REG-GS-2223-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. $\boxed{\text{A}}$

$$\text{Required sum} = \frac{\frac{3}{4}(4^n - 1)}{4 - 1} = \frac{1}{4}(4^n - 1)$$

2. $\boxed{\text{A}}$

$$\text{Required sum} = \frac{2\left[1 - \left(\frac{\sqrt{2}}{2}\right)^{10}\right]}{1 - \frac{\sqrt{2}}{2}} = \frac{2\left(1 - \frac{1}{32}\right)}{\left(\frac{2-\sqrt{2}}{2}\right)} = \frac{31}{8(2 - \sqrt{2})}$$

3. $\boxed{\text{C}}$

$$\text{Required sum} = \frac{-1[1 - (-4)^7]}{1 - (-4)} = -3277$$

4. $\boxed{\text{A}}$

$$\text{5th term} = \left(32 - 2^{5-5}\right) - \left(32 - 2^{5-4}\right) = 31 - 30 = 1$$

5. $\boxed{\text{A}}$

$$640\left(-\frac{1}{2}\right)^{n-1} = -5$$
$$\left(-\frac{1}{2}\right)^{n-1} = -\frac{1}{128} = \left(-\frac{1}{2}\right)^7$$
$$n = 8$$

There are 8 terms.

$$\text{Required sum} = \frac{640\left[1 - \left(-\frac{1}{2}\right)^8\right]}{1 - \left(-\frac{1}{2}\right)} = 425$$

6. $\boxed{\text{D}}$

$$\text{Required sum} = \frac{7(5^7 - 1)}{5 - 1}$$
$$= 136\,717$$

7. $\boxed{\text{A}}$

$$\frac{-1(3^k - 1)}{3 - 1} < -200$$
$$3^k > 401$$
$$k \log 3 > \log 401$$
$$k > 5.46$$

Minimum value of k is 6.

8. $\boxed{\text{B}}$

$$\frac{3(3^k - 1)}{3 - 1} > 780$$
$$3^k > 521$$
$$k \log 3 > \log 521$$
$$k > 5.69$$

Minimum value of k is 6.

9. $\boxed{\text{D}}$

$$\text{Required sum} = \frac{18}{1 - \frac{1}{3}}$$
$$= 27$$

10. $\boxed{\text{A}}$

Let the common ratio be r.

$$\frac{30}{1 - r} = 25$$
$$r = \frac{1}{6}$$

11. $\boxed{\text{A}}$

Let the first term be a.

$$\frac{a}{1 - \left(-\frac{1}{2}\right)} = \frac{1}{6}$$
$$a = \frac{1}{4}$$

$$\text{Required sum} = \frac{\frac{1}{4}\left[1 - \left(-\frac{1}{2}\right)^5\right]}{1 - \left(-\frac{1}{2}\right)}$$
$$= \frac{11}{64}$$

12. $\boxed{\text{C}}$

Sum of first four terms $= -10 - 5 - 2.5 - 1.25$

$$= -18.75$$

Sum to infinity $= \dfrac{-10}{1 - \frac{1}{2}}$

$$= -20$$

Error $= -18.75 - (-20)$

$$= \frac{5}{4}$$

13. $\boxed{\text{C}}$

Let the first term be a.

$$\frac{a}{1 - \frac{1}{2}} = 16$$

$$a = 8$$

14. $\boxed{\text{B}}$

$$\frac{2x - 1}{5x + 2} = \frac{x - 2}{2x - 1}$$

$$(2x - 1)^2 = (x - 2)(5x + 2)$$

$$4x^2 - 4x + 1 = 5x^2 - 8x - 4$$

$$0 = x^2 - 4x - 5$$

$$x = 5 \quad \text{or} \quad -1 \text{ (rejected)}$$

Required sum $= \dfrac{27}{1 - \frac{9}{27}}$

$$= \frac{81}{2}$$

15. $\boxed{\text{A}}$

Common ratio $= \dfrac{q}{p}$

First term $= \dfrac{p}{\left(\frac{q}{p}\right)^2} = \dfrac{p^3}{q^2}$

Required sum $= \dfrac{\frac{p^3}{q^2}}{1 - \frac{q}{p}}$

$$= \frac{p^3}{q^2} \div \frac{p - q}{p}$$

$$= \frac{p^4}{q^2(p - q)}$$

16. $\boxed{\text{A}}$

$$\frac{36}{2k} = \frac{72k}{36}$$
$$36 = 4k^2$$
$$k = 3 \quad \text{or} \quad -3 \text{ (rejected)}$$

The geometric sequence is 216, 36, 6, . . .

$$\text{Required sum} = \frac{216}{1 - \frac{6}{36}} = \frac{1296}{5}$$

17. $\boxed{\text{C}}$

$$\frac{a}{1 - \left(-\frac{1}{3}\right)} = \frac{45}{8}$$
$$a = \frac{15}{2}$$

18. $\boxed{\text{B}}$

$$\text{Ratio} = \frac{-24}{72} = -\frac{1}{3}$$
$$\text{Sum to infinity} = \frac{72}{1 - \left(-\frac{1}{3}\right)} = 54$$

19. $\boxed{\text{B}}$

Let the first term and common ratio be a and r respectively.

$$\begin{cases} \dfrac{a(1-r^3)}{1-r} = -\dfrac{3}{8} \\ \dfrac{a}{1-r} = -\dfrac{1}{3} \end{cases}$$

$$\left(-\frac{1}{3}\right)(1-r^3) = -\frac{3}{8}$$
$$r^3 = -\frac{1}{8}$$
$$r = -\frac{1}{2}$$

20. $\boxed{\text{C}}$

All positive terms: $36, 9, \frac{9}{4}, \ldots$.

$$\text{Sum} = \frac{36}{1 - \frac{1}{4}} = 48$$

21. $\boxed{\text{B}}$

$$\text{Required sum} = \left(\frac{1}{3} + \frac{1}{3^5} + \ldots\right) + \left(\frac{2}{3^3} + \frac{2}{3^7} + \ldots\right) \quad = \frac{\frac{1}{3}}{1 - \frac{1}{3^4}} + \frac{\frac{2}{27}}{1 - \frac{1}{3^4}}$$
$$= \frac{33}{80}$$

22. $\boxed{\text{A}}$

Let the first term and common ratio be a and r respectively.

$$\begin{aligned} \frac{ar^7}{ar^5} &= \frac{3}{48} \\ r^2 &= \frac{1}{16} \\ r &= \pm\frac{1}{4} \end{aligned}$$

I. ✓. $x_4 = \dfrac{48}{r^2} = 768$.

II. ✗. $\dfrac{x_9}{x_{12}} = \dfrac{1}{r^3} < 0 < 1$ when $r = -\dfrac{1}{4}$.

III. ✗. There are n terms.

$$x_4 + x_6 + \ldots + x_{2n+2} = \frac{768\left[1 - \left(\frac{1}{16}\right)^n\right]}{1 - \frac{1}{16}} = 819.2\left[1 - \left(\frac{1}{16}\right)^n\right] < 819.2 < 1000$$

23. $\boxed{\text{B}}$

I. ✓. $\begin{aligned}[t] \text{General term} &= (1 - 2^{-n}) - (1 - 2^{-(n-1)}) \\ &= 2^{-n}(-1 + 2) \\ &= 2^{-n} \end{aligned}$

We have $2^{-n} < 1$ for all positive integers n.

II. ✗. The nth term $= \dfrac{1}{2^n}$ is a rational number for all positive integers n.

III. ✓. $\begin{aligned}[t] \log T_{n+1} - \log T_n &= \log 2^{-n-1} - \log 2^{-n} \\ &= (-n - 1)\log 2 + n \log 2 \\ &= -\log 2 = \text{constant} \end{aligned}$

Thus, it is an arithmetic sequence.

24. $\boxed{\text{A}}$

$$\begin{aligned} \text{General term} &= (2^{n+1} - 2) - (2^n - 2) \\ &= 2^n(2 - 1) \\ &= 2^n \end{aligned}$$

I. ✓. $\dfrac{T(n+1)}{T(n)} = \dfrac{2^{n+1}}{2^n} = 2 = \text{constant}$.

II. ✗. Second term $= 2^2 = 4 \neq 6$.

III. ✗.

25. $\boxed{\text{D}}$

Let the first term and common ratio be a and r respectively.

$$\begin{aligned} \frac{ar^7}{ar^2} &= \frac{6}{192} \\ r^5 &= \frac{1}{32} \\ r &= \frac{1}{2} \end{aligned}$$

I. ✗.

II. ✓. $a = \dfrac{192}{r^2} = 768.$

$$\begin{aligned} 768\left(\frac{1}{2}\right)^{n-1} &> 10^{-2} \\ (n-1)\log\frac{1}{2} &> \log\frac{1}{76\,800} \\ n &< 17.2 \end{aligned}$$

17 terms are greater than 10^{-2}.

III. ✓. Sum $= \dfrac{768\left(1-\left(\frac{1}{2}\right)^{13}\right)}{1-\frac{1}{2}} \approx 1535.8$

> 1535

26. $\boxed{\text{A}}$

I. ✓. 4th term $= 2 \times 3^4 = 162$.

II. ✗. Sum of first n terms $= \dfrac{2(3)(3^n-1)}{3-1} = 3(3^n-1)$.

III. ✗. Sum to infinity does not exists as common ratio > 1.

27. $\boxed{\text{D}}$

Let first term and common ratio be a and r respectively.

$$\frac{ar^7}{ar^3} = \frac{4}{243} \div \frac{1}{12}$$
$$r^4 = \frac{16}{81}$$
$$r = \pm\frac{2}{3}$$

When $r = \frac{2}{3}$, $a = \frac{1}{12} \div r^3 = \frac{9}{32}$ and $S(\infty) = \frac{a}{1-r} = \frac{27}{32} > \frac{1}{2}$ (rejected).
When $r = -\frac{2}{3}$, $a = \frac{1}{12} \div r^3 = -\frac{9}{32}$.

I. ✗.

II. ✓. $a_1 + a_2 + \ldots + a_{10} = \frac{a(r^9-1)}{r-1} \approx -0.173 < -\frac{1}{10}$.

III. ✓. $a_2 + a_4 + \ldots = \frac{ar}{1-r^2} = \frac{27}{80}$.

28. $\boxed{\text{A}}$

Let the first term and common ratio be a and r respectively.

$$\frac{ar^6}{ar^4} = \frac{15}{375}$$
$$r^2 = \frac{1}{25}$$
$$r = \pm\frac{1}{5}$$

I. ✓. $a_1 = \frac{375}{r^4} = 234\,375$.

II. ✗. When $r = -\frac{1}{5}$, $a_2 = \frac{ar^4}{r^3} < 0$ and $a_2 \neq 46\,875$.

III. ✗. When $r = -\frac{1}{5}$,

$$\begin{aligned} a_1 + a_2 + a_3 + \ldots &= \frac{234\,375}{1 - \left(-\frac{1}{5}\right)} \\ &= 195\,312.5 < 290\,000 \end{aligned}$$

29. $\boxed{\text{A}}$

Let the first term and common ratio be a and r respectively.

$$\begin{aligned}\frac{ar^6}{ar^2} &= \frac{48}{3}\\ r^4 &= 16\\ r &= \pm 2\end{aligned}$$

I. ✓. 5th term $= 3 \times r^2 = 12$.

II. ✗. The sequence has negative terms when $r = -2$.

III. ✗. Sum to infinity does not exist.

30. $\boxed{\text{D}}$

Let the common ratio be r.

$$\begin{aligned}r^{6-2} &= \frac{1}{18} \div \frac{9}{32}\\ r^4 &= \frac{16}{81}\\ r &= \pm\frac{2}{3}\end{aligned}$$

When $r = \frac{2}{3}$, $a_1 = \frac{a_2}{r} = \frac{27}{64}$.

Sum to infinity $= \frac{a_1}{1-r} = \frac{81}{64} > \frac{1}{2}$.

When $r = -\frac{2}{3}$, $a_1 = \frac{a_2}{r} = -\frac{27}{64}$.

Sum to infinity $= \frac{a_1}{1-r} = -\frac{81}{320} < \frac{1}{2}$.

Thus, $r = -\frac{2}{3}$.

I. ✓. $a_4 = a_2 \times r^2 = \frac{1}{8}$

II. ✓.

$$\begin{aligned}a_1 - a_2 + a_3 - a_4 + \ldots + a_9 - a_{10} &= \frac{a_1[1-(-r)^{10}]}{1-(-r)}\\ &= \frac{\frac{a_2}{r}(1-r^{10})}{1+r}\\ &\approx -1.24 < -1\end{aligned}$$

III. ✓.

$$\begin{aligned}a_2 + a_4 + \ldots + &= \frac{a_2}{1-r^2}\\ &= \frac{81}{160}\\ &= \frac{9}{320\left(\frac{1}{18}\right)}\end{aligned}$$

Conventional Questions

31. (a) $A(1)+A(2)+A(3)+\ldots+A(n) = 1000(2+2^2+2^3+\ldots+2^n)$

$$= 1000\times\frac{2(2^n-1)}{2-1} \quad \text{1M}$$

$$= 2000(2^n-1) \quad \text{1}$$

(b) $B(1)+B(2)+B(3)+\ldots+B(n) > A(1)+A(2)+A(3)+\ldots+A(n)$

$$\frac{4(4^n-1)}{4-1} > 2000(2^n-1)$$

$$\frac{4}{3}(2^n)^2 - 2000(2^n) + \frac{5996}{3} > 0 \quad \text{1A}$$

$2^n < 1$ (rejected) or $2^n > 1499$

$$n\log 2 > \log 1499 \quad \text{1M}$$

$$n > 10.5$$

The least value of n is 11. 1A

32. (a) $S(n) = \dfrac{16\left[1-\left(\frac{3}{4}\right)^n\right]}{1-\frac{3}{4}}$ 1M

$$= 64\left[1-\left(\frac{3}{4}\right)^n\right] \quad \text{1A}$$

(b) Required sum $= 64\left[1-\left(\frac{3}{4}\right)^7\right]$

$$= \frac{14\,197}{256} \quad \text{1A}$$

(c) $64\left[1-\left(\frac{3}{4}\right)^k\right] > 60$ 1M

$$\left(\frac{3}{4}\right)^k < \frac{1}{16}$$

$$k\log\frac{3}{4} < \log\frac{1}{16} \quad \text{1M}$$

$$k > 9.64$$

The least value of k is 10. 1A

33. (a) Let the first term and common ratio be a and r respectively.

$$\begin{cases} ar^4 + ar^5 + ar^6 = 6318 \\ a + ar + ar^2 = 78 \end{cases} \quad \text{1M}$$

$$\frac{ar^4(1+r+r^2)}{a(1+r+r^2)} = \frac{6318}{78} \quad \text{1M}$$

$$r^4 = 81$$

$$r = \pm 3 \quad \text{1A}$$

When $r = 3$, $a = \dfrac{78}{1+r+r^2} = 6$ and $T(n) = 6(3)^{n-1}$.

When $r = -3$, $a = \dfrac{78}{1+r+r^2} = \dfrac{78}{7}$ and $T(n) = \dfrac{78}{7}(-3)^{n-1}$. 1A

(b) (i) $T(n) = 6(3)^{n-1}$

$$\frac{6(3^m - 1)}{3 - 1} > 5000 \quad \text{1M}$$

$$3^m > \frac{5003}{3}$$

$$m \log 3 > \log \frac{5003}{3} \quad \text{1M}$$

$$m > 6.75$$

Least value of m is 7. 1A

(ii) Required sum $= \dfrac{6[(3^3)^8 - 1]}{3^3 - 1}$ 1M+1A

$\approx 6.52 \times 10^{10}$ 1A

34. (a) Let the common ratio be r.

$$\begin{cases} G_1 r + G_1 r^2 = 1944 \\ G_1 r^4 + G_1 r^5 = 72 \end{cases}$$

$$\frac{G_1 r^4(1+r)}{G_1 r(1+r)} = \frac{72}{1944} \quad \text{1M}$$

$$r^3 = \frac{1}{27}$$

$$r = \frac{1}{3} \quad \text{1A}$$

So, $G_1 = \dfrac{1944}{r + r^2} = 4374$. 1A

(b) $$\frac{4374}{1 - \frac{1}{3}} - \frac{4374\left[1 - \left(\frac{1}{3}\right)^n\right]}{1 - \frac{1}{3}} < 5 \times 10^{-19} \quad \text{1M+1M}$$

$$6561\left(\frac{1}{3}\right)^n < 5 \times 10^{-19}$$

$$\log 6561 + n \log \frac{1}{3} < \log 5 \times 10^{-19} \quad \text{1M}$$

$$n > 46.4$$

The least value of n is 47. 1A

35. Let first term and common ratio be a and r respectively, and 243 be the nth term.

Consider the sum from the term 243 to infinity,

$$\frac{243}{1-r} = 4096 - 3367 + 243$$ 1M+1A

$$243 = 972 - 972r$$

$$r = \frac{3}{4}$$ 1A

Consider the sum to infinity,

$$\frac{a}{1-r} = 4096$$ 1M

$$a = 1024$$ 1A

The first term is 1024.

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