Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Compulsory Part

S4 – S5 Core Assignment Set 8

Name: ____________________ Centre: __________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S4 – S5 Core
Phase 2 – Lesson 4

1. $\boxed{\text{B}}$

Solve the system $\begin{cases} x - y + 9 = 0 \\ x^2 + y^2 - 4x + cy - 45 = 0 \end{cases}$ using the calculator program.

Value of c	Number of intersections	Sign of Δ
2	0	−

Required range does not contain 2 and 2 is not a boundary value.

The answer is B.

2. $\boxed{\text{C}}$

Consider the system

$$\begin{cases} x - y + 13 = 0 \\ x^2 + y^2 - 14x + cy - 223 = 0 \end{cases}$$

Put $y = x + 13$ into equation of circle gives a quadratic equation.

Discriminant of the quadratic equation should be positive (2 distinct real roots) $\Rightarrow$ options A or C

When $c = 0$, by calculator program, there are two intersections $\Rightarrow$ required range contains 0 $\Rightarrow$ the answer is C.

3. $\boxed{\text{B}}$

PR (equation $x = 3$) is vertical and is the tangent to the circle.

Consider the centre and radius of the circles.

A. ✗. Centre $\left(9, \frac{9}{2}\right)$, radius $= \sqrt{9^2 + \left(\frac{81}{4}\right)^2 - 59} = 6.5$.
Distance between centre and $x = 3$ is $6 \neq 6.5$.

B. ✓. Centre $(5, 4)$, radius $= \sqrt{5^2 + 4^2 - 37} = 2$.
Distance between centre and $x = 3$ is 2.

C. ✗. Centre $\left(\frac{5}{2}, 2\right)$, radius $= \sqrt{\left(\frac{5}{2}\right)^2 + 2^2 - 37}$, which is not a real number.

D. ✗. Centre $\left(\frac{9}{2}, \frac{9}{4}\right)$, radius $= \sqrt{\left(\frac{9}{2}\right)^2 + \left(\frac{9}{4}\right)^2 - 59}$, which is not a real number.

4. A

$(1)^2 + y^2 - 8(1) + 4y - 5 = 0$

$y^2 + 4y - 12 = 0$

$y = -6 \quad \text{or} \quad 2$

Coordinates of A are $(1,\ -6)$.

Coordinates of centre G are $(4,\ -2)$.

Slope of $AG = \dfrac{-2+6}{4-1}$

$= \dfrac{4}{3}$

Slope of $L_1 = -\dfrac{3}{4}$

Equation of L_1 is

$$y + 6 = -\frac{3}{4}(x - 1)$$

$$3x + 4y + 21 = 0$$

5. A

Coordinates of centre G are $(-4,\ 1)$.

Slope of $GP = \dfrac{1+1}{-4+2}$

$= -1$

Slope of tangent = 1

Required equation is

$$y + 1 = 1(x + 2)$$

$$x - y + 1 = 0$$

6. B

$x^2 + (x-1)^2 - 4x + 4(x-1) + c = 0$

$2x^2 + (-2 - 4 + 4)x + (1 - 4 + c) = 0$

$2x^2 - 2x + (c - 3) = 0$

$\Delta = 2^2 - 4(2)(c - 3) < 0$

$-8c + 28 < 0$

$c > \dfrac{7}{2}$

7. A

$x^2 + (x-1)^2 - 2x + 4(x-1) + k = 0$

$2x^2 + (-2-2+4)x + (1-4+k) = 0$

$2x^2 + (k-3) = 0$

$\Delta = 0^2 - 4(2)(k-3) \geq 0$

$24 - 8k \geq 0$

$k \leq 3$

The largest value of k is 3.

8. B

Solve the system $\begin{cases} 2x + y - 5 = 0 \\ x^2 + y^2 - kx + 6y - 10 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
2	2	+

Required range does not contain 2 and 2 is not a boundary value of the range.

The answer is B.

9. B

Coordinates of centre G are $(-1, -2)$.

Slope of $GP = \dfrac{1+2}{-3+1} = -\dfrac{3}{2}$

Slope of tangent $= \dfrac{2}{3}$

Required equation is

$$y - 1 = \frac{2}{3}(x+3)$$

$$2x - 3y + 9 = 0$$

10. D

Solve the system $\begin{cases} x - y + k = 0 \\ x^2 + y^2 - 2x + 4y - 3 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
−7	1	0
0	2	+

Required range has −7 as a boundary value and excludes 0.

The answer is D.

11. C

Let the radius of the inscribed circle be r.

Consider the area of $\triangle OAB$.

$$\frac{(OA)r}{2}+\frac{(OB)r}{2}+\frac{(AB)r}{2}=\frac{(12)(5)}{2}$$

$$\frac{12r}{2}+\frac{5r}{2}+\frac{\sqrt{12^2+5^2}r}{2}=30$$

$$r=2$$

Coordinates of the centre of inscribed circle are $(2,\ 2)$.

Required equation is $(x-2)^2+(y-2)^2=4$.

12. B

Check by the calculator program.

A. MATH ERROR

B. $(-0.354,\ 8.65)$ and $(-5.65,\ 3.35)$

C. MATH ERROR

D. MATH ERROR

The answer is B. Note that the "two parts" do not have to be equal.

13. D

Solve the system $\begin{cases} mx-y-1=0 \\ x^2+y^2-16x-2y+31=0 \end{cases}$ using the calculator program.

The system has repeated solutions when $m=\frac{5}{3}$ and when $m=-\frac{3}{5}$.

Thus, $m=\frac{5}{3}$ or $-\frac{3}{5}$.

14. A

$$x^2+(-2x-1)^2-6x-2(-2x-1)+k=0$$

$$5x^2+2x+(k+3)=0$$

$$\Delta=2^2-4(5)(k+3)<0$$

$$-20k-56<0$$

$$k>-\frac{14}{5}$$

15. D

Coordinates of centre G are $(-h,\ -k)$.

Slope of $AG=\frac{b+k}{a+h}$

$$m\times\frac{b+k}{a+h}=-1$$

$$m=-\frac{h+a}{k+b}$$

16. D

Solve the system $\begin{cases} x - y + m = 0 \\ x^2 + y^2 + 2x - 4y - 13 = 0 \end{cases}$ using the calculator program.

Value of m	Number of intersections	Sign of Δ
-9	0	$-$

Required range does not contain $m = -9$ and -9 is not a boundary value of the required range.
The answer is D.

17. A

Solve the system $\begin{cases} 3x + 4y + k = 0 \\ x^2 + y^4 - \frac{9}{4} = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
$-\frac{15}{2}$	1	0
0	2	$+$

Required range has $-\frac{15}{2}$ as a boundary value (not equal to) and includes 0.
The answer is A.

18. D

$$0^2 + y^2 + 6(0) - 2y - 3 = 0$$
$$y^2 - 2y - 3 = 0$$
$$y = 3 \quad \text{or} \quad -1$$

Coordinates of centre G are $(-3,\ 1)$.
Since A, B, G are collinear, the slope of AG is also positive.
The coordinates of B are $(0,\ 3)$.
Let the coordinates of A be $(a,\ b)$.

$$\frac{a+0}{2} = -3 \quad \text{and} \quad \frac{b+3}{2} = 1$$
$$a = -6 \qquad\qquad b = -1$$

The coordinates of A are $(-6,\ -1)$.

19. D

A. ✗. Using the calculator program, the system $\begin{cases} x+y-9=0 \\ x^2+y^2+6x+6y+9=0 \end{cases}$ does not have real solutions.

B. ✗. $x+y+9=0$ does not pass through $(3,\ 6)$.

C. ✗. Same reason as A.

D. ✓.

20. A

Put $(3,\ b)$ into the equation of L.

$$(3)-4(b)-11=0$$
$$b=-2$$

Put $(-1,\ -3)$ and $(3,\ -2)$ into the equation of C.

$$\begin{cases} 1+9-a-15+c=0 \\ 9+4+3a-10+c=0 \end{cases}$$

Solving, we have $a=-2$ and $c=3$.

The answer is A.

21. A

$$x^2+(kx-2)^2+4kx+4=0$$
$$(1+k^2)x^2+(-4k+4k)x+4+4=0$$
$$(1+k^2)x^2+8=0$$

$$\Delta = 0^2-4(1+k^2)(8)$$
$$=-32k^2-32$$
$$<0$$

L and S do not intersect.

22. D

Solve the system $\begin{cases} x-2y+1=0 \\ x^2+y^2-6x+k=0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
0	2	+

Required range does not contain $k=0$ and 0 is not a boundary value of the required range.

The answer is D.

23. D

$(x+3)^2+(3-1)^2=5$

$(x+3)^2=1$

$x=-4$ or -2

Required area $=\dfrac{(-2+4)(3)}{2}$

$=3$

24. D

$x^2+0^2-8x-6(0)+12=0$

$x^2-8x+12=0$

$x=2$ or 6

Coordinates of the centre G are $(4,\ 3)$.

L passes through centre G and A, and has a positive slope.

The coordinates of A are $(2,\ 0)$.

Slope of $L=\dfrac{3-0}{4-2}=\dfrac{3}{2}$

Required equation is

$y-3=\dfrac{3}{2}(x-4)$

$y=\dfrac{3}{2}x-3$

25. C

Centre $(-2,\ 4)$. Slope of radius $=\dfrac{4-3}{-2+7}=\dfrac{1}{5}$

Slope of tangent $=-5$

Equation of tangent is

$y-3=-5(x+7)$

$5x+y+32=0$

26. C

$x^2+k^2-2x-2k+1=0$

$x^2-2x+(k^2-2k+1)=0$

$\Delta=2^2-4(1)(k^2-2k+1)=0$

$-4k^2+8k=0$

$k=0$ or 2

27. A

Solve the system $\begin{cases} mx - y = 0 \\ x^2 + y^2 - 6x + 4 = 0 \end{cases}$ using the calculator program.

A. 1 intersection $\longrightarrow$ tangent

B. No intersections

C. No intersections

D. No intersections

28. C

$x^2 + 16^2 - 2kx - 2k(16) + k^2 = 0$

$x^2 - 2kx + (k^2 - 32k + 256) = 0$

$\Delta = (2k)^2 - 4(1)(k^2 - 32k + 256) = 0$

$128k - 1024 = 0$

$k = 8$

29. C

$0^2 + y^2 + 8(0) - 10y + 16 = 0$ and $x^2 + 0^2 + 8x - 10(0) + 16 = 0$

$y^2 - 10y + 16 = 0$ $x^2 + 8x + 16 = 0$

$y = 2$ or 8 $x = -4$

Required area $= \dfrac{(8-2)(4)}{2}$

$= 12$

30. A

Solve the system $\begin{cases} 3x + 4y + k = 0 \\ x^2 + y^2 - 12x - 14y + 60 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
-71	1	0
0	0	$-$

Required range has -71 as a boundary value and excludes 0.
The answer is A.

31. (a) $(x+2)(x-4)+(y+8)(y-2)=0$

$x^2+y^2-2x+6y-24=0$ 1M

Coordinates of centre are $(1,\ -3)$ and radius $=\sqrt{1^2+3^2+24}=\sqrt{34}$ 1A+1A

(b) $\sqrt{(-2-1)^2+(k+3)^2}<\sqrt{34}$ 1M

$k^2+6k-16<0$

$-8<k<2$ 1A

(c) The equation of L is

$y-k=1(x+2)$ 1M

$y=x+2+k$

$x^2+(x+2+k)^2-2x+6(x+2+k)-24=0$ 1M

$(1+1)x^2+[2(2+k)-2+6]x+[(2+k)^2+12+6k-24]=0$

$2x^2+(2k+8)x+(k^2+10k-8)=0$

x-coordinate of midpoint $=\frac{1}{2}\left(-\frac{2k+8}{2}\right)=-\frac{k+4}{2}$ 1M

When $x=-\frac{k+4}{2}$, $y=-\frac{k+4}{2}+2+k=\frac{k}{2}$.

The coordinates of the midpoint are $\left(-\frac{k+4}{2},\ \frac{k}{2}\right)$. 1A

32. (a) $Q\ (8,\ 6)$ 1A

Coordinates of $C=\left(\frac{0+8}{2},\ \frac{0+6}{2}\right)$ 1M

$=(4,\ 3)$ 1A

(b) Slope of $RC=\frac{6-3}{0-4}=-\frac{3}{4}$

Slope of tangent to the circle at $R=\frac{4}{3}$ 1M

The equation of tangent to the circle at P is

$\frac{y-0}{x-8}=\frac{4}{3}$ 1M

$3y=4x-32$ 1A

The equation of tangent to the circle at R is $y=\frac{4}{3}x+6$ 1A

(c) Slope of $OC=\frac{3-0}{4-0}=\frac{3}{4}$

Slope of tangent to the circle at $O=-\frac{4}{3}$ 1M

Product of slopes of two tangents $=-\frac{4}{3}\times\frac{4}{3}=-\frac{16}{9}\neq-1$ 1M

So, the tangents to the circle at R and at O are not perpendicular.

Thus, the quadrilateral formed is not a rectangle. The claim is not correct. 1A

33. (a) $\angle ABC = \angle AOC$

$\angle ADC = 2\angle ABC = 2\angle AOC$ 1M

$\angle OCD = \angle OAD = 90^\circ$ 1M

In quadrilateral $OACD$,

$$\angle AOC + 2 \times 90^\circ + \angle ADC = 360^\circ$$ 1M

$$3\angle AOC = 180^\circ$$

$$\angle AOC = 60^\circ$$ 1A

(b) (i) $\angle AOD = \frac{1}{2}\angle AOC = 30^\circ$ 1M

In $\triangle AOD$,

$$\sin 30^\circ = \frac{AD}{OD} = \frac{BD}{OD}$$

$$OD = 2BD$$ 1M

Therefore, the coordinates of D are $\left(\frac{0 + 2 \times 9}{3}, \ \frac{0 + 2 \times 9}{3}\right) = (6,\ 6)$. 1A

Required equation is

$$(x - 6)^2 + (y - 6)^2 = (9 - 6)^2 + (9 - 6)^2$$

$$(x - 6)^2 + (y - 6)^2 = 18$$ 1A

(ii) Let $y = mx$ be the equation of tangent from O.

$$(x - 6)^2 + (mx - 6)^2 = 18$$ 1M

$$(m^2 + 1)x^2 - 12(m + 1)x + 54 = 0$$

Since $y = mx$ is a tangent,

$$\Delta = [-12(m + 1)]^2 - 4(m^2 + 1)(54) = 0$$ 1M

$$-72m^2 + 288m - 72 = 0$$ 1A

$$m = \frac{4 \pm \sqrt{4^2 - 4}}{2} = 2 \pm \sqrt{3}$$

The equation of OA and OC are $y = (2 - \sqrt{3})x$ and $y = (2 + \sqrt{3})x$ respectively. 1A+1A

34. (a) $CE \perp AB$ *(property of orthocentre)*

$BD \perp AC$ *(property of orthocentre)*

$\angle BEC = \angle BDC = 90°$

Thus, $BCDE$ is a cyclic quadrilateral. *(converse of ∠s in the same segment)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) Coordinates of centre $= \left(\frac{-6+14}{2}, \frac{-6-6}{2}\right) = (4, -6)$ 1A

The equation of the circle is

$(x-4)^2 + (y+6)^2 = (0-4)^2 + (8+6)^2$ 1M

$(x-4)^2 + (y+6)^2 = 100$ 1A

(ii) Distance between A and centre $= \sqrt{4^2 + (-6-8)^2} = \sqrt{212}$

Radius of circle = 10

Angle between two tangents $= 2 \times \tan^{-1}\frac{10}{\sqrt{212}} \approx 86.8° \neq 90°$ 1M+1A

The claim is not agreed. 1A

35. (a) $AB^2 = 9^2 + 18^2 = 405$

$AC^2 = 13^2 + 16^2 = 425$

$BC^2 = 4^2 + 2^2 = 20$

Therefore, $AB^2 + BC^2 = AC^2$. 1M

Thus, $\angle ABC = 90°$.

$\triangle ABC$ is a right-angled triangle. 1A

(b) The coordinates of the centre $= \left(\frac{-9+4}{2}, \frac{-8+8}{2}\right) = \left(-\frac{5}{2}, 0\right)$ 1A

The equation of Ω is

$$\left(x+\frac{5}{2}\right)^2 + y^2 = \left(0+\frac{5}{2}\right)^2 + 10^2$$ 1M

$$x^2 + y^2 + 5x - 100 = 0$$ 1A

(c) (i) The coordinates of D are $(10, 0)$. 1A

$$(10)^2 + 0^2 + 5(10) - 100 = 50 \neq 0$$

Therefore, D does not lie on the circle passing through A, B and C. 1M

Thus, $ABCD$ is not a cyclic quadrilateral. 1A

(ii) Let the slope of L be m.

The equation of L is $y = m(x-10)$. 1M

Substitute L into Ω,

$$x^2 + (mx - 10m)^2 + 5x - 100 = 0$$ 1M

$$(1+m^2)x^2 + (-20m^2 + 5)x + (100m^2 - 100) = 0$$

Since L is tangent, $\Delta = 0$.

$$(20m^2 - 5)^2 - 4(1+m^2)(100m^2 - 100) = 0$$ 1M

$$-200m^2 + 425 = 0$$

$$m = \pm\frac{\sqrt{34}}{4}$$ 1A

The equations of L are $y = \frac{\sqrt{34}}{4}(x-10)$ and $y = -\frac{\sqrt{34}}{4}(x-10)$.

 END OF PAPER