Dexter Wong & His Mathematics Team

Summer Course   2022 – 2023

MATHEMATICS   Compulsory Part

S4 – S5 Core   Assignment   Set 5

Name: ____________________   Centre: ____________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S4 – S5 Core
Phase 2 – Lesson 1

1. A

I. ✓. Radius of $C_1 = \sqrt{3^2 + 4^2} = 5$; radius of $C_2 = \sqrt{25} = 5$

II. ✓. Distance between centres $= \sqrt{(3-0)^2 + (0+4)^2} = 5$

III. ✗. $(0,\ 0)$ does not satisfy the equation of C_2. C_2 does not pass through the origin.

2. D

$C: x^2 + y^2 - 3x + y - \frac{13}{2} = 0$

I. ✗. Centre $\left(\frac{3}{2},\ -\frac{1}{2}\right)$

II. ✓. $AB = \sqrt{(2-1)^2 + (1+2)^2} = \sqrt{10}$

Radius $= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \frac{13}{2}} = 3 < AB$

III. ✓. Slope of $AB = \frac{1+2}{2-1} = 3$

Slope of $AG = \frac{-2 + \frac{1}{2}}{1 - \frac{3}{2}} = 3$, where G is the centre.

Thus, three points are collinear.

3. D

$3^2 + 2^2 - 4(3) + 8(-2) + k = 0$

$k = 15$

Centre $(2,\ -4)$ and radius $= \sqrt{2^2 + 4^2 - 15} = \sqrt{5}$

Area $= (\sqrt{5})^2\pi = 5\pi$ sq. units

4. C

Centre $(-8,\ -5)$ lies in the third quadrant.

The answer is C.

5. D

Centre $(p,\ q)$ lies in quadrant IV. So, $p > 0$ and $q < 0$.

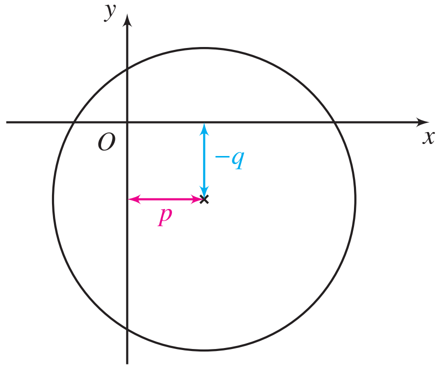
I. ✓.

II. ✓. $p - r < 0$

(length p is shorter than the radius)

III. ✓. $\sqrt{p^2 + q^2} < r$

(distance between origin and centre is smaller than the radius)



6. A

$x^2 + y^2 - 9x + 8y - \frac{1}{2} = 0$

I. ✓. $0 + y^2 - 0 + 8y - \frac{1}{2} = 0$

$y \approx 0.0620 \quad \text{or} \quad -8.06$

The circle intersect y-axis at two points.

II. ✗. Coordinates of centre are $\left(\frac{9}{2},\ -4\right)$.

III. ✗. Sub $(0,\ 0)$, L.H.S. $= -\frac{1}{2} < 0$. Origin lies inside the circle.

7. A

C: $x^2 + y^2 - 6x - 2y + \frac{5}{3} = 0$

A. ✓. Centre $(3,\ 1)$ and radius $= \sqrt{3^2 + 1^2 - \frac{5}{3}} = \frac{5}{\sqrt{3}} < 3$.

The circle lies on the right of the y-axis.

B. ✗. Sub $(0,\ 0)$ into L.H.S. of equation of C, L.H.S. $= 0 + 5 = 5 > 0$.

$(0,\ 0)$ lies outside C.

C. ✗. Centre is at $(3,\ 1)$.

D. ✗. Area $= \left(\frac{5}{\sqrt{3}}\right)^2 \pi = \frac{25\pi}{3} < \frac{25\pi}{2}$.

8. D

I. ✓. $G_1\ (-2,\ 6)$, $G_2\ (2,\ 4)$. Slope of $OG_2 \times$ slope of $G_1G_2 = \frac{4}{2} \times \frac{6-4}{-2-2} = -1$

II. ✓. Distance between centres $= \sqrt{(2+2)^2 + (6-4)^2} = \sqrt{20}$

Radius of $C_1 = \sqrt{2^2 + 6^2 + 40} = \sqrt{80} = 2\sqrt{20}$; radius of $C_2 = \sqrt{2^2 + 4^2} = \sqrt{20}$

Since distance between centres = difference in radii, the circles touch each other internally.

III. ✓. Area ratio $= \left(\frac{\sqrt{80}}{\sqrt{20}}\right)^2 = 4$

9. B

A. ✗. Coefficients of x^2 and y^2 must equal.

B. ✓.

C. ✗. No xy term in equation of circle.

D. ✗. Coefficients of x^2 and y^2 must equal.

10. D

Substitute the coordinates of the points into L.H.S. of the equation.

A. $10^2 + 6^2 - 8(10) + 4(6) - 16 = 64 > 0$. W lies outside the circle.

B. $8^2 + 8^2 - 8(8) + 4(8) - 16 = 80 > 0$. X lies outside the circle.

C. $6^2 + 6^2 - 8(6) + 4(6) - 16 = 32 > 0$. Y lies outside the circle.

D. $9^2 + 0 - 8(9) - 16 = -7 < 0$. Z lies inside the circle.

11. C

C: $x^2 + y^2 - 2x + 8y - \dfrac{534}{5} = 0$

I. ✓. Centre $(1,\ -4)$. Since $3(1) + 7(-4) + 25 = 0$, the line passes through centre of circle.

II. ✓. $2^2 + 16^2 - 2(2) + 8(-16) - \dfrac{534}{5} = \dfrac{106}{5} > 0$. $(2,\ -16)$ lies outside the circle.

III. ✗.

12. B

Coordinates of centre are $\left(4,\ -\dfrac{k}{2}\right)$.

$$\frac{-\frac{k}{2} - 2}{4 - 6} = 2$$
$$k = 4$$

13. A

Centre $= \left(\dfrac{-8}{-2},\ \dfrac{0}{-2}\right) = (4,\ 0)$

Radius $= \sqrt{4^2 + 0^2 + 8} = \sqrt{24} = 2\sqrt{6}$

14. A

I. ✓. Both centres are at $(-4,\ 3)$.

II. ✗. Radius of $C_1 = \sqrt{4^2 + 3^2 - 9} = 4$ and radius of $C_2 = \sqrt{4^2 + 3^2} = 5 > 4$.

III. ✗. Put $x = 0$.

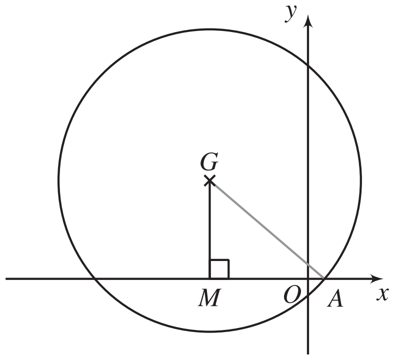
$$y^2 - 6y + 9 = 0 \quad \text{and} \quad y^2 - 6y = 0$$
$$y = 3 \qquad\qquad y = 0 \quad \text{or} \quad 6$$

C_1 cuts the y-axis at one point only.

15. C

Denote the centre by G.

Let M be a point on the x-axis such that GM is perpendicular to the x-axis.
Let A be the intersection of the circle and the positive x-axis.
The coordinates of M are $(-3,\ 0)$.
$AM = \frac{8}{2} = 4$ and the coordinates of A are $(1,\ 0)$.
Radius of circle $= \sqrt{(1+3)^2 + (0-3)^2} = 5$
Required equation is $(x+3)^2 + (y-3)^2 = 25$.



16. D

L passes through the centre of C.
Coordinates of centre of C are $\left(\frac{5}{2},\ -\frac{k}{2}\right)$.

$$2\left(\frac{5}{2}\right) - 3\left(\frac{-k}{2}\right) + 3 = 0$$
$$k = -\frac{16}{3}$$

17. A

Let the centre of S be G. The coordinates of G are $(1,\ 2)$. Note that $GM \perp AB$.
Slope of $GM = \frac{2+2}{1-3} = -2$
Slope of $AB = \frac{1}{2}$
Required equation is

$$y + 2 = \frac{1}{2}(x-3)$$
$$x - 2y - 7 = 0$$

18. B

Coordinates of O are $(1,\ 3)$.
$x^2 + 0 - 2x - 0 - 8 = 0$

$$x = 4 \quad \text{or} \quad -2$$

Required area $= \frac{(4+2)(3)}{2}$

$= 9$ sq units

19. D

C: $x^2 + y^2 + 3x + 4y - \frac{25}{2} = 0$

Centre $\left(-\frac{3}{2},\ -2\right)$

Radius $= \sqrt{\left(\frac{3}{2}\right)^2 + 2^2 + \frac{25}{2}} = \frac{5\sqrt{3}}{2}$

20. C

Centre $(2,\ k)$

L passes through the centre of C.

$$(2) + (k) - 7 = 0$$
$$k = 5$$

21. D

Centre $(2,\ -1)$

Distance from P to centre $= \sqrt{(2+2)^2 + (1+1)^2} = \sqrt{20}$

Radius $= \sqrt{2^2 + 1^2 + 31} = 6$

Required length $= 2\sqrt{6^2 - (\sqrt{20})^2}$

$= 8$

22. A

L passes through centre of circle $\left(\frac{k}{-2},\ -2\right)$.

$$\frac{3 - (-2)}{16 - \frac{k}{-2}} = \frac{1}{2}$$
$$k = -12$$

23. C

$$3^2 + 4^2 - 6(3) - 4k + 7k + 2 > 0$$
$$9 + 3k > 0$$
$$k > -3$$

24. D

I. ✗. Coordinates of centres are $\left(-\frac{a}{2},\ \frac{b}{2}\right)$ and $\left(\frac{a}{2},\ -\frac{b}{2}\right)$ respectively.

II. ✓. Both radius $= \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 - 0} = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$

III. ✓. Origin is indeed the midpoint of two centres.

25. (a) Coordinates of centres of C_1 and C_2 are $(2,\ -1)$ and $(-2,\ -3)$ respectively. 1M

Required distance $= \sqrt{(2+2)^2 + (-1+3)^2} = 2\sqrt{5}$ 1A

(b) $\sqrt{2^2 + (-1)^2} + \sqrt{(-2)^2 + (-3)^2 - k} = 2\sqrt{5}$ 1M

$$\sqrt{13 - k} = \sqrt{5}$$

$$k = 8$$ 1A

26. (a) $3^2 + (-1)^2 - 6(3) - 8(-1) + k = 0$

$$k = 0$$ 1A

(b) $x^2 + \left(-\frac{3x}{4}\right)^2 - 6x - 8\left(-\frac{3x}{4}\right) + k = 0$

$$\frac{25x^2}{16} + k = 0$$

Since S touches L,

$$\Delta = 0^2 - 4\left(\frac{25}{16}\right)(k) = 0$$ 1M

$$k = 0$$ 1A

(c) Radius $= \sqrt{\frac{10\pi}{\pi}} = \sqrt{10}$

$$\sqrt{(-3)^2 + (-4)^2 - k} = \sqrt{10}$$ 1M

$$25 - k = 10$$

$$k = 15$$ 1A

27. Radius of $C_1 = \sqrt{1^2 + 2^2 + 4} = 3$. 1M

Radius of $C_2 = 3 \times \sqrt{\frac{16}{9}} = 4$. 1M

The coordinates of centre of C_1 are $(1,\ 2)$. 1M

The equation of C_2 is

$$(x-1)^2 + (y-2)^2 = 4^2$$

$$x^2 + y^2 - 2x - 4y - 11 = 0$$ 1A

28. (a) (i) Let the coordinates of C be $(0,\ c)$.

$$\frac{2+c}{2} = 3 \qquad \text{1M}$$

$$c = 4 \qquad \text{1A}$$

The coordinates of C are $(0,\ 4)$.

(ii) $\sqrt{(a-0)^2 + (3-2)^2} = \sqrt{2}$ 1M

$$a^2 + 1 = 2$$

$$a = 1 \quad \text{or} \quad -1 \text{ (rejected)} \qquad \text{1A}$$

The equation of the circle is $(x-1)^2 + (y-3)^2 = 2$. 1A

(b) The area of $\triangle ABC = \frac{1}{2}(1)(4-2) = 1$ 1M+1A

29. (a) $(6,\ 17)$ 1A

(b) (i) Let $(h,\ k)$ be the coordinates of P.

Since P lies on L, we have $4h + 3k + 50 = 0$. 1M

Since $RP \perp L$,

$$\frac{k-17}{h-6} \times \frac{-4}{3} = -1 \qquad \text{1M}$$

$$3h - 4k + 50 = 0$$

Solving, $h = -14$ and $k = 2$.

Required distance $= \sqrt{(-14-6)^2 + (2-17)^2}$ 1M

$= 25$ 1A

(ii) (1) P, Q and R are collinear. 1A

(2) $QR = 10$ 1M

$PQ = 25 - 10 = 15$ 1M

Required ratio $= PQ : QR$

$= 3 : 2$ 1A

END OF PAPER