Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Extended Part

S5 – S6 M2 Differentiation Assignment $\boxed{\text{Set 6}}$

Name: ______________________ Centre: ____________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S5 – S6 M2 Differentiation
Phase 2 – Lesson 2

1. (a) Vertical asymptote is $x = 1$. 1A

$$f(x) = \frac{3x^2 - 3x + 1}{x - 1}$$
$$= 3x + \frac{1}{x - 1}$$ 1M

Oblique asymptote is $y = 3x$. 1A

(b) $f'(x) = 3 - \frac{1}{(x-1)^2}$ 1A

$f'(2) = 3 - \frac{1}{(2-1)^2}$

$= 2$ 1A

Required equation is

$$y - 7 = 2(x - 2)$$

$2x - y + 3 = 0$ 1A

2. (a) 4 1A

(b) $y = \frac{x^2 - 5x}{x + 4}$

$= x - 9 + \frac{36}{x + 4}$ 1M

Oblique asymptote is $y = x - 9$. 1A

(c) $\frac{dy}{dx} = 1 - \frac{36}{(x+4)^2}$ 1A

$= \frac{(x+10)(x-2)}{(x+4)^2}$

When $\frac{dy}{dx} = 0$, $x = -10$ or 2. 1A

x	$x < -10$	$-10 < x < -4$	$-4 < x < 2$	$x > 2$
$\frac{dy}{dx}$	+	−	−	+

1M

There is only one maximum point at $x = -10$.

The claim is agreed. 1A

3. (a) $g'(x) = \dfrac{-k(2x-6)}{(x^2-6x+21)^2}$ 1A

When $g'(x) = 0$, $x = 3$.

x	$x < 3$	$x > 3$
$g'(x)$	$-$	$+$

1M

Minimum point is $\left(3,\ \dfrac{k}{12}\right)$. 1A

(b) No vertical asymptote.

Horizontal asymptote is $y = 0$. 1A

(c) $\dfrac{k}{12} = -2$ 1M

$k = -24$

$g'(x) = \dfrac{48x-144}{(x^2-6x+21)^2}$

$g''(x) = \dfrac{(x^2-6x+21)^2(48)-(48x-144)(2)(x^2-6x+21)(2x-6)}{(x^2-6x+21)^4}$ 1M

$= \dfrac{-144(x-1)(x-5)}{(x^2-6x+21)^3}$

When $g''(x) = 0$, $x = 1$ or 5.

x	$x < 1$	$1 < x < 5$	$x > 5$
$g''(x)$	$-$	$+$	$-$

1M

Points of inflexion are $\left(1,\ -\dfrac{3}{2}\right)$ and $\left(5,\ -\dfrac{3}{2}\right)$. 1A+1A

4. (a) $f'(x) = 2x + \dfrac{8}{(x-1)^2}$ 1A

$f''(x) = 2 - \dfrac{16}{(x-1)^3}$ 1A

(b) (i) When $f'(x) = 0$,

$$2x = -\frac{8}{(x-1)^2}$$ 1M

$$2x(x-1)^2 = -8$$

$$2x^3 - 4x^2 + 2x - 8 = 0$$

$$2(x+1)(x^2 - 3x + 4) = 0$$ 1M

$x = -1$ or $x^2 - 3x + 4 = 0$ (rejected)

x	$x < -1$	$-1 < x < 1$	$x > 1$
$f'(x)$	$-$	$+$	$+$

Required range is $-1 < x < 1$ or $x > 1$. 1A

(ii) When $f''(x) = 0$,

$$2(x-1)^3 = 16$$

$$x - 1 = 2$$

$$x = 3$$

x	$x < 1$	$1 < x < 3$	$x > 3$
$f''(x)$	$+$	$-$	$+$

Required range is $x < 1$ or $x > 3$. 1A

(c) $f(-1) = 5$ and $f(3) = 5$.

The relative minimum point is $(-1,\ 5)$ and no relative maximum point. 1A+1A

Point of inflexion is $(3,\ 5)$. 1A

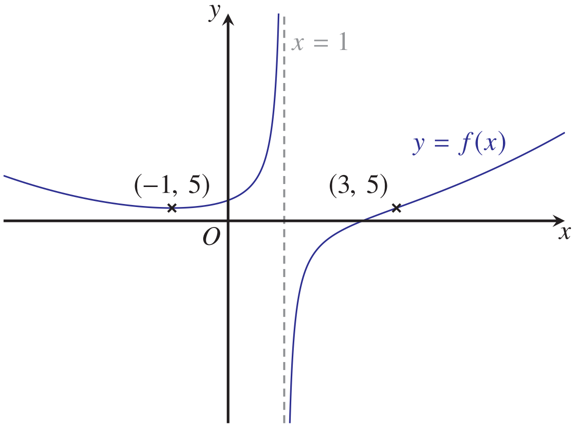
(d) Vertical asymptote is $x = 1$. 1A

No horizontal or oblique asymptotes.

(e) (Correct points and asymptotes) 1A

(Correct shape) 1A

(All correct) 1A



5. (a) $f(x) = x - 1 + \dfrac{1}{x+1}$

$f'(x) = 1 - \dfrac{1}{(x+1)^2}$ 1A

$f''(x) = \dfrac{2}{(x+1)^3}$ 1A

(b) (i) When $f'(x) = 0$, $x = 0$ or -2 1M

x	$x < -2$	$-2 < x < -1$	$-1 < x < 0$	$x > 0$
$f'(x)$	$+$	$-$	$-$	$+$

Required range is $x < -2$ or $x > 0$ 1A

(ii) $-2 < x < -1$ or $-1 < x < 0$ 1A

(iii) $f''(x) \neq 0$ for all real values of x.

x	$x < -1$	$x > -1$
$f''(x)$	$-$	$+$

Required range is $x > -1$. 1A

(iv) $x < -1$ 1A

(c) (i) Maximum point is $(-2,\ -4)$ and minimum point is $(0,\ 0)$ 1A+1A

(ii) There is no point of inflexion. 1A

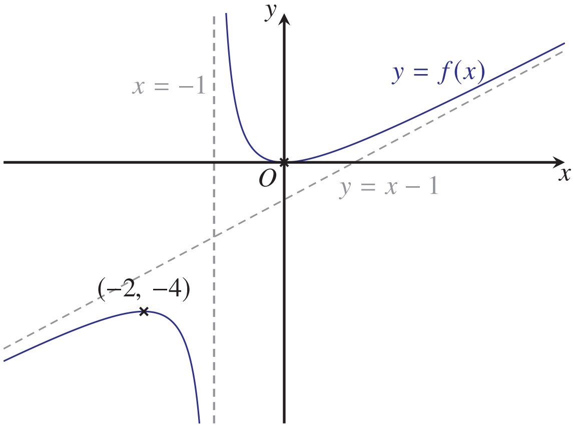
(d) Vertical asymptote is $x = -1$. 1A

$f(x) = x - 1 + \dfrac{1}{x+1}$ 1M

Oblique asymptote is $y = x - 1$. 1A

(e) (Correct shape and points) 1M+1A

(All correct) 1A



6. (a) The x-intercept is 0 and the y-intercept is 0. 1A

(b) $f'(x) = \dfrac{(x^3+2)(4x^3) - x^4(3x^2)}{(x^3+2)^2}$ 1M

$= \dfrac{x^6+8x^3}{(x^3+2)^2}$ 1A

$f''(x) = \dfrac{(x^3+2)^2(6x^5+24x^2) - (x^6+8x^3)\times 2(x^3+2)(3x^2)}{(x^3+2)^4}$

$= \dfrac{6x^2(x^3+2)[(x^3+2)(x^3+4) - (x^6+8x^3)]}{(x^3+2)^4}$ 1M

$= \dfrac{12x^2(4-x^3)}{(x^3+2)^3}$ 1

(c) (i) When $f'(x) = 0$, $x = 0$ or -2

x	$x < -2$	$-2 < x < -2^{\frac{1}{3}}$	$-2^{\frac{1}{3}} < x < 0$	$x > 0$
$f'(x)$	$+$	$-$	$-$	$+$

1M

The maximum point is $\left(-2,\ -\dfrac{8}{3}\right)$ and the minimum point is $(0,\ 0)$. 1A

When $f''(x) = 0$, $x = 0$ or $2^{\frac{2}{3}}$

x	$-2^{\frac{1}{3}} < x < 0$	$0 < x < 2^{\frac{2}{3}}$	$x > 2^{\frac{2}{3}}$
$f''(x)$	$+$	$+$	$-$

The point of inflexion is $\left(2^{\frac{2}{3}},\ \dfrac{2^{\frac{5}{3}}}{3}\right)$. 1A

(ii) Vertical asymptote is $x = -2^{\frac{1}{3}}$. 1A

$f(x) = \dfrac{x^4}{x^3+2} = x - \dfrac{2x}{x^3+2}$

The oblique asymptote is $y = x$. 1A

(iii) (Shape of the graph) 1A

(Asymptotes) 1A

(All correct) 1A