Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Compulsory Part

S5 – S6 Core Assignment Set 7

Name: ____________________ Centre: ____________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S5 – S6 Core
Phase 2 – Lesson 3

1. C

Let length of each edge be 2. Let K be a point on BV such that $AK \perp BV$ and $CK \perp BV$.
$AK = CK = 2\sin 60° = \sqrt{3}$ and $AC = \sqrt{2^2 + 2^2} = 2\sqrt{2}$
In $\triangle AKC$, $\angle AKC$ is the required angle.

$$(2\sqrt{2})^2 = (\sqrt{3})^2 + (\sqrt{3})^2 - 2(\sqrt{3})^2 \cos \angle AKC$$
$$\angle AKC \approx 109°$$

2. D

Let E be a point on CD such that $BE \perp CD$. Then $\theta = \angle AEB$.
$CD = \sqrt{24^2 + 7^2} = 25$ cm. By considering the area of $\triangle BCD$,

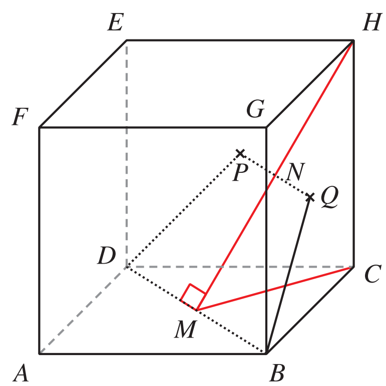
$$\frac{(25)(BE)}{2} = \frac{(24)(7)}{2}$$
$$BE = \frac{168}{25}$$

$$\tan\theta = \frac{7}{BE} = \frac{25}{24}$$

3. C

Let the length of cube be 2. M and N be midpoints of BD and PQ respectively.

Note that since D, P, H are collinear and B, Q, H are collinear, we have M, N and H are collinear.
Note that $MH \perp BD$ and $CM \perp BD$.
The angle required is $\angle CMH$.
$CM = \frac{1}{2}\sqrt{2^2 + 2^2} = \sqrt{2}$.
$\angle CMH = \tan^{-1}\frac{2}{\sqrt{2}} \approx 55°$.



4. D

Let K be a point on EG such that $AK \perp EG$.
Required angle is $\angle AKF$.
Consider the area of $\triangle EFG$.

$$\frac{1}{2}(8)(6) = \frac{1}{2}(EG)(FK)$$
$$24 = \frac{1}{2}\sqrt{8^2 + 6^2}(EK)$$
$$EK = 4.8\text{ cm}$$

$$\tan\theta = \frac{12}{4.8}$$
$$\theta \approx 68°$$

5. D

Let E be a point on AC such that $VE \perp AC$.
Required angle is $\angle VED$.
Since the pyramid is symmetric about the plane VAC,
we have $\angle VEB = \angle VED = \dfrac{180^\circ}{2} = 90^\circ$.

6. B

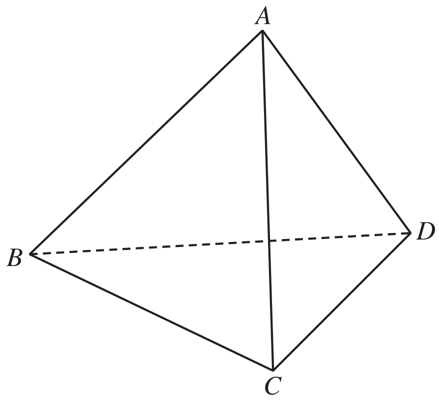
Since BV is perpendicular to the plane VAC, $\angle BVA = \angle BVC = 90^\circ$.
$AB = BC = \sqrt{6^2 + 8^2} = 10\,\text{cm}$ and $BM = BN = 5\,\text{cm}$
$\triangle BMN$ is equilateral. So, $MN = 5\,\text{cm}$.
$\angle VBM = \tan^{-1}\dfrac{8}{6}$

$$VM^2 = 6^2 + 5^2 - (2)(6)(5)\cos\angle VBM$$
$$VM = 5$$

So, $VM = VN = MN = 5\,\text{cm}$, and the required area is $\dfrac{1}{2}(5)^2 \sin 60^\circ = \dfrac{25\sqrt{3}}{4}\,\text{cm}^2$.

7. B

Refer to the figure. Let E be the midpoint of BC and length of each side be 12 cm.
In $\triangle AED$, $AE = DE = 12\sin 60^\circ = 6\sqrt{3}\,\text{cm}$
$$12^2 = AE^2 + DE^2 - 2(AE)(DE)\cos\angle AED$$
$\angle AED = \cos^{-1}\dfrac{1}{3}$
Let X be the projection of A on the plane BCD. Then
X is the centroid of $\triangle BCD$ and it lies on DE.
In $\triangle AEX$, $\angle AXE = 90^\circ$ and
height $= AE\sin\angle AED$
$= \sqrt{96}\,\text{cm}$
Volume $= \dfrac{1}{2}(12)^2 \sin 60^\circ(\sqrt{96}) \times \dfrac{1}{3}$
$= 144\sqrt{2}\,\text{cm}^3$



8. C

Let K be a point on VB such that $AK \perp VB$. We have $CK \perp VB$ also.
Required angle is $\angle AKC$.
$AC = \sqrt{12^2 + 12^2} = 12\sqrt{2}\,\text{cm}$
$VA = VB = VC = \sqrt{8^2 + \left(\frac{12\sqrt{2}}{2}\right)^2} = \sqrt{136}\,\text{cm}$
In $\triangle VAB$,

$$\cos\angle ABV = \frac{\left(\frac{12}{2}\right)}{VB}$$
$$\angle ABV \approx 59.0^\circ$$

$AK = CK = 12\sin\angle ABV \approx 10.3\,\text{cm}$
In $\triangle AKC$,

$$AC^2 = AK^2 + CK^2 - 2(AK)(CK)\cos\angle AKC$$
$$\angle AKC \approx 111^\circ$$

9. A

Required angle is $\angle PHF$.
$PF = 12 \times \frac{2}{1+2} = 8\,\text{cm}$ and $FH = \sqrt{8^2 + 6^2} = 10\,\text{cm}$
$\tan\angle PHF = \frac{8}{10}$
$\angle PHF \approx 39^\circ$

10. C

Let K be a point on AM such that $DK \perp AM$.
Then $\alpha = \angle EMD$ and $\beta = \angle EKD$.

I. ✓. Since $\angle DKM = 90^\circ$, we have $DK < DM$ and $\alpha < \beta$.

II. ✗. Since $\alpha < \beta$, we have $\cos\alpha > \cos\beta$.

III. ✓. $\angle DAM = \angle BMA = \tan^{-1}\frac{5}{2.5} \approx 63.4^\circ$
$DK = AD\sin\angle DAM \approx 4.47\,\text{cm}$
$\tan\beta = \frac{DE}{DK} = \frac{4}{\sqrt{5}}$

11. A

Let K be a point on ME such that $FK \perp ME$. [In fact, K is at the position of point M.]
Since $AF \perp EM$, we also have $AK \perp ME$. The angle required is therefore $\angle AKF$.
Since $MH = EH = 12\,\text{cm}$, $\angle EMH = 45^\circ$ and so $\angle FEM = 45^\circ$
$FK = FE\sin\angle FEM = 12\sqrt{2}\,\text{cm}$
Required angle $= \tan^{-1}\frac{AF}{FK} = \frac{10}{12\sqrt{2}} \approx 31^\circ$

12. D

Let D be a point on BC such that $PD \perp BC$.

Required angle is $\angle ADP$.

$BC = \sqrt{4^2 + 3^2} = 5\text{ m}$

$$\frac{(PD)(BC)}{2} = \frac{(PB)(PC)}{2}$$

$$PD = 2.4\text{ m}$$

$$\tan \angle ADP = \frac{5}{2.4} = \frac{25}{12}$$

13. B

Let M and N be midpoints of BC and AD respectively.

Required angle is $\angle MVN$.

$$MV = NV = \sqrt{8^2 - \left(\frac{4}{2}\right)^2} = \sqrt{60}\text{ cm}$$

$$6^2 = MV^2 + NV^2 - 2(MV)(NV)\cos \angle MVN$$

$\angle MVN \approx 46°$

14. C

$15^2 = 9^2 + 12^2 \quad \Rightarrow \quad \angle BDC = 90°$.

Since $\triangle ABD$ is vertical and CD is horizontal, $AD \perp CD$.

Therefore, $\theta = \angle ADB = \tan^{-1} \frac{12}{9}$ and so $\sin \theta = \frac{4}{5}$.

15. C

Let M be a point on BC such that $AM \perp BC$.

Required angle is $\angle AMD$.

Let the length of each edge be 2.

$AM = DM = \sqrt{2^2 - 1^2} = \sqrt{3}$

In $\triangle ADM$,

$$2^2 = (\sqrt{3})^2 + (\sqrt{3})^2 - 2(\sqrt{3})(\sqrt{3})\cos \angle AMD$$

$$\angle AMD \approx 71°$$

16. B

Required angle is $\angle CBH$ (or $\angle DAE$).

$\angle CBH = 45°$

17. B

Let $BC = 1$.

$AE = BF = 1\sin 60^\circ = \frac{\sqrt{3}}{2}$

$AC = \frac{AE}{\sin 30^\circ} = \sqrt{3}$

$\cos\angle ACB = \frac{BC}{AC} = \frac{1}{\sqrt{3}}$

18. B

We have $\angle VAC = 60^\circ$.

Since $VA = VC$, we have $\angle VCA = \angle VAC = 60^\circ$ and $\triangle VAC$ is equilateral.

$AC = \sqrt{1^2 + 1^2} = \sqrt{2}$ m

$VB = VA = VC = AC = \sqrt{2}$ m

In $\triangle VAB$,

$$1^2 = VA^2 + VB^2 - 2(VA)(VB)\cos\angle AVB$$

$$\angle AVB \approx 41^\circ$$

19. (a) In $\triangle BEF$,

$$FE^2 = k^2 + (rk)^2 - 2k(rk)\cos 60° \quad \text{1M}$$
$$= k^2(1 - r + r^2) \quad \text{1A}$$

In $\triangle AFG$, $FG \perp AC$.

$$FG^2 = (AF \sin 45°)^2 \quad \text{1M}$$
$$= [(1-r)k]^2 \sin^2 45°$$
$$= \frac{k^2(1-r)^2}{2} \quad \text{1A}$$

(b) $EG = \sqrt{FE^2 - FG^2}$

$$= \sqrt{k^2(1-r+r^2) - \frac{k^2}{2}(1-r)^2} \quad \text{1M}$$
$$= k\sqrt{\frac{1+r^2}{2}} \quad \text{1A}$$

$EN = AE \cos 45° = \frac{k}{\sqrt{2}}$ 1A

$\sin\theta = \frac{EN}{EG} = \frac{1}{\sqrt{1+r^2}}$ 1

(c) As r varies from 0 to 1, $\frac{1}{\sqrt{2}} \le \sin\theta \le 1$. 1M

Thus, $45° \le \theta \le 90°$. 1A

20. (a) $VM = \sqrt{12.5^2 - \left(\frac{20}{2}\right)^2} = 7.5\text{ cm}$ 1M+1A

$AM = \sqrt{26^2 - 10^2} = 24\text{ cm}$ 1M+1A

(b) Required angle is $\angle VMA$. 1A

In $\triangle VMA$,

$$18^2 = 7.5^2 + 24^2 - 2(7.5)(24)\cos\angle VMA \quad \text{1M}$$
$$\angle VMA \approx 31.1° \quad \text{1A}$$

(c) Let VH be the height of the tetrahedron. Then H lies on AM.

$$\sin\angle VMH = \frac{VH}{7.5} \quad \text{1M}$$
$$VH \approx 3.87\text{ cm}$$

Area of $\triangle ABC = \frac{1}{2}(20)(24) = 240\text{ cm}^2$

Volume of tetrahedron $= \frac{1}{3}(240)(VH)$ 1M

$\approx 310\text{ cm}^3$ 1A

21. (a) Let O be the circumcentre of circle $ABCD$.

$\angle BAC = \angle DAC$ (given)

$\angle BOC = 2\angle BAC$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$\angle DOC = 2\angle DAC$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)

$= \angle BOC$

$BC = CD$ (equal $\angle$s, equal chords)

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(b) (i) Let the coordinates of M be $(a,\ -a)$ such that it lies on $y = -x$.

$$\sqrt{(a-0)^2 + (-a-0)^2} = \sqrt{(a+200)^2 + (-a+600)^2}$$ 1M

$$2a^2 = 2a^2 - 800a + 400\,000$$

$$a = 500$$

Required equation is

$$(x-500)^2 + (y+500)^2 = (0-500)^2 + (0+500)^2$$

$$(x-500)^2 + (y+500)^2 = 500\,000$$ 1A

Coordinates of M are $(500,\ -500)$. 1A

(ii) $(0-500)^2 + (y+500)^2 = 500\,000$

$$(y+500)^2 = 250\,000$$

$$y = -1000 \quad \text{or} \quad 0 \text{ (rejected)}$$

Coordinates of C are $(0,\ -1000)$. 1A

(c) Let K be a point on VC such that $BK \perp VC$.

Then $DK \perp VC$ and the required angle is $\angle BKD$.

$BM = CM = DM = \sqrt{(500+200)^2 + (-500+600)^2} = 500\sqrt{2}$ 1M

$BC = CD = \sqrt{200^2 + (-1000+600)^2} = 200\sqrt{5}$

$VB = VC = VD = \sqrt{MB^2 + 50^2} = 50\sqrt{201}$

$$MB^2 = MC^2 + BC^2 - 2(MC)(BC)\cos\angle BCM$$

$\angle BCM \approx 71.6°$

$BD = 2 \times BC \sin\angle BCM \approx 849$ 1M

$$VB^2 = VC^2 + BC^2 - 2(VC)(BC)\cos\angle VCB$$

$\angle VCB \approx 71.6°$

$DK = BK = BC \sin\angle VCB \approx 424$

$$BD^2 = BK^2 + DK^2 - 2(BK)(DK)\cos\angle BKD$$ 1M

$\angle BKD \approx 177°$ 1A

END OF PAPER