Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Compulsory Part

S4 – S5 Core Assignment Set 6

Name: ____________________ Centre: __________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S4 – S5 Core
Phase 2 – Lesson 2

1. $\boxed{\text{A}}$

$x^2 + y^2 - x - \frac{y}{2} + \frac{1}{4} = 0$

I. ✗. Centre $\left(\frac{1}{2}, \frac{1}{4}\right)$

II. ✗. Radius $= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 - \frac{1}{4}} = \frac{1}{4} \neq 2$

III. ✓. Radius $= \frac{1}{4} = y$-coordinate of centre.
So, the circle touches the x-axis.

2. $\boxed{\text{A}}$

$x^2 + y^2 + 2x + 4y + \frac{4}{3} = 0$

I. ✗. x-coordinate of centre $= -1$

II. ✓. $0^2 + 0^2 + 0 + 0 + \frac{4}{3} > 0$. The origin lies outside C.

III. ✗. Radius $= \sqrt{1^2 + 2^2 - \frac{4}{3}} \neq 1$

3. $\boxed{\text{C}}$

The coordinates of the centre are $(-5,\ 5)$.
Required equation is

$$(x + 5)^2 + (y - 5)^2 = 5^2$$
$$x^2 + y^2 + 10x - 10y + 25 = 0$$

4. $\boxed{\text{B}}$

$x^2 + y^2 - 2x + 4y - 3 = 0$

I. ✓.

II. ✗. Radius $= \sqrt{1^2 + 2^2 + 3} = \sqrt{8} \neq 3$

III. ✓. $2^2 + 1^2 - 2(2) + 4(1) - 3 = 2$
> 0
$(2,\ 1)$ lies outside the circle.

5. $\boxed{\text{C}}$

Radius of the circle is 3.
Required equation is $(x - 3)^2 + (y + 5)^2 = 9$.

6. $\boxed{\text{D}}$

$\frac{h}{-2} = 2$ and $\frac{k}{-2} = -1$

$h = -4$ $\quad$ $k = 2$

7. C

The coordinates of the centre are $(-3,\ -1)$.

Radius $= \sqrt{16} = 4$

8. B

Radius $= \sqrt{2^2 + 1^2 + 4} = 3$

Required area $= 3^2\pi = 9\pi$ sq. units

9. A

$1^2 + 4^2 + k > 0$

$k > -17$

10. D

Diameter passes through centre $(4,\ 3)$.

$\frac{-5-3}{k-4} = -4$

$k = 6$

11. A

Radius $= \sqrt{2^2 + 3^2 + 12} = 5$

Circumference $= 2(5)\pi = 10\pi$

12. C

The circle passes through points $(5 \pm 12,\ 0)$, i.e., $(-7,\ 0)$ and $(17,\ 0)$.

Centre $(5,\ -7) \Rightarrow$ The circle is in the form $x^2 + y^2 - 10x + 14y + F = 0$.

Substitute $(-7,\ 0)$, $F = -119$.

13. B

Line segment joining $(0,\ 8)$ and $(-6,\ 0)$ is a diameter.

Coordinates of centre are $(-3,\ 4)$.

The equation is $x^2 + y^2 + 6x - 8y = 0$ (passes through origin).

14. A

A. ✓.

B. ✗. Centre $(1,\ -7)$ does not lie on L.

C. ✗. Centre $\left(\frac{5}{2},\ \frac{7}{2}\right)$ does not lie on L.

D. ✗. $(3,\ 4)$ does not lie on the circle.

15. A

Required equation is

$$(x+2)^2 + (y-4)^2 = (3+2)^2 + (5-4)^2$$

$$x^2 + y^2 + 4x - 8y - 6 = 0$$

16. D

A. ✗. Centre $\left(-\frac{5}{2},\ 5\right)$ does not lie on $x + y - 1 = 0$.

B. ✗. Centre $\left(\frac{19}{2},\ -4\right)$ does not lie on $x + y - 1 = 0$.

C. ✗. Centre $\left(\frac{17}{2},\ -\frac{19}{2}\right)$ does not lie on $x + y - 1 = 0$.

D. ✓. Centre $\left(-\frac{17}{2},\ \frac{19}{2}\right)$ lie on $x + y - 1 = 0$,
and both $(0,\ 0)$ and $(3,\ 4)$ satisfy the equation.

17. D

Let the radius be r.
The coordinates of the centre are $(r,\ -r)$.
Centre lies on L.

$$\begin{aligned}(r) + 2(-r) + 4 &= 0\\ r &= 4\end{aligned}$$

Required equation is

$$\begin{aligned}(x-4)^2 + (y+4)^2 &= 4^2\\ x^2 + y^2 - 8x + 8y + 16 &= 0\end{aligned}$$

18. A

Since $\angle AOB = 90°$, where O is the origin.
Centre of the circle is the midpoint of AB.
The coordinates of the centre are $\left(\frac{-3}{2},\ 2\right)$.
Required equation is

$$\begin{aligned}\left(x + \frac{3}{2}\right)^2 + (y-2)^2 &= \left(0 + \frac{3}{2}\right)^2 + (0-2)^2\\ x^2 + y^2 + 3x - 4y &= 0\end{aligned}$$

19. A

Radius of circle is 2. Let the centre of C_3 be $(h,\ k)$. Note that the polygon formed by joining the centres is an equilateral triangle.
By considering the line segment joining centres of C_1 and C_3,

$$h - 2 = (2+2)\cos 60° \quad \text{and} \quad k - 2 = (2+2)\sin 60°$$
$$h = 4 \qquad\qquad k = 2 + 2\sqrt{3}$$

Only option A has its centre at $\left(4,\ 2 + 2\sqrt{3}\right)$.

20. C

Centre $(-10,\ 12)$. The equation is in the form $x^2 + y^2 + 20x - 24y + F = 0$, where F is a constant.
The coordinates of midpoint of AB are $(-10,\ 0)$.
Thus, the x-coordinates of A and B are $-10 \pm 16 = 6$ or -26.

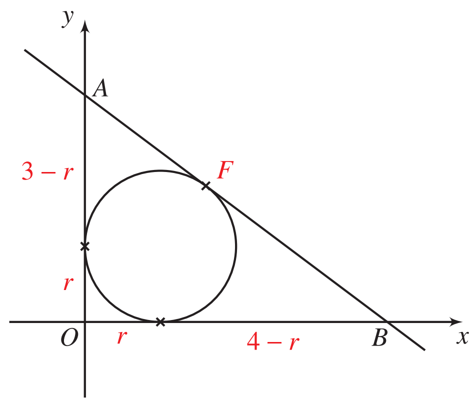
$$(6)^2 + (0)^2 + 20(6) - 24(0) + F = 0$$
$$F = -156$$

C: $x^2 + y^2 + 20x - 24y - 156 = 0$

21. C

Let F be the point of contact of AB and the circle. Let r be the radius.

$AF = 3 - r$ and $BF = 4 - r$
$(3 - r) + (4 - r) = \sqrt{3^2 + 4^2}$
$r = 1$
The equation of circle is $(x - 1)^2 + (y - 1)^2 = 1$.



22. C

y-coordinate of centre $= 2$
x-coordinate of centre $= \dfrac{1 + 4}{2} = \dfrac{5}{2}$
Radius $= \dfrac{5}{2}$
Required equation is

$$\left(x - \frac{5}{2}\right)^2 + (y - 2)^2 = \left(\frac{5}{2}\right)^2$$
$$x^2 + y^2 - 5x - 4y + 4 = 0$$

23. B

Let the coordinates of C be $(h,\ 0)$, where $h < 0$.
Since $y = 3$ is a tangent to the circle, radius of the circle is 3.
Since $x = 1$ is a tangent to the circle, we have

$$1 - h = 3$$
$$h = -2$$

Required equation is $(x + 2)^2 + y^2 = 9$.

24. A

Centre is the midpoint of AB.

The coordinates of the centre are $(-1,\ -1)$.

Required equation is

$$(x+1)^2+(y+1)^2=(2+1)^2+(3+1)^2$$
$$x^2+y^2+2x+2y-23=0$$

25. B

y-coordinate of centre $=\dfrac{2\sqrt{3}+8\sqrt{3}}{2}$

$=5\sqrt{3}$

Centre of circle is the centroid/orthocentre/circumcentre/incentre of $\triangle CAB$.

(Note: four centres coincide when it is equilateral triangle.)

A. ✗. Centre $\left(-5,\ 5\sqrt{3}\right)$ should not lie on the line AB.

B. ✓.

C. ✗. y-coordinate of centre $=-5\sqrt{3}\neq 5\sqrt{3}$

D. ✗. y-coordinate of centre $=-5\sqrt{3}\neq 5\sqrt{3}$

26. B

Radius = 3

Required equation is

$$(x+5)^2+(y+3)^2=3^2$$
$$x^2+y^2+10x+6y+25=0$$

27. D

Distance between the two points $=\sqrt{(0+8)^2+(0+6)^2}=10=2(5)$

The line segment joining the two points is therefore a diameter of the required circle.

The coordinates of the centre are $(-4,\ -3)$.

Required equation is

$$(x+4)^2+(y+3)^2=5^2$$
$$x^2+y^2+8x+6y=0$$

28. B

x-coordinate of centre $=\dfrac{1+5}{2}=3$

y-coordinate of centre $=\dfrac{1+(-3)}{2}=-1$

Radius $=5-3=2$

Required equation is $(x-3)^2+(y+1)^2=4$.

29. A

Radius of circle is 2. Let the centre of C_3 be $(h,\ k)$. Note that the polygon formed by joining the centres is an equilateral triangle.
By considering the line segment joining centres of C_1 and C_3,

$$h - 2 = (2+2)\cos 60° \quad \text{and} \quad k - 2 = (2+2)\sin 60°$$
$$h = 4 \qquad\qquad k = 2 + 2\sqrt{3}$$

Only option A has its centre at $\left(4,\ 2+2\sqrt{3}\right)$.

30. A

x-coordinate of centre $= \dfrac{2+8}{2} = 5$
Radius = 5 and y-coordinate of centre = 4
Required equation is

$$(x-5)^2 + (y-4)^2 = 16$$
$$x^2 + y^2 - 10x - 8y + 16 = 0$$

31. (a) $y^2 - 12y + 32 = 0$

$y = 4 \quad \text{or} \quad 8$

Coordinates of A are $(0,\ 4)$. 1A

(b) $c = 4$ 1A

Coordinates of P are $(6,\ 6)$. 1M

Slope of $AP = \dfrac{6-4}{6-0} = \dfrac{1}{3}$

Slope of $L = m = -3$ 1A

(c) Let the x-coordinate of B be b.

Since B lies on $y = -3x + 4$, the coordinates of B are $(b,\ -3b + 4)$.

$$\sqrt{b^2 + (-3b + 4 - 4)^2} = \sqrt{(0-6)^2 + (4-6)^2}$$ 1M

$$b^2 + 9b^2 = 40$$

$$b = 2 \quad \text{or} \quad -2 \text{ (rejected)}$$

The coordinates of B are $(2,\ -2)$. 1A

The equation of C_2 is

$$(x+10)^2 + (y+6)^2 = (2+10)^2 + (-2+6)^2$$ 1M

$$(x+10)^2 + (y+6)^2 = 160$$ 1A

32. (a) Slope of $L_2 = \dfrac{-6-6}{0+6} = -2$.

Slope of $L_1 = \dfrac{-1}{-2} = \dfrac{1}{2}$. 1M

Equation of L_1 is

$$y - 6 = \frac{1}{2}(x+6)$$ 1M

$$x - 2y + 18 = 0$$

Coordinates of A and B are $(-18,\ 0)$ and $(0,\ 9)$ respectively. 1A+1A

(b) (i) $\angle APQ = 90°$ *(given)*

$\angle AOQ = 90°$

Thus, A, P, O and Q are concyclic. *(converse of ∠s in the same segment)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(ii) Since $\angle AOQ = 90°$, AQ is a diameter of the circle.

Coordinates of centre $= \left(\dfrac{-18+0}{2},\ \dfrac{0-6}{2}\right) = (-9,\ -3)$ 1M

Equation of circle is

$$(x+9)^2 + (y+3)^2 = (0+9)^2 + (0+3)^2$$ 1M

$$(x+9)^2 + (y+3)^2 = 90$$ 1A

33. (a) Let the equation of circle be $x^2 + y^2 + Dx + Ey + F = 0$, where D, E and F are constants. 1A

$$\begin{cases} 1 + 4 + D + 2E + F = 0 & (1) \\ 9 + 3E + F = 0 & (2) \\ 16 + 4D + F = 0 & (3) \end{cases}$$ 1M

Consider (2) − (1) and (3) − (2).

$$\begin{cases} -D + E = -4 \\ 4D - 3E = -7 \end{cases}$$ 1M

Solving, we have $D = -19$, $E = -23$. 1A

When $D = -19$, $E = -23$, $F = -9 - 3(-23) = 60$.

The equation of the circle is $x^2 + y^2 - 19x - 23y + 60 = 0$. 1A

(b) Centre of the circle $= \left(\frac{19}{2}, \frac{23}{2}\right)$ 1A

Radius of the cirlce $= \sqrt{\left(\frac{19}{2}\right)^2 + \left(\frac{23}{2}\right)^2 - 60} = \frac{5\sqrt{26}}{2}$ 1A

(c) If two points on the circle form a diameter, then the midpoint of them must be at the centre of circle.

Midpoint of $AB = \left(\frac{1}{2}, \frac{5}{2}\right)$ 1M

Midpoint of $BC = \left(2, \frac{3}{2}\right)$

Midpoint of $CA = \left(\frac{5}{2}, 1\right)$

None of the above is at the centre of the circle.

Thus, the claim is incorrect. 1A

34. (a) (i) Slope of $PQ = \frac{13-1}{1+5} = 2$

Slope of $L = -\frac{1}{2}$ 1M

Coordinates of midpoint of PQ are $(-2,\ 7)$.

The equation of L is

$$y - 7 = -\frac{1}{2}(x+2)$$ 1M

$$x + 2y - 12 = 0$$ 1A

(ii) x-coordinate of $G = 12 - 2k$ 1M

The equation of C is in the form $x^2 + y^2 - 2(12-2k)x - 2ky + F = 0$, where F is a constant.

$$12^2 + 10^2 - (24-4k)(12) - 2k(10) + F = 0$$ 1M

$$F = -28k + 44$$

The equation of C is $x^2 + y^2 + (4k-24)x - 2ky - 28k + 44 = 0$. 1

(b) The area of C is smallest when $GR \perp L$.

$$\frac{10-k}{12-(-2k+12)} \times \frac{-1}{2} = -1$$ 1M

$$-10 + k = -4k$$

$$k = 2$$ 1A