Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Compulsory Part

S5 – S6 Core Assignment Set 6

Name: ____________________ Centre: ____________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S5 – S6 Core
Phase 2 – Lesson 2

1. B

I. ✓. Note that $\triangle AHF \cong \triangle DGE$, we have $\angle AHF = \angle DGE$.

II. ✗. $\angle AGH = 90°$ while $\angle DGE < 90°$.

III. ✓. Note that $\triangle BEG \cong \triangle DGE$, we have $\angle BEG = \angle DGE$.

2. B

A. Required angle is $\angle EBF$.

B. Required angle is $\angle ENF$.

C. Let Q be a point on $ABGF$ such that PQ is perpendicular to $ABGF$.
Required angle is $\angle PFQ$, which is equal to $\angle FPE$.

D. Let K be the midpoint of BG.
Required angle is $\angle MNK$, which is equal to $\angle CAB$.

By simple observation, we have $\angle ENF$ being the greatest angle among all.
The answer is B.

3. D

Let Q be the midpoint of AD.
Then $\theta = \angle PEQ$.

$$\tan\theta = \frac{PQ}{EQ} = \frac{y}{\sqrt{x^2+(2z)^2}} = \frac{y}{\sqrt{x^2+4z^2}}$$

4. B

Let $AB = 1$.
In $\triangle ABD$, $AD = \frac{1}{\sin 30°} = 2$ and $BD = \frac{1}{\tan 30°} = \sqrt{3}$.
In $\triangle ABC$, $AC = \frac{1}{\sin 45°} = \sqrt{2}$ and $BC = \frac{1}{\tan 45°} = 1$.

$CD^2 = 1^2 + (\sqrt{3})^2 - 2(1)(\sqrt{3})\cos 150°$

$CD = \sqrt{7}$

$CD^2 = AD^2 + AC^2 - 2(AD)(AC)\cos\angle CAD$

$\angle CAD \approx 100°$

5. C

Let N be the midpoint of GH.

Required angle is MFN.

$MN = 24\text{ cm}$

$FN = \sqrt{5^2 + 8^2}$

$= \sqrt{89}\text{ cm}$

$\tan\angle MFN = \dfrac{24}{\sqrt{89}}$

$\angle MFN \approx 69^\circ$

6. C

A. $\tan\angle ACE = \dfrac{AE}{CE}$

B. $\tan\angle AQE = \dfrac{AE}{EQ}$

C. $\tan\angle ADE = \dfrac{AE}{DE}$

D. $\tan\angle PCR = \dfrac{PR}{CR} = \dfrac{AE}{CR}$

Since $DE < EQ = RC < CE$, we have $\tan\angle ADE$ being the greatest among all.

$\angle ADE$ is therefore the greatest angle among all.

The answer is C.

7. B

Since $VA = VB$, we have $\angle VAB = \dfrac{180^\circ - 60^\circ}{2} = 60^\circ$ and so all lateral faces are equilateral triangles.

Required angle is $\angle VAM$, where M is the projection of V on $ABCD$.

Let $AB = 2$. Then $VA = 2$ and $AM = \dfrac{1}{2}\sqrt{2^2 + 2^2} = \sqrt{2}$

Required angle $= \angle VAM = \cos^{-1}\dfrac{\sqrt{2}}{2} = 45^\circ$

8. D

$PQ = CE = 40\sin 10^\circ$

$DP = \dfrac{90}{1+2} = 30\text{ m}$ and $AP = \sqrt{30^2 + 40^2} = 50\text{ m}$.

$\sin\theta = \dfrac{40\sin 10^\circ}{50}$

$= \dfrac{4\sin 10^\circ}{5}$

9. $\boxed{\text{C}}$

We have $\theta = \angle BEG$.
$EG = \sqrt{8^2 + 6^2} = 10\,\text{cm}$
$BE = \sqrt{10^2 + 4^2} = \sqrt{116}\,\text{cm}$
$$\cos\theta = \frac{10}{\sqrt{116}} = \frac{5}{\sqrt{29}}$$

10. $\boxed{\text{B}}$

Required angle is $\angle ACF$.
$AC = \dfrac{AD}{\sin 60^\circ} = \dfrac{10}{\sqrt{3}}\,\text{cm}$
$CD = \dfrac{AD}{\tan 60^\circ} = \dfrac{5}{\sqrt{3}}\,\text{cm}$
$AF = DE = CD\sin 30^\circ = \dfrac{5}{2\sqrt{3}}\,\text{cm}$
$\sin\angle ACF = \dfrac{AF}{AC}$
$\angle ACF \approx 14^\circ$

11. $\boxed{\text{D}}$

Note that $DE < EG < FH$.
Since $\tan\alpha = \dfrac{AE}{EG}$, $\tan\beta = \dfrac{AE}{DE}$ and $\tan\gamma = \dfrac{BF}{FH} = \dfrac{AE}{FH}$,
we have $\tan\beta > \tan\alpha > \tan\gamma$.
Therefore, we have $\beta > \alpha > \gamma$.

12. $\boxed{\text{A}}$

A. Angle between AC and $BCHG$ is $\angle ACB$.
B. Angle between DH and $BCHG$ is $\angle DHC$.
C. Angle between DG and $BCHG$ is $\angle DGC$.
D. Let Y be the midpoint of GH.
   Angle between XB and $BCHG$ is $\angle XBY$.

By simple observation, we have $\angle ACB$ being the greatest angle among all.
The answer is A.

13. $\boxed{\text{C}}$

I. ✓. Note that AD is perpendicular to the plane $CDEH$, we have $AD \perp DH$.
II. ✗. Note that $\angle BAE = 90^\circ$, we have $\angle ABE = 180^\circ - 90^\circ - \angle BEA < 90^\circ$.
III. ✓. Note that CH is perpendicular to the plane $ABCD$, we have $CH \perp AC$.

14. C

Let K be the midpoint of EF. The required angle is $\angle MNK$.

In $\triangle DEF$,

$$\frac{7}{\sin 50^\circ} = \frac{6}{\sin \angle DFE}$$

$$\angle DFE \approx 41.0^\circ \quad \text{or} \quad 139^\circ \text{ (rejected)}$$

$\angle DEF = 180^\circ - \angle DFE - 50^\circ \approx 89.0^\circ$

$$NK^2 = \left(\frac{6}{2}\right)^2 + \left(\frac{7}{2}\right)^2 - 2\left(\frac{6}{2}\right)\left(\frac{7}{2}\right)\cos \angle DEF$$

$$NK \approx 4.57\,\text{cm}$$

In $\triangle MNK$,

$$\tan \angle MNK = \frac{5}{NK}$$

$$\angle MNK \approx 48^\circ$$

Required angle = 48°

15. D

Let G be the midpoint of BC.

Required angle is $\angle ADG$.

$AG = 6\sin 60^\circ = 3\sqrt{3}\,\text{cm}$

$DG = \sqrt{3^2 + 10^2} = \sqrt{109}\,\text{cm}$

$$\tan \angle ADG = \frac{3\sqrt{3}}{\sqrt{109}}$$

$$\angle ADG \approx 26.5^\circ$$

16. B

Required angle is $\angle YXH$.

Let $CH = 2\,\text{cm}$.

We have $YH = \frac{2}{2} = 1\,\text{cm}$ and $XH = \sqrt{1^2 + 2^2} = \sqrt{5}\,\text{cm}$.

$$\tan \angle YXH = \frac{1}{\sqrt{5}}$$

$$\angle YXH \approx 24^\circ$$

17. B

Let H be a point on CF such that $GH \perp CF$. The angle required is $\angle GEH$.

$GH = BC \times \frac{3}{2+3} = 28.8\,\text{cm}$

$FH = FC \times \frac{3}{5} = 8.4\,\text{cm}$ and $EH = \sqrt{40^2 + 8.4^2} = \sqrt{1670.56}\,\text{cm}$

Required angle $= \tan^{-1} \frac{28.8}{EH} \approx 35^\circ$

18. [A]

Let the length of side of the cube be 2.

I. ✓. EF is perpendicular to $ABFG$. So, $\angle BFE = 90°$.

II. ✓. AB is perpendicular to $ADEF$. So, $\angle BAE = 90°$.

III. ✗. $BE = \sqrt{2^2 + 2^2 + 2^2} = \sqrt{12}$. $EM = BM\sqrt{2^2 + 1^2} = \sqrt{5}$
$EM^2 + BM^2 = 5 + 5 = 10 \neq BE^2$. So, $\angle BME \neq 90°$.

19. [C]

$BD = \sqrt{25^2 - 15^2} = 20\,\text{m}$

$$32^2 = 22^2 + 20^2 - 2(22)(20)\cos\angle BDC$$
$$\angle BDC \approx 99.2°$$

Let E be a point on CD produced such that $BE \perp CD$.
Required angle is $\angle BAE$.
$BE = BD\sin(180° - \angle BDC) \approx 19.7\,\text{m}$

$$\sin\angle BAE = \frac{BE}{25}$$
$$\angle BAE \approx 52°$$

20. [B]

Let E be a point on $ABCD$ such that VE is perpendicular to the plane $ABCD$.
Required angle is $\angle VCE$.

$CE = \frac{1}{2}\sqrt{5^2 + 6^2} = \frac{\sqrt{61}}{2}\,\text{cm}$

$\cos\angle VCE = \frac{CE}{8}$

$\angle VCE \approx 61°$

21. (a) $BC^2 = 30^2 + 30^2 - 2(30)(30)\cos 40°$ 1M

$BC \approx 20.5\text{ cm}$ 1A

(b) Since $\triangle ABC$ is equilateral, the circumcentre of $\triangle ABC$ coincides with centroid of $\triangle ABC$.

$r = \frac{2}{3} \times BC \sin 60°$ 1M

$\approx 11.8\text{ cm}$ 1A

(c) Required angle $= \cos^{-1} \frac{r}{30}$ 1M

$\approx 66.7°$ 1A

22. (a) Let R be a point on GH such that $RS \perp GH$.

Required angle is $\angle RAS$. 1A

$AD = \frac{30}{\sin 30°} = 60\text{ m}$ 1M

$AS = \sqrt{60^2 + 10^2} = 10\sqrt{37}\text{ m}$ 1M

$\sin \angle RAS = \frac{30}{10\sqrt{37}}$ 1M

$\angle RAS \approx 29.6°$ 1A

(b) Angle between DA and the plane $ABHG$ is $\angle DAG = 30°$. 1A

Angle between CA and the plane $ABHG$ is $\angle CAH$. 1A

$AC = \sqrt{60^2 + 20^2} = 20\sqrt{10}\text{ m}$

$\sin \angle CAH = \frac{30}{20\sqrt{10}}$ 1M

$\angle CAH \approx 28.3°$

Mean of two angles $= \frac{30° + \angle CAH}{2} \approx 29.2° \neq \angle RAS$

The claim is disagreed. 1A

23. (a) $BF = 200\cos 40° \approx 153\text{ m}$ 1M

$DE = CF = BF\sin 35° \approx 87.9\text{ m}$

$\sin \angle EBD = \frac{DE}{200}$ 1M

$\angle EBD \approx 26.1°$

The inclination of BE is $26.1°$. 1A

(b) Consider $\triangle ETX$, $\angle ETX = 180° - 90° - 70° = 20°$

$\angle TXE = 70° - \angle EBD \approx 43.9°$ 1A

$\frac{EX}{\sin 20°} = \frac{60}{\sin \angle TXE}$ 1M

$EX \approx 29.6\text{ m}$ 1A

$BX = 200 - EX \approx 170\text{ m}$ 1A

24. (a) (i) $CD^2 = 25^2 + 6^2 - 2(25)(6)\cos 57°$ 1M

$CD \approx 22.3\text{ cm}$ 1A

(ii) $\dfrac{\sin \angle BAC}{25} = \dfrac{\sin 57°}{28}$ 1M

$\angle BAC \approx 48.5°$ or $132°$ (rejected) 1A

(iii) Area of $\triangle ABC = \dfrac{1}{2}(28)(25)\sin(180° - 57° - \angle BAC)$ 1M

$\approx 337\text{ cm}^2$ 1A

(iv) $CE = \sqrt{25^2 - 24^2} = 7\text{ cm}$ 1A

$AE = \sqrt{28^2 - 7^2} = \sqrt{735}\text{ cm}$

$AB^2 = 28^2 + 25^2 - 2(28)(25)\cos \angle ACB$

$AB \approx 32.2\text{ cm}$

Let $s = \dfrac{AB + AE + BE}{2}$.

Area of $\triangle ABE = \sqrt{s(s - AB)(s - AE)(s - BE)}$

$\approx 318\text{ cm}^2$

Let h cm be the shortest distance from E to the horizontal ground.

$\dfrac{h}{3}(\text{area of } \triangle ABC) = \dfrac{1}{3}(\text{area of } \triangle ABE)(CE)$ 1M

$h \approx 6.60$ 1A

Required distance is 6.60 cm.

(b) $DE = \sqrt{CD^2 - 7^2} \approx 21.2\text{ cm}$

Let d cm be the perpendicular distance from E to CD.

$\dfrac{d(CD)}{2} = \dfrac{(CE)(DE)}{2}$ 1M

$d = \dfrac{(CE)(DE)}{CD}$

$\approx 6.65 \neq 6.60$

So, the perpendicular distance from E to CD and the shortest distance from E to the horizontal ground are different.

Thus, the claim is disagreed. 1A

END OF PAPER