Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Extended Part

S5 – S6 M2 Differentiation Assignment $\boxed{\text{Set 5}}$

Name: ______________________ Centre: ____________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course S5 – S6 M2 Differentiation Phase 2 – Lesson 1

1. (a) $9 = 8a + 4b + 1$ 1M

$\frac{dy}{dx} = 3ax^2 + 2bx$

$0 = 3a(4) + 2b(2)$ 1M

Solving, we have $a = -2$ and $b = 6$. 1A

(b) $\frac{dy}{dx} = -6x^2 + 12x$

$\frac{d^2y}{dx^2} = -12x + 12$

$\left.\frac{d^2y}{dx^2}\right|_{(2,\ 9)} = -12(2) + 12 = -12 < 0$ 1M

Thus, (2, 9) is a maximum point. The claim is agreed. 1A

(c) When $\frac{d^2y}{dx^2} = 0$, $x = 1$.

x	$x < 1$	$x > 1$
$\frac{d^2y}{dx^2}$	+	−

1M

Point of inflexion is (1, 5). 1A

2. (a) $\dfrac{x^2-4}{x^2+4}=\dfrac{(x+2)(x-2)}{x^2+4}$

x-intercepts are 2 and -2 and y-intercept is -1. 1A

(b) $f(x)=1-\dfrac{8}{x^2+4}$

$f'(x)=\dfrac{16x}{(x^2+4)^2}$ 1A

$f''(x)=\dfrac{16(x^2+4)^2-16x(2)(x^2+4)(2x)}{(x^2+4)^4}$

$=\dfrac{-48x^2+64}{(x^2+4)^3}$ 1A

(c) (i) When $f'(x)=0$, $x=0$.

When $x=0$, $f''(x)=1>0$ and $f(x)=-1$. 1M

Minimum point is $(0,\ -1)$. 1A

(ii) When $f''(x)=0$, $x=\pm\dfrac{2}{\sqrt{3}}$.

x	$x<-\dfrac{2}{\sqrt{3}}$	$-\dfrac{2}{\sqrt{3}}<x<\dfrac{2}{\sqrt{3}}$	$x>\dfrac{2}{\sqrt{3}}$
$f''(x)$	$-$	$+$	$-$

1M

When $x=\pm\dfrac{2}{\sqrt{3}}$, $y=-\dfrac{1}{2}$.

The points of inflexion are $\left(\dfrac{2}{\sqrt{3}},\ -\dfrac{1}{2}\right)$ and $\left(-\dfrac{2}{\sqrt{3}},\ -\dfrac{1}{2}\right)$. 1A

(d) $f(x)=1-\dfrac{8}{x^2+4}$ 1M

There is no vertical asymptote and the horizontal asymptote is $y=1$. 1A

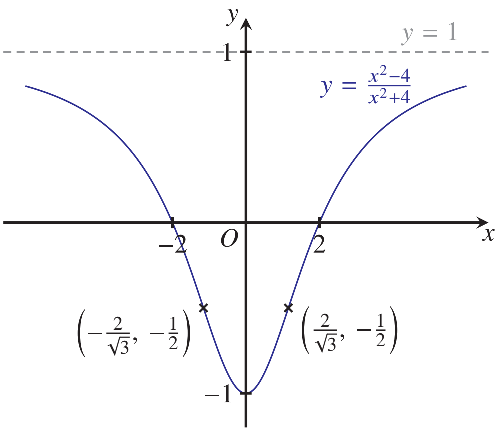
(e) $f(-x)=\dfrac{(-x)^2-4}{(-x)^2+4}=\dfrac{x^2-4}{x^2+4}=f(x)$.

Thus, $f(x)$ is an even function. 1

(f) (Correct shape) 1A

(Asymptotes + intercepts + stationary points) 1A

(All correct) 1A



3. (a) Vertical asymptote is $x = 1$. 1A

$$f(x) = x + 2 + \frac{3x - 2}{(x-1)^2}$$ 1M

Oblique asymptote is $y = x + 2$. 1A

(b) $f'(x) = 1 + \dfrac{3(x-1)^2 - (3x-2)(2)(x-1)}{(x-1)^4}$ 1M

$= 1 + \dfrac{-3x+1}{(x-1)^3}$ 1A

$1 + \dfrac{-3x+1}{(x-1)^3} = 0$

$(x-1)^3 - 3x + 1 = 0$

$x^2(x-3) = 0$

$x = 0$ or 3 1A

(c)

x	$x < 0$	$0 < x < 1$	$1 < x < 3$	$x > 3$
$f'(x)$	$+$	$+$	$-$	$+$

1M

Minimum point is $\left(3, \dfrac{27}{4}\right)$ and no maximum point. 1A

(d) $f''(x) = \dfrac{-3(x-1)^3 - (-3x+1)(3)(x-1)^2}{(x-1)^3}$

$= \dfrac{6x}{(x-1)^4}$ 1

(e) When $f''(x) = 0$, $x = 0$.

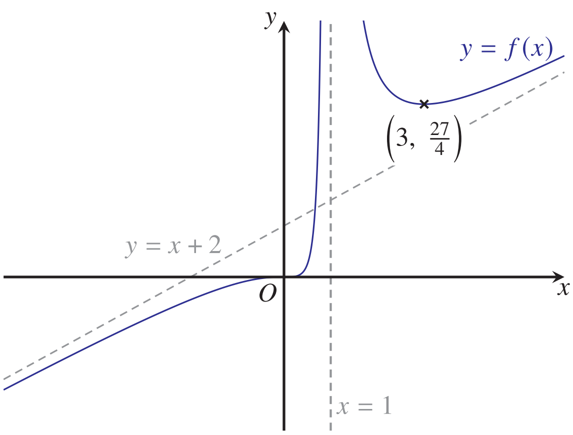
x	$x < 0$	$0 < x < 1$	$x > 1$
$f''(x)$	$-$	$+$	$+$

1M

Point of inflexion is $(0,\ 0)$. 1A

(f) (Correct shape) 1A

(All correct) 1A



4. (a) (i) $f'(x) = \dfrac{(2x+k)(x-1)-(x^2+kx-2)(1)}{(x-1)^2}$ 1M

$= \dfrac{x^2-2x+2-k}{(x-1)^2}$ 1A

(ii) $f'(-1) = \dfrac{1+2+2-k}{(-1-1)^2} = 0$

$k = 5$ 1

When $f'(x) = 0$,

$$\frac{x^2-2x+2-5}{(x-1)^2} = 0$$

$$\frac{(x-3)(x+1)}{(x-1)^2} = 0$$

$$x = -1 \quad \text{or} \quad 3$$

Thus, $h = 3$. 1A

(b) (i) We have $f(x) = \dfrac{x^2+5x-2}{x-1}$. So, $f(-1) = 3$ and $f(3) = 11$.

Maximum point is $(-1,\ 3)$. 1A

Minimum point is $(3,\ 11)$. 1A

(ii) $f''(x) = \dfrac{(2x-2)(x-1)^2-(x^2-2x-3)(2)(x-1)}{(x-1)^4}$

$= \dfrac{2(x-1)[(x-1)^2-(x^2-2x-3)]}{(x-1)^4}$

$= \dfrac{8}{(x-1)^3}$ 1A

We have $f''(x) \neq 0$ for all values of x.

Thus, there is no point of inflexion. 1

(c) Vertical asymptote is $x = 1$. 1A

$f(x) = \dfrac{x^2+5x-2}{x-1}$

$= x+6+\dfrac{4}{x-1}$ 1M

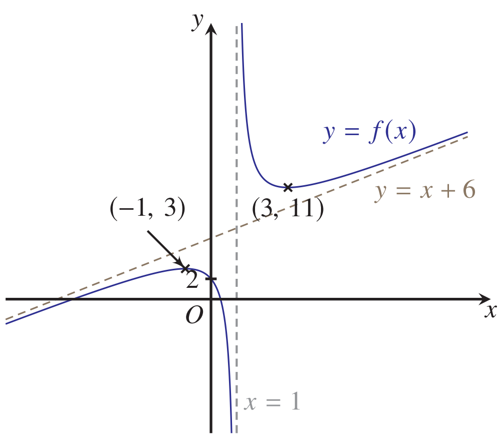
Oblique asymptote is $y = x+6$. 1A

(d) y-intercept $= 2$. 1A

(Correct shape) 1A

(Correct asymptote and points) 1A

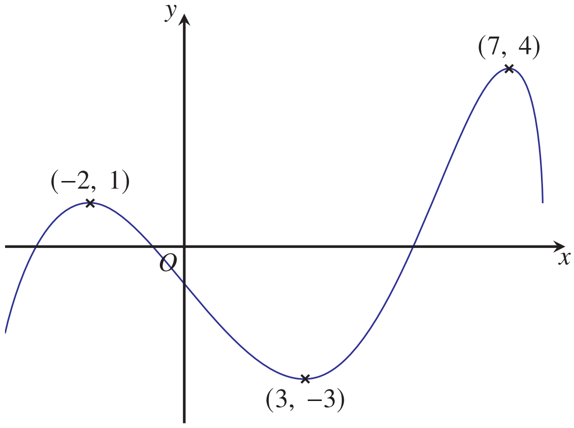
(All correct) 1A



5. (Correct points) 1A

(Correct shape) 1A

(All correct) 1A



6. (a) (i) $f'(x) = \dfrac{(2x-4)(x^2+1)-(x^2-4x+1)(2x)}{(x^2+1)^2}$ 1M

$= \dfrac{4x^2-4}{(x^2+1)^2}$ 1A

(ii) $f'(x) = \dfrac{4(x+1)(x-1)}{(x^2+1)^2}$.

When $f'(x) = 0$, $x = \pm 1$. 1M

x	$x < -1$	$-1 < x < 1$	$x > 1$
$f'(x)$	$+$	$-$	$+$

1M

$f(-1) = 3$ and $f(1) = -1$.

Thus, the maximum and minimum value of $f(x)$ are 3 and -1 respectively. 1

(b) $f''(x) = \dfrac{8x(x^2+1)^2-(4x^2-4)(2)(x^2+1)(2x)}{(x^2+1)^4}$ 1M

$= \dfrac{8x(x^2+1)-4x(4x^2-4)}{(x^2+1)^3}$

$= \dfrac{-8x^3+24x}{(x^2+1)^3}$ 1A

$= \dfrac{-8x(x^2-3)}{(x^2+1)^3}$

When $f''(x) = 0$, $x = 0$ or $\pm\sqrt{3}$.

x	$x < -\sqrt{3}$	$-\sqrt{3} < x < 0$	$0 < x < \sqrt{3}$	$x > \sqrt{3}$
$f''(x)$	$+$	$-$	$+$	$-$

1M

$f(-\sqrt{3}) = 1+\sqrt{3}$, $f(0) = 1$ and $f(\sqrt{3}) = 1-\sqrt{3}$

The points of inflexion are $\left(-\sqrt{3},\ 1+\sqrt{3}\right)$, $(0,\ 1)$ and $\left(\sqrt{3},\ 1-\sqrt{3}\right)$. 1A

(c) There are no vertical asymptotes. 1A

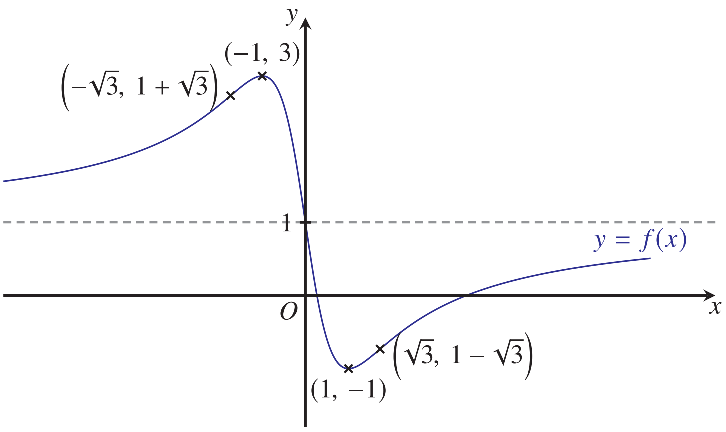
$f(x) = 1 - \dfrac{4x}{x^2+1}$. 1M

Horizontal asymptote is $y = 1$. 1A

(d) (Correct shape) 1A

(Correct points and asymptotes) 1A

(All correct) 1A



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