}\ln 2 - \ln 6$ 1A

3. (a) Let $x = 2\sin\theta$. Then $dx = 2\cos\theta\, d\theta$. 1M

$$\int_0^2 \sqrt{4-x^2}\, dx = \int_0^{\frac{\pi}{2}} \sqrt{4-4\sin^2\theta}(2\cos\theta)\, d\theta$$

$$= 4\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$= 2\int_0^{\frac{\pi}{2}} (1+\cos 2\theta)\, d\theta \quad \text{1M}$$

$$= 2\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}} \quad \text{1M}$$

$$= \pi \quad \text{1A}$$

(b) $$\int_0^1 xe^x\, dx = \left[xe^x\right]_0^1 - \int_0^1 e^x\, dx \quad \text{1M+1A}$$

$$= e - \left[e^x\right]_0^1$$

$$= 1 \quad \text{1A}$$

4. (a) Let $x = 3\tan\theta$. Then $dx = 3\sec^2\theta\, d\theta$. 1M

$$\int_0^{\sqrt{3}} \frac{1}{x^2+9}\, dx = \int_0^{\frac{\pi}{6}} \frac{3\sec^2\theta}{9\tan^2\theta+9}\, d\theta \quad \text{1A}$$

$$= \frac{1}{3}\int_0^{\frac{\pi}{6}} d\theta$$

$$= \frac{1}{3}\left[\theta\right]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{18} \quad \text{1A}$$

(b) $$\int_0^{\sqrt{3}} \ln(x^2+9)\, dx = \left[x\ln(x^2+9)\right]_0^{\sqrt{3}} - \int_0^{\sqrt{3}} \frac{2x^2}{x^2+9}\, dx \quad \text{1M}$$

$$= \sqrt{3}\ln 12 - 2\int_0^{\sqrt{3}} \left(1 - \frac{9}{x^2+9}\right) dx \quad \text{1M}$$

$$= \sqrt{3}\ln 12 - 2\int_0^{\sqrt{3}} dx + 18\int_0^{\sqrt{3}} \frac{1}{x^2+9}\, dx$$

$$= \sqrt{3}\ln 12 - 2\left[x\right]_0^{\sqrt{3}} + 18\left(\frac{\pi}{18}\right) \quad \text{1M}$$

$$= \sqrt{3}\ln 12 - 2\sqrt{3} + \pi \quad \text{1A}$$

5. (a) $$\int x\cos kx\,dx = \frac{x\sin kx}{k} - \frac{1}{k}\int \sin kx\,dx \qquad \text{1M}$$

$$= \frac{x\sin kx}{x} + \frac{\cos kx}{k^2} + \text{constant} \qquad \text{1A}$$

(b) $$\int_0^{\frac{\pi}{2}} x\cos x\cos 2x\,dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} x(\cos 3x + \cos x)\,dx \qquad \text{1M}$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} x\cos 3x\,dx + \frac{1}{2}\int_0^{\frac{\pi}{2}} x\cos x\,dx$$

$$= \frac{1}{2}\left[\frac{x\sin 3x}{3} + \frac{\cos 3x}{9}\right]_0^{\frac{\pi}{2}} + \frac{1}{2}\Big[x\sin x + \cos x\Big]_0^{\frac{\pi}{2}} \qquad \text{1M}$$

$$= \frac{\pi}{6} - \frac{5}{9} \qquad \text{1A}$$

6. (a) Let $u = \sin x$. Then $du = \cos x\,dx$. 1M

$$\int_0^{\frac{\pi}{2}} e^{\sin x}\cos^3 x\,dx = \int_0^1 e^u(1-u^2)\,du$$

$$= \Big[e^u(1-u^2)\Big]_0^1 + 2\int_0^1 ue^u\,du \qquad \text{1M}$$

$$= -1 + 2\Big[ue^u\Big]_0^1 - 2\int_0^1 e^u\,du$$

$$= 1 + 2e - 2\Big[e^u\Big]_0^1$$

$$= 1 \qquad \text{1A}$$

(b) $$\cos 3x = \cos(2x + x)$$

$$= \cos 2x\cos x - \sin 2x\sin x \qquad \text{1M}$$

$$= (2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x \qquad \text{1M}$$

$$= 4\cos^3 x - 3\cos x \qquad \text{1A}$$

$$\int_0^{\frac{\pi}{2}} e^{\sin 3x}\,dx = \int_0^{\frac{\pi}{2}} e^{\sin x}(4\cos^3 x - 3\cos x)\,dx \qquad \text{1M}$$

$$= 4\int_0^{\frac{\pi}{2}} e^{\sin x}\cos^3 x\,dx - 3\int_0^{\frac{\pi}{2}} e^{\sin x}\cos x\,dx$$

$$= 4(1) - 3\Big[e^{\sin x}\Big]_0^{\frac{\pi}{2}} \qquad \text{1A}$$

$$= 7 - 3e \qquad \text{1A}$$

7. (a) (i) $\tan\alpha = \dfrac{\cos\frac{2\pi}{13} - 1}{\sin\frac{2\pi}{13}}$

$= \dfrac{1 - 2\sin^2\frac{\pi}{13} - 1}{2\sin\frac{\pi}{13}\cos\frac{\pi}{13}}$ 1M

$= -\tan\dfrac{\pi}{13}$

$= \tan\left(-\dfrac{\pi}{13}\right)$

$\alpha = -\dfrac{\pi}{13}$ 1

(ii) $\tan\beta = \dfrac{\cos\frac{2\pi}{13} + 1}{\sin\frac{2\pi}{13}}$

$= \dfrac{2\cos^2\frac{\pi}{13} - 1 + 1}{2\sin\frac{\pi}{13}\cos\frac{\pi}{13}}$ 1M

$= \cot\dfrac{\pi}{13}$

$= \tan\left(\dfrac{\pi}{2} - \dfrac{\pi}{13}\right)$

$= \tan\dfrac{11\pi}{26}$

$\beta = \dfrac{11\pi}{26}$ 1A

(b) (i) $x^2 + 2x\cos\dfrac{2\pi}{13} + 1 = \left(x + \cos\dfrac{2\pi}{13}\right)^2 + \sin^2\dfrac{2\pi}{13}$ 1A

(ii) Let $x + \cos\dfrac{2\pi}{13} = \sin\dfrac{2\pi}{13}\tan\theta$. Then $dx = \sin\dfrac{2\pi}{13}\sec^2\theta\, d\theta$. 1M+1A

When $x = -1$,

$\tan\theta = \dfrac{-1 + \cos\frac{2\pi}{13}}{\sin\frac{2\pi}{13}}$

$\theta = -\dfrac{\pi}{13}$ 1M

When $x = 1$,

$\tan\theta = \dfrac{1 + \cos\frac{2\pi}{13}}{\sin\frac{2\pi}{13}}$

$\theta = \dfrac{11\pi}{26}$

$$\int_{-1}^{1} \frac{\sin \frac{2\pi}{13}}{x^2 + 2x\cos\frac{2\pi}{13} + 1}\,\mathrm{d}x = \int_{-1}^{2} \frac{\sin \frac{2\pi}{13}}{\left(x + \cos\frac{2\pi}{13}\right)^2 + \sin^2 \frac{2\pi}{13}}\,\mathrm{d}x \qquad \text{1A}$$

$$= \int_{-\frac{\pi}{13}}^{\frac{11\pi}{26}} \frac{\sin^2 \frac{2\pi}{13} \sec^2\theta}{\sin^2 \frac{2\pi}{13} \tan^2\theta + \sin^2 \frac{2\pi}{13}}\,\mathrm{d}\theta$$

$$= \int_{-\frac{\pi}{13}}^{\frac{11\pi}{26}} \mathrm{d}\theta$$

$$= \Big[\theta\Big]_{-\frac{\pi}{13}}^{\frac{11\pi}{26}}$$

$$= \frac{\pi}{2} \qquad \text{1A}$$

(c) Let $u = -x$. Then $\mathrm{d}u = -\,\mathrm{d}x$. 1M

$$\int_{-1}^{1} \frac{\sin \frac{11\pi}{13}}{x^2 + 2x\cos\frac{11\pi}{13} + 1}\,\mathrm{d}x = \int_{-1}^{1} \frac{\sin \frac{2\pi}{13}}{x^2 - 2x\cos\frac{2\pi}{13} + 1}\,\mathrm{d}x \qquad \text{1A}$$

$$= -\int_{1}^{-1} \frac{\sin \frac{2\pi}{13}}{u^2 + 2u\cos\frac{2\pi}{13} + 1}\,\mathrm{d}u$$

$$= \int_{-1}^{1} \frac{\sin \frac{2\pi}{13}}{u^2 + 2u\cos\frac{2\pi}{13} + 1}\,\mathrm{d}u$$

$$= \frac{\pi}{2} \qquad \text{1A}$$

8. (a) $\displaystyle\int x(x+2)e^x\,\mathrm{d}x = (x^2+2x)e^x - \int (2x+2)e^x\,\mathrm{d}x$ 1M

$$= (x^2+2x)e^x - 2(x+1)e^x + 2\int e^x\,\mathrm{d}x \qquad \text{1M}$$

$$= (x^2-2)e^x + 2e^x + \text{constant}$$

$$= x^2e^x + \text{constant} \qquad \text{1A}$$

(b) $\displaystyle\int_{1}^{e} x(x+2)e^x \ln x\,\mathrm{d}x = \Big[x^2e^x\ln x\Big]_1^e - \int_1^e x^2e^x \cdot \frac{1}{x}\,\mathrm{d}x$ 1M+1A

$$= e^{e+2} - \int_1^e xe^x\,\mathrm{d}x$$

$$= e^{e+2} - \Big[xe^x\Big]_1^e + \int_1^3 e^x\,\mathrm{d}x \qquad \text{1M}$$

$$= e^{e+2} - (e^{e+1} - e) + \Big[e^x\Big]_1^e$$

$$= e^{e+2} - e^{e+1} + e^e \qquad \text{1A}$$

9. (a) $\dfrac{A}{x+1} + \dfrac{B}{x+3} = \dfrac{A(x+3) + B(x+1)}{(x+1)(x+3)}$

$= \dfrac{(A+B)x + (3A+B)}{(x+1)(x+3)}$

We have $A + B = 1$ and $3A + B = 0$. 1M

Solving, we have $A = -\dfrac{1}{2}$ and $B = \dfrac{3}{2}$. 1A

(b) $\displaystyle\int \frac{x}{(x+1)(x+3)}\,\mathrm{d}x = \int \left(-\frac{1}{2(x+1)} + \frac{3}{2(x+3)}\right)\mathrm{d}x$ 1M

$= -\dfrac{1}{2}\ln|x+1| + \dfrac{3}{2}\ln|x+3| + \text{constant}$ 1A

(c) Let $u = \sqrt{x}$. Then $\mathrm{d}u = \dfrac{1}{2\sqrt{x}}\,\mathrm{d}x$. 1M

$$\int_1^9 \frac{1}{(\sqrt{x}+1)(\sqrt{x}+3)}\,\mathrm{d}x = 2\int_1^9 \frac{\sqrt{x}}{2\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+3)}\,\mathrm{d}x$$

$$= 2\int_1^3 \frac{u}{(u+1)(u+3)}\,\mathrm{d}u$$

$$= 2\left[-\frac{1}{2}\ln|u+1| + \frac{3}{2}\ln|u+3|\right]_1^3$$ 1M

$$= 3\ln 3 - 4\ln 2$$ 1A

10. (a) $\displaystyle\int_{\frac{5\pi}{4}}^{2\pi} x\sin x\,dx = \Big[-x\cos x\Big]_{\frac{5\pi}{4}}^{2\pi} + \int_{\frac{5\pi}{4}}^{2\pi} \cos x\,dx$ 1M

$$= -2\pi - \frac{5\sqrt{2}\pi}{8} + \Big[\sin x\Big]_{\frac{5\pi}{4}}^{2\pi}$$ 1A

$$= -\frac{(5\sqrt{2}+16)\pi}{8} + \frac{\sqrt{2}}{2}$$ 1A

(b) $\displaystyle\int_{\frac{5\pi}{4}}^{2\pi} x^2\sin x\,dx = \Big[-x^2\cos x\Big]_{\frac{5\pi}{4}}^{2\pi} + 2\int_{\frac{5\pi}{4}}^{2\pi} x\cos x\,dx$ 1M

$$= -4\pi^2 - \frac{25\sqrt{2}\pi^2}{32} + 2\Big[x\sin x\Big]_{\frac{5\pi}{4}}^{2\pi} - 2\int_{\frac{5\pi}{4}}^{2\pi} \sin x\,dx$$ 1M

$$= -\frac{(25\sqrt{2}+128)\pi^2}{32} + \frac{5\sqrt{2}\pi}{4} + 2\Big[\cos x\Big]_{\frac{5\pi}{4}}^{2\pi}$$

$$= -\frac{(25\sqrt{2}+128)\pi^2}{32} + \frac{5\sqrt{2}\pi}{4} + 2 + \sqrt{2}$$ 1A

(c) Let $u = e^x + \pi$. Then $du = e^x\,dx$. 1M

$$\int_{\ln\frac{\pi}{4}}^{\ln\pi} e^{2x}(e^x+\pi)\sin(e^x+\pi)\,dx$$

$$= \int_{\frac{5\pi}{4}}^{2\pi} (u-\pi)u\sin u\,du$$ 1A

$$= \int_{\frac{5\pi}{4}}^{2\pi} u^2\sin u\,dx - \pi\int_{\frac{5\pi}{4}}^{2\pi} u\sin u\,du$$

$$= -\frac{(25\sqrt{2}+128)\pi^2}{32} + \frac{5\sqrt{2}\pi}{4} + 2 + \sqrt{2} - \pi\left[-\frac{(5\sqrt{2}+16)\pi}{8} + \frac{\sqrt{2}}{2}\right]$$ 1M

$$= -\frac{(5\sqrt{2}+64)\pi^2}{32} + \frac{3\sqrt{2}\pi}{4} + 2 + \sqrt{2}$$ 1A

11. (a) $2x^2 - x + 2 = 2\left(x - \frac{1}{4}\right)^2 + \frac{15}{8}$ 1A

Let $x - \frac{1}{4} = \frac{\sqrt{15}}{4}\tan\theta$. Then $\mathrm{d}x = \frac{\sqrt{15}}{4}\sec^2\theta\,\mathrm{d}\theta$. 1M

$$\int \frac{1}{2x^2 - x + 2}\,\mathrm{d}x = \int \frac{1}{2\left(x - \frac{1}{4}\right)^2 + \frac{15}{8}}\,\mathrm{d}x$$

$$= \int \frac{\frac{\sqrt{15}}{4}\sec^2\theta}{\frac{15}{8}\tan^2\theta + \frac{15}{8}}\,\mathrm{d}\theta \quad \text{1A}$$

$$= \frac{2\sqrt{15}}{15}\int \mathrm{d}\theta$$

$$= \frac{2\sqrt{15}}{15}\theta + \text{constant}$$

$$= \frac{2\sqrt{15}}{15}\tan^{-1}\frac{4x - 1}{\sqrt{15}} + \text{constant} \quad \text{1A}$$

(b) Let $u = \frac{1}{x}$. Then $\mathrm{d}u = -\frac{1}{x^2}\,\mathrm{d}x$.

$$\int_{\frac{1}{2}}^{2} \frac{1}{(x^5 + 1)(2x^2 - x + 2)}\,\mathrm{d}x = \int_{\frac{1}{2}}^{2} \frac{x^2}{(x^5 + 1)(2x^2 - x + 2)}\,\mathrm{d}x$$

$$= -\int_{2}^{\frac{1}{2}} \frac{\frac{1}{u^2}}{den}\left(\frac{1}{u^5} + 1\right)\left(\frac{2}{u^2} - \frac{1}{u} + 2\right)\mathrm{d}u \quad \text{1A}$$

$$= \int_{\frac{1}{2}}^{2} \frac{u^5}{(u^5 + 1)(2u^2 - u + 2)}\,\mathrm{d}u$$

$$= \int_{\frac{1}{2}}^{2} \frac{x^5}{(x^5 + 1)(2x^2 - x + 2)}\,\mathrm{d}x$$

$$\int_{\frac{1}{2}}^{2} \frac{1}{(x^5 + 1)(2x^2 - x + 2)}\,\mathrm{d}x$$

$$= \frac{1}{2}\left[\int_{\frac{1}{2}}^{2} \frac{1}{(x^5 + 1)(x^2 - x + 2)}\,\mathrm{d}x + \int_{\frac{1}{2}}^{2} \frac{x^5}{(x^5 + 1)(x^2 - x + 2)}\,\mathrm{d}x\right]$$

$$= \frac{1}{2}\int_{\frac{1}{2}}^{2} \frac{1 + x^5}{(x^5 + 1)(x^2 - x + 2)}\,\mathrm{d}x$$

$$= \frac{1}{2}\int_{\frac{1}{2}}^{2} \frac{1}{2x^2 - x + 2}\,\mathrm{d}x \quad \text{1A}$$

$$= \frac{\sqrt{15}}{15}\left[\tan^{-1}\frac{4x - 1}{\sqrt{15}}\right]_{\frac{1}{2}}^{2} \quad \text{1M}$$

$$= \frac{\sqrt{15}}{15}\left(\tan^{-1}\frac{7\sqrt{15}}{15} - \tan^{-1}\frac{\sqrt{15}}{15}\right) \quad \text{1A}$$

12. (a) $\int e^{3x}(x^2+1)\,dx = \dfrac{e^{3x}(x^2+1)}{3} - \dfrac{2}{3}\int xe^{3x}\,dx$ 1M

$= \dfrac{e^{3x}(x^2+1)}{3} - \dfrac{2xe^{3x}}{9} + \dfrac{2}{9}\int e^{3x}\,dx$ 1M

$= \dfrac{x^2e^{3x}}{3} - \dfrac{2xe^{3x}}{9} + \dfrac{e^{3x}}{3} + \dfrac{2}{27}e^{3x} + \text{constant}$

$= \dfrac{x^2e^{3x}}{3} - \dfrac{2xe^{3x}}{9} + \dfrac{11e^{3x}}{27} + \text{constant}$ 1A

(b) Let $u = \tan\theta$. Then $du = \sec^2\theta\,d\theta$. 1M

$\int_0^{\frac{\pi}{4}} e^{3\tan\theta}\sec^4\theta\,d\theta = \int_0^{\frac{\pi}{4}} e^{3\tan\theta}\sec^2\theta(\tan^2\theta+1)\,d\theta$ 1A

$= \int_0^1 e^{3u}(u^2+1)\,du$

$= \left[\dfrac{u^2e^{3u}}{3} - \dfrac{2ue^{3u}}{9} + \dfrac{11e^{3u}}{27}\right]_0^1$ 1M

$= \dfrac{14e^3-11}{27}$ 1A

13. (a) Let $u = 1-e^{-x}$. Then $du = e^{-x}\,dx$. 1M

$\int_{\ln 2}^{\ln 3} \dfrac{e^{-x}}{1-e^{-x}}\,dx = \int_{\frac{1}{2}}^{\frac{2}{3}} \dfrac{du}{u}$ 1A

$= \Big[\ln|u|\Big]_{\frac{1}{2}}^{\frac{1}{3}}$

$= \ln\dfrac{4}{3}$ 1A

(b) $\dfrac{A}{u} + \dfrac{B}{u-1} = \dfrac{A(u-1)+Bu}{u(u-1)}$

$= \dfrac{(A+B)u-A}{u(u-1)}$

We have $A+B=0$ and $-A=1$. 1M

Solving, we have $A=-1$ and $B=1$. 1A

(c) $\int_{\ln 2}^{\ln 3} \dfrac{1}{e^x(e^x-1)}\,dx = \int_{\ln 2}^{\ln 3}\left(-\dfrac{1}{e^x} + \dfrac{1}{e^x-1}\right)dx$ 1M

$= -\int_{\ln 2}^{\ln 3} e^{-x}\,dx + \int_{\ln 2}^{\ln 3} \dfrac{1}{e^x-1}\,dx$

$= -\int_{\ln 2}^{\ln 3} e^{-x}\,dx + \int_{\ln 2}^{\ln 3} \dfrac{e^{-x}}{1-e^{-x}}\,dx$

$= \Big[e^{-x}\Big]_{\ln 2}^{\ln 3} + \ln\dfrac{4}{3}$ 1M

$= -\dfrac{1}{6} + \ln\dfrac{4}{3}$ 1A

END OF PAPER