Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Compulsory Part

S5 – S6 Core Assignment $\boxed{\text{Set 5}}$

Name: ____________________ Centre: __________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course S5 – S6 Core Phase 2 – Lesson 1

1. $\boxed{\text{B}}$

Let $OA = r$ cm.

$$r^2\pi \times \frac{60^\circ}{360^\circ} - \frac{1}{2}r^2 \sin 60^\circ = 10$$
$$r \approx 10.5$$

2. $\boxed{\text{D}}$

$$\frac{AC}{\sin 30^\circ} = \frac{AB}{\sin(180^\circ - 30^\circ - 105^\circ)}$$
$$\frac{AC}{AB} = \frac{\sin 30^\circ}{\sin 45^\circ}$$
$$= \frac{1}{2} \div \frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}}{2}$$

3. $\boxed{\text{A}}$

Let $BP = 1$. So, $PQ = QC = 1$ and $AB = AC = 3$.

In $\triangle ABP$, $AP = \sqrt{3^2 + 1^2 - 2(1)(3)\cos 60^\circ} \approx 2.65$. Similarly, $AQ = AP \approx 2.65$

In $\triangle APQ$, $\angle PAQ = \cos^{-1} \dfrac{AP^2 + AQ^2 - PQ^2}{2(AP)(AQ)} \approx 22^\circ$

4. $\boxed{\text{D}}$

$a = 36 \times \dfrac{3}{3+4+5} = 9$ cm

$b = 9 \times \dfrac{4}{3} = 12$ cm and $c = 9 \times \dfrac{5}{3} = 15$ cm

Let $s = \dfrac{a+b+c}{2} = 18$ cm.

$$\text{Area} = \sqrt{18(18-9)(18-12)(18-15)}$$
$$= 54 \text{ cm}^2$$

5. $\boxed{\text{D}}$

$CD = AB = 5$ cm. Consider $\triangle ACD$.

Let $s = \dfrac{4+5+7}{2} = 8$.

$$\text{Area of } \triangle ACD = \sqrt{8(8-4)(8-5)(8-7)}$$
$$= \sqrt{96} \text{ cm}^2$$

$$\text{Required area} = 2 \times \sqrt{96}$$
$$= 8\sqrt{6} \text{ cm}^2$$

6. $\boxed{\text{C}}$

Let O be the midpoint of BD.

Note that $\triangle ABO$, $\triangle AEO$ and $\triangle DEO$ are equilateral, we have $BD = 16$ cm.

$$16^2 = 10^2 + 10^2 - 2(10)(10)\cos\angle BCD$$
$$\angle BCD \approx 106°$$

7. $\boxed{\text{B}}$

Put $AB = 5$ cm. Then $BC = 7$ cm and $CA = 9$ cm.

$$9^2 = 5^2 + 7^2 - 2(5)(7)\cos\angle B$$
$$\angle B \approx 95.7°$$

8. $\boxed{\text{B}}$

$$\begin{aligned}\text{Required area} &= 6^2\pi \times \frac{40°}{360°} - \frac{1}{2}(6)(3)\sin 40° \\ &\approx 6.78\text{ cm}^2\end{aligned}$$

9. $\boxed{\text{B}}$

$\angle BAC = 180° - 90° - 60° = 30°$

$AE = 6\cos 30° = 3\sqrt{3}$ cm

$AD = BC = \dfrac{6}{\tan 60°} = 2\sqrt{3}$ cm

$\angle DAE = \angle ACB = 60°$

$$\begin{aligned}\text{Required area} &= \frac{1}{2}(AD)(AE)\sin 60° \\ &= \frac{9\sqrt{3}}{2}\text{ cm}^2\end{aligned}$$

10. $\boxed{\text{B}}$

$$\frac{1}{2}(x)(x)\sin 60° = 9\sqrt{3}$$
$$x^2 = 36$$
$$x = 6 \quad \text{or} \quad -6 \text{ (rejected)}$$

11. $\boxed{\text{B}}$

$\angle AOB = \angle BOC = \angle COA = 120°$

$$\begin{aligned}\text{Required area} &= 6^2\pi - 3 \times \frac{1}{2}(6)(6)\sin 120° \\ &\approx 66.3\text{ cm}^2\end{aligned}$$

12. $\boxed{\text{C}}$

Let E be a point on AD such that $CE // AB$.

$\angle CED = 180^\circ - \beta$ and $DE = x - y$.

$$\frac{CD}{\sin(180^\circ - \beta)} = \frac{x-y}{\sin(\beta - \phi)}$$

$$CD = \frac{(x-y)\sin\beta}{\sin(\beta - \phi)}$$

13. $\boxed{\text{D}}$

$\angle BAC = 180^\circ - 60^\circ - 70^\circ = 50^\circ$

$$\frac{AB}{\sin 70^\circ} = \frac{8}{\sin 50^\circ}$$

$$AB \approx 9.81\,\text{cm}$$

$$\text{Required area} = \frac{1}{2}(AB)(8)\sin 60^\circ$$

$$\approx 34.0\,\text{cm}^2$$

14. $\boxed{\text{A}}$

$$\text{Required area} = 4^2\pi \times \frac{60^\circ}{360^\circ} - \frac{1}{2}(4)(4)\sin 60^\circ$$

$$\approx 1.45\,\text{cm}^2$$

15. $\boxed{\text{D}}$

Let $OB = x$.

$$\text{Required ratio} = \left[\frac{1}{2}(x)^2\sin 45^\circ\right] : \left(x^2\pi \times \frac{45^\circ}{360^\circ}\right) : \left(\frac{1}{2}(x)(x\tan 45^\circ)\right)$$

$$= \frac{x^2\sqrt{2}}{4} : \frac{x^2\pi}{8} : \frac{x^2}{2}$$

$$= 2\sqrt{2} : \pi : 4$$

16. $\boxed{\text{C}}$

$$12\sqrt{3} = 2 \times \frac{1}{2}(AB)(AD)\sin 120^\circ$$

$$= AB(6) \cdot \frac{\sqrt{3}}{2}$$

$$AB = 4$$

17. $\boxed{\text{A}}$

Let E be a point on CD such that $AE // BC$.

Then $ABCE$ is a parallelogram.

$AE = 5\,\text{cm}$, $DE = 9 - 4 = 5\,\text{cm}$ and $\angle AED = 56^\circ$.

$$x^2 = 5^2 + 5^2 - 2(5)(5)\cos 56^\circ$$

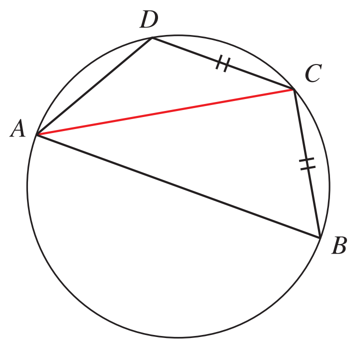
$$x \approx 4.69$$

18. $\boxed{\text{B}}$

Join AC as shown.

In $\triangle ABC$,

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos 60°$$
$$AC = \sqrt{21}$$



And

$$BC^2 = AC^2 + AB^2 - 2(AC)(AB)\cos \angle BAC$$
$$\angle BAC \approx 49.1°$$

Since $CD = BC$, $\angle CAD = \angle BAC \approx 49.1°$.

In $\triangle ACD$, $\angle ADC = 120°$ and $\angle ACD = 180° - 120° - \angle BAC \approx 10.9°$.

Area of quadrilateral $= \frac{1}{2}(CD)(AC)\sin \angle ACD + \frac{1}{2}(AB)(BC)\sin 60° \approx 10.4$ and it refers to $6\sqrt{3}$.

19. $\boxed{\text{C}}$

In $\triangle PSQ$,

$$\frac{3}{\sin \angle PQS} = \frac{7}{\sin 120°}$$
$$\angle PQS \approx 21.8°$$
$$\angle SPQ = 180° - 120° - \angle PQS \approx 38.2°$$

In $\triangle PQR$, let $RS = x$ cm.

$$QR^2 = PQ^2 + PR^2 - 2(PQ)(PR)\cos \angle QPR$$
$$19^2 = 7^2 + (3+x)^2 - 2(7)(3+x)\cos \angle SPQ$$
$$0 = (3+x)^2 - 14\cos \angle SPQ(3+x) - 312$$
$$x + 3 = 24 \quad \text{or} \quad -13 \text{ (rejected)}$$
$$x = 21$$

20. $\boxed{\text{B}}$

$$1.5^2 = 2.8^2 + 4.1^2 - 2(2.8)(4.1)\cos \angle ACB$$

$\angle ACB \approx 12.7°$

$h = 4.1 \sin \angle ACB = 0.9$

21. $\boxed{\text{B}}$

Consider $\triangle BCD$.

$$10^2 = 5^2 + 6^2 - 2(5)(6)\cos \angle BCD$$
$$\angle BCD \approx 131°$$

$\angle ABC = 180° - \angle BCD \approx 49°$

22. $\boxed{\text{C}}$

Let $s = \dfrac{8+12+18}{2} = 19.$

Area of $\triangle BCD = \sqrt{19(19-8)(19-12)(19-18)}$

$\approx 38.2\,\text{cm}^2$

Required area $= 2(\text{area of } \triangle BCD)$

$\approx 76.5\,\text{cm}^2$

23. $\boxed{\text{B}}$

Let $AC = 2$. Then $BC = 2$, $CD = 1$ and $AE = \dfrac{AB}{2} = \sqrt{2}$

Note that $CE \perp AB$, $\angle ACE = \sin^{-1}\dfrac{\sqrt{2}}{2} = 45°$

$\angle CAD = \tan^{-1}\dfrac{1}{2} \approx 26.6°$

$\theta = 180° - 45° - \tan^{-1}\dfrac{1}{2} \approx 108°$

$\sin\theta \approx 0.948683.$

By checking all options, we see that $\sin\theta = \dfrac{3\sqrt{10}}{10}$

24. $\boxed{\text{D}}$

Let E be a point on CD such that $AE // BC$.

Then $ABCE$ is a parallelogram.

$DE = 100 - 60 = 40\,\text{cm}$, $\angle ADE = 180° - 120° = 60°$ and $\angle AED = \angle BCD = 180°0110° = 70°$

$\dfrac{AD}{\sin 70°} = \dfrac{40}{\sin(180° - 60° - 70°)}$

$AD \approx 49\,\text{cm}$

25. In $\triangle ABD$,

$$BD^2 = 10^2 + 9^2 - 2(10)(9)\cos 36° \qquad \text{1M}$$

$$BD \approx 5.95 \text{ cm} \qquad \text{1A}$$

$$\text{Area of } \triangle ABD = \frac{1}{2}(9)(10)\sin 36° \qquad \text{1M}$$

$$\approx 26.5 \text{ cm}^2$$

In $\triangle BCD$,

$$s = \frac{BD + BD + 7}{2}$$

$$\approx 9.45 \text{ cm}$$

$$\text{Area of } \triangle BCD = \sqrt{s(s - BD)^2(s - 7)} \qquad \text{1M}$$

$$\approx 16.8 \text{ cm}^2$$

Area of the quadrilateral $ABCD$ = (area of $\triangle BCD$) + (area of $\triangle ABD$)

$$\approx 43.3 \text{ cm}^2 \qquad \text{1A}$$

26. Consider $\triangle ABC$, let $s = \dfrac{12 + 5 + 15}{2} = 16 \text{ cm}$

$$\text{Area of } \triangle ABC = \sqrt{16(16 - 12)(16 - 5)(16 - 15)} \qquad \text{1M}$$

$$= \sqrt{704} \text{ cm}^2$$

$$\text{Area of } ABCDE = \sqrt{704} + \frac{1}{2}(5)(3 + 8)\sin 60° - \frac{1}{2}(5)(3)\sin 60° \qquad \text{1M+1M}$$

$$\approx 43.9 \text{ cm}^2 \qquad \text{1A}$$

27. (a) $\angle BAC = 180° - 110° = 70°$ and $\angle ABC = 110° - 25° = 85°$.

$$\frac{AB}{\sin 25°} = \frac{4}{\sin 70°} \qquad \text{1M}$$

$$AB \approx 1.80 \text{ cm} \qquad \text{1A}$$

$$\frac{AC}{\sin 85°} = \frac{4}{\sin 70°}$$

$$AC \approx 4.24 \text{ cm} \qquad \text{1A}$$

(b) $$\text{Required area} = \pi(AB)^2 \frac{110°}{360°} + \frac{1}{2}(4)(AC)\sin 25° \qquad \text{1M+1M}$$

$$\approx 6.69 \text{ cm}^2 \qquad \text{1A}$$

28. (a) $BE = DE$ 1M

$$\frac{15}{\sin \angle ECD} = \frac{16}{\sin 53°} \qquad \text{1M}$$

$$\angle ECD \approx 48.5° \quad \text{or} \quad 132° \text{ (rejected)} \qquad \text{1A}$$

(b) $\angle CDB = 180° - 53° - \angle ECD \approx 78.5°$

$$BC^2 = 30^2 + 16^2 - 2(30)(16)\cos \angle CDB \qquad \text{1M}$$

$$BC \approx 31.1 \text{ cm}$$

$$\text{Required perimeter} = 2(BC + 16) \qquad \text{1M}$$

$$\approx 94.1 \text{ cm} \qquad \text{1A}$$

29. $AE^2 = 12^2 + 10^2 - 2(12)(10)\cos 110°$ 1M

$AE \approx 18.1$ cm 1A

$12^2 = 10^2 + AE^2 - 2(10)(AE)\cos\angle AEB$ 1M

$\angle AEB \approx 38.6°$

$DE^2 = 5^2 + 3^2 - 2(5)(3)\cos 130°$

$DE \approx 7.30$ cm

$3^2 = 5^2 + DE^2 - 2(5)(DE)\cos\angle DEC$

$\angle DEC \approx 18.4°$ 1A

$\angle AED = 180° - \angle DEC - \angle AEB \approx 123°$

Area of $\triangle AED = \frac{1}{2}(AE)(DE)\sin\angle AED$ 1M

$\approx 55.3\ \text{cm}^2$ 1A

30. $AB^2 = 10^2 + 8^2 - 2(10)(8)\cos 75°$ 1M

$AB \approx 11.1$ cm

$\frac{\sin\angle BAE}{8} = \frac{\sin 75°}{AB}$ 1M

$\angle BAE \approx 44.3°$ or $136°$ (rejected)

$AC^2 = 12^2 + 9^2 - 2(12)(9)\cos 82°$ 1M

$AC \approx 14.0$ cm

$\frac{\sin\angle CAD}{9} = \frac{\sin 82°}{AC}$ 1M

$\angle CAD \approx 39.7°$ or $140°$ (rejected)

$\angle BAC = 180° - \angle BAE - \angle CAD \approx 96.1°$

Area of $\triangle ABC = \frac{1}{2}(AB)(AC)\sin\angle BAC$ 1M

$\approx 76.9\ \text{cm}^2$ 1A

 END OF PAPER