M2REG-MI-2223-ASM-SET 1-MATH
Suggested solutions

1. (a) $\sum_{i=1}^{5}(a_i^2 + 2a_i + 1) = \sum_{i=1}^{5} a_i^2 + 2\sum_{i=1}^{5} a_i + \sum_{i=1}^{5} 1$

$= 8 + 2(4) + 5(1)$ 1M

$= 21$ 1A

(b) $\sum_{i=1}^{5}(a_i^2 + 1)(2a_i - 3) = 2\sum_{i=1}^{5} a_i^3 - 3\sum_{i=1}^{5} a_i^2 + 2\sum_{i=1}^{5} a_i + \sum_{i=1}^{5}(-3)$

$= 2(-3) - 3(8) + 2(4) + 5(-3)$ 1M

$= -37$ 1A

2. $\sum_{i=1}^{8}(3a_i + 2)(2b_i - 5) = 6\sum_{i=1}^{8} a_i b_i - 15\sum_{i=1}^{8} a_i + 4\sum_{r=1}^{8} b_i + \sum_{i=1}^{8}(-10)$

$= 6(12) - 15(-4) + 4(6) + 8(-10)$ 1M

$= 76$ 1A

3. (a) $\sum_{i=5}^{9} a_i(a_i^2 + 2) = \sum_{i=5}^{9} a_i^3 + 2\sum_{i=5}^{9} a_i$

$= 5 + 2(1)$ 1M

$= 7$ 1A

(b) $\sum_{j=5}^{9} \frac{a_j^6 - 1}{a_j^3 + 1} = \sum_{i=5}^{9} \frac{a_i^6 - 1}{a_i^3 + 1}$

$= \sum_{i=5}^{9}(a_i^3 - 1)$

$= \sum_{i=5}^{9} a_i^3 - \sum_{i=5}^{9}(1)$

$= 5 - (9 - 5 + 1)(1)$ 1M

$= 0$ 1A

4. (a) $\sum_{i=3}^{7} m_i^2(1 - m_i^3) = \sum_{i=3}^{7} m_i^2 - \sum_{i=3}^{7} m_i^5$ 1M

$= 2 - 6$

$= -4$ 1A

(b) $\sum_{k=3}^{7} \frac{m_k^{10} - 1}{m_k^5 + 1} = \sum_{i=3}^{7} \frac{m_i^{10} - 1}{m_i^5 + 1}$

$= \sum_{i=3}^{7} m_i^5 - \sum_{i=3}^{7}(1)$

$= 6 - (7 - 3 + 1)(1)$ 1M

$= 1$ 1A

5. When $n = 1$,

L.H.S. = 1 and R.H.S. = $\frac{3^1 - 1}{2} = 1 =$ L.H.S.

The statement is true for $n = 1$. 1

Assume $1 + 3 + 9 + \ldots + 3^{k-1} = \frac{3^k - 1}{2}$, where $k \in \mathbb{Z}^+$. 1

$1 + 3 + 9 + \ldots + 3^{k-1} + 3^{(k+1)-1}$

$= \frac{3^k - 1}{2} + 3^k$ 1

$= \frac{3^k - 1 + 2 \times 3^k}{2}$

$= \frac{3^{k+1} - 1}{2}$ 1

The statement is true for $n = k + 1$.

By M.I., the statement is true $\forall n \in \mathbb{Z}^+$. 1

6. For $n = 1$,

L.H.S. = $1 \times 11 = 11$ and R.H.S. = $\frac{1}{6}(1)(1 + 1)(2(1 + 31) = 11 =$ L.H.S.

The statement is true for $n = 1$. 1

Assume $1 \times 11 + 2 \times 12 + 3 \times 13 + \ldots + k(k + 10) = \frac{1}{6}k(k + 1)(2k + 31)$, where $k \in \mathbb{Z}^+$. 1

$1 \times 11 + 2 \times 12 + 3 \times 13 + \ldots + k(k + 10) + (k + 1)[(k + 1) + 10]$

$= \frac{1}{6}k(k + 1)(2k + 31) + (k + 1)(k + 11)$ 1

$= \frac{k + 1}{6}[2k^2 + 31k + 6(k + 11)]$

$= \frac{k + 1}{6}(2k^2 + 37k + 66)$

$= \frac{k + 1}{6}(k + 2)(2k + 33)$

$= \frac{1}{6}(k + 1)[(k + 1) + 1][2(k + 1) + 31]$ 1

The statement is true for $n = k + 1$.

By the principle of M.I., the statement is true $\forall n \in \mathbb{Z}^+$. 1

7. For $n = 1$,

L.H.S. $= 1 \times 2 \times 5 = 10$ and R.H.S. $= \frac{1}{3}(1)(2)(3)(5) = 10 =$ L.H.S.

The statement is true for $n = 1$. 1

Assume $1 \times 2 \times 5 + 2 \times 3 \times 13 + \ldots + k(k+1)(8k-3) = \frac{1}{3}k(k+1)(k+2)(6k-1)$ for some $k \in \mathbb{Z}^+$. 1M

$$1 \times 2 \times 5 + 2 \times 3 \times 13 + 3 \times 4 \times 21 + \ldots + k(k+1)(8k-3) + (k+1)(k+2)(8k+5)$$

$$= \frac{1}{3}k(k+1)(k+2)(6k-1) + (k+1)(k+2)(8k+5) \quad \text{1M}$$

$$= \frac{1}{3}(k+1)(k+2)[(6k^2-k)+3(8k+5)]$$

$$= \frac{1}{3}(k+1)(k+2)(6k^2+23k+15)$$

$$= \frac{1}{3}(k+1)(k+2)(k+3)(6k+5) \quad \text{1}$$

The statement is true for $n = k + 1$.

By mathematical induction, the statement is true $\forall n \in \mathbb{Z}^+$. 1

8. For $n = 1$,

L.H.S. $= (2)(2)(3) = 12$ and R.H.S. $= 1(2)^2(3) = 12 =$ L.H.S.

The statement is true for $n = 1$. 1

Assume $\sum_{i=1}^{k} 2i(i+1)(2i+1) = k(k+1)^2(k+2)$ for some $k \in \mathbb{Z}^+$. 1M

$$\sum_{i=1}^{k+1} 2i(i+1)(2i+1) = \sum_{i=1}^{k} 2i(i+1)(2i+1) + 2(k+1)(k+2)(2k+3)$$

$$= k(k+1)^2(k+2) + 2(k+1)(k+2)(2k+3) \quad \text{1M}$$

$$= (k+1)(k+2)(k^2+k+4k+6)$$

$$= (k+1)(k+2)(k^2+5k+6)$$

$$= (k+1)(k+2)^2(k+3) \quad \text{1}$$

The statement is true for $n = k + 1$.

By mathematical induction, the statement is true $\forall n \in \mathbb{Z}^+$. 1

9. For $n = 1$,

L.H.S. $= \dfrac{1^2}{1 \cdot 3} = \dfrac{1}{3}$ and R.H.S. $= \dfrac{1(1+1)}{2(2+1)} = \dfrac{1}{3} =$ L.H.S.

The statement is true for $n = 1$. 1

Assume $\displaystyle\sum_{i=1}^{k} \frac{i^2}{(2i-1)(2i+1)} = \frac{k(k+1)}{2(2k+1)}$ for some $k \in \mathbb{Z}^+$. 1M

$$\begin{aligned}
\sum_{i=1}^{k+1} \frac{i^2}{(2i-1)(2i+1)} &= \sum_{i=1}^{k} \frac{i^2}{(2i-1)(2i+1)} + \frac{(k+1)^2}{(2k+1)(2k+3)} \\
&= \frac{k(k+1)}{2(2k+1)} + \frac{(k+1)^2}{(2k+1)(2k+3)} && \text{1M} \\
&= \frac{k+1}{2(2k+1)(2k+3)}[k(2k+3)+2(k+1)] \\
&= \frac{k+1}{2(2k+1)(2k+3)}(2k^2+5k+2) \\
&= \frac{k+1}{2(2k+1)(2k+3)}(2k+1)(k+2) \\
&= \frac{(k+1)(k+2)}{2(2k+3)} && \text{1}
\end{aligned}$$

The statement is true for $n = k+1$.

By mathematical induction, the statement is true $\forall n \in \mathbb{Z}^+$. 1

10. When $n = 1$,

L.H.S. $= 4$ and R.H.S. $= \dfrac{2^2 + 3^2 - 5}{2} = 4 =$ L.H.S.

It is true for $n = 1$. 1

Assume $4 + 11 + 31 + \ldots + (2^{k-1} + 3^k) = \dfrac{2^{k+1} + 3^{k+1} - 5}{2}$, where $k \in \mathbb{Z}^+$. 1

$$\begin{aligned}
&4 + 11 + 31 + \ldots + (2^{k-1} + 3^k) + (2^k + 3^{k+1}) \\
&= \frac{2^{k+1} + 3^{k+1} - 5}{2} + 2^k + 3^{k+1} && \text{1} \\
&= \frac{2^{k+1} + 3^{k+1} - 5 + 2^{k+1} + 2 \cdot 3^{k+1}}{2} \\
&= \frac{2^{k+2} + 3^{k+2} - 5}{2} && \text{1}
\end{aligned}$$

It is true for $n = k+1$.

By M.I., it is true $\forall n \in \mathbb{Z}^+$. 1