Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Compulsory Part

S5 – S6 Core Assignment Set 8

Name: ______________________ Centre: ____________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S5 – S6 Core
Phase 2 – Lesson 4

1. (a) $VB = 5\tan 40° \approx 4.20\text{ cm}$ 1M+1A

$VC = \dfrac{5}{\cos 40°} \approx 6.53\text{ cm}$ 1A

(b) $AB^2 = (5\tan 40°)^2 + (5\tan 40°)^2 - 2(5\tan 40°)(5\tan 40°)\cos 25°$ 1M

$AB \approx 1.82\text{ cm}$ 1A

(c) Let M be a point on VC such that $AM \perp VC$ and $BM \perp VC$.

The angle required is $\angle AMB$. 1A

$BM = 5\sin 40° \approx 3.21\text{ cm}$ 1M

Since $\triangle VAM \cong \triangle VBM$, $AM = BM \approx 3.21\text{ cm}$ 1M

In $\triangle AMB$,

$$AB^2 = (AM)^2 + (BM)^2 - 2(AM)(BM)\cos\angle AMB$$ 1M

$$\angle AMB \approx 32.8°$$ 1A

Required angle is 32.8°.

(d) Let N be the midpoint of AB.

Required angle is $\angle VCN$. 1A

$CN = \sqrt{5^2 - \left(\dfrac{AB}{2}\right)^2} \approx 4.92\text{ cm}$ 1M

$VN = VA\cos\dfrac{25°}{2} \approx 4.10\text{ cm}$ 1M

In $\triangle VCN$,

$$VN^2 = CN^2 + VC^2 - 2(CN)(VC)\cos\angle VCN$$ 1M

$$\angle VCN \approx 38.8°$$ 1A

Required angle is 38.8°.

2. (a) The paper card is symmetric about AC.

So, $\angle BAC = 60°$ and $\angle ACB = 180° - 60° - 70° = 50°$. 1A

$$\frac{AC}{\sin 70°} = \frac{10}{\sin 50°}$$ 1M

$$AC \approx 12.3\text{ cm}$$ 1A

$BC = \dfrac{10\sin 60°}{\sin 50°} \approx 11.3\text{ cm}$ 1A

(b) (i) $AE = \sqrt{AB^2 - BE^2}$ and $CE^2 = \sqrt{BC^2 - BE^2}$

$AC^2 = AE^2 + CE^2$

$= BC^2 + AB^2 - 2BE^2$ 1M

$BE \approx 6.22\text{ cm}$ 1A

(ii) Let h cm be the required distance.

$AE = \sqrt{10^2 - BE^2} \approx 7.83\text{ cm}$ and $CE = \sqrt{BC^2 - BE^2} \approx 9.4\text{ cm}$

$$\frac{1}{3} \times \frac{(AE)(DB)}{2} \times CE = \frac{1}{3} \times \frac{(AD)(AC)\sin 60°}{2} \times h$$ 1M+1M

$$h \approx 8.66$$ 1A

Required distance is 8.66 cm.

3. (a) (i) Let $s = \dfrac{6+7+5}{2} = 9$.

Area of $\triangle ABC = \sqrt{9(9-5)(9-6)(9-7)}$ 1M

$= 6\sqrt{6}$ 1A

(ii) $\dfrac{1}{2}(6)(r) + \dfrac{1}{2}(7)(r) + \dfrac{1}{2}(5)(r) = 6\sqrt{6}$ 1M

$r = \dfrac{2\sqrt{6}}{3}$ 1

(b) (i) Let X be the point of contact of AB and the inscribed circle in $\triangle ABC$.

Then $\angle VXO = 60°$. 1A

In $\triangle VXO$, $VO = r\tan 60° = \dfrac{2\sqrt{6}}{3} \times \sqrt{3} = 2\sqrt{2}$

Volume of $VABC = \dfrac{1}{3}(6\sqrt{6})(2\sqrt{2})$ 1M

$= 8\sqrt{3}$ 1A

(ii) Let Y be the point of contact of BC and the inscribed circle.

Since $\triangle VOX \cong \triangle VOY$ (SAS)

$\angle VYO = \angle VXO = 60°$ and $VY = \dfrac{r}{\cos 60°} = \dfrac{4\sqrt{6}}{3}$.

Area of $\triangle VBC = \dfrac{1}{2}(7)\left(\dfrac{4\sqrt{6}}{3}\right)$ 1M

$= \dfrac{14\sqrt{6}}{3}$ 1A

(iii) Let F be the foot of perpendicular from A to the plane VBC.

Required angle is $\angle ABF$. 1A

By considering the volume of tetrahedron $VABC$,

$8\sqrt{3} = \dfrac{1}{3}\left(\dfrac{14\sqrt{6}}{3}\right)(AF)$ 1M

$AF = \dfrac{18\sqrt{2}}{7}$

In $\triangle ABF$,

$\sin\angle ABF = \dfrac{18\sqrt{2}}{7} \div 6$

$\angle ABF \approx 37°$ 1A

4. (a) $PS^2 = 24^2 + 18^2 - 2(24)(18)\cos 50°$ 1M

$PS \approx 18.6$ cm 1A

(b) Let M be the midpoint of QR. The angle required is $\angle SMP$. 1M

$SM = \sqrt{18^2 - \left(\frac{20}{2}\right)^2} = 4\sqrt{14}$ cm 1M

$PM = \sqrt{24^2 - 10^2} = 2\sqrt{119}$ cm

$PS^2 = SM^2 + PM^2 - 2(SM)(PM)\cos\angle SMP$ 1M

$\angle SMP \approx 57.0°$ 1A

(c) $\angle PAS = \cos^{-1}\dfrac{AP^2 + AS^2 - SP^2}{2(AP)(AS)}$

By symmetry of the figure, $\angle PAS$ is the largest when A is at the midpoint of QR such that AP and AS are perpendicular to QR and are shortest.

When A moves from Q to midpoint M, $\angle PAS$ increases from 50° to 57.0°. 1A

When A moves from midpoint M to R, $\angle PAS$ decreases from 57.0° to 50°. 1A

5. (a) Let M be the midpoint of AC.

$BM = 20\sin 60° = 10\sqrt{3}$ cm

$BE = BM\sin 60° = 15$ cm 1A

$\angle BEC = 90°$. So, BC is a diameter of the circumcircle of $\triangle BCE$. 1M

Thus, $DE = DB = DC = \frac{20}{2} = 10$ cm 1A

(b) $AE = \sqrt{AB^2 - BE^2}$ 1M

$= \sqrt{175}$ cm

$AD = BM = 10\sqrt{3}$ cm

$AE^2 = AD^2 + DE^2 - 2(AD)(DE)\cos\angle ADE$ 1M

$\angle ADE \approx 49.5° \neq 90°$

The claim is disagreed. 1A