M2REG-MAD-2223-ASM-SET 2-MATH
Suggested solutions

1. $\begin{vmatrix} x-1 & x+\sqrt{2} \\ x-\sqrt{2} & 2x \end{vmatrix} = 2x(x-1)-(x+\sqrt{2})(x-\sqrt{2})$ 1M

$= 2x^2-2x-(x^2-2)$

$= x^2-2x+2$

$= (x-1)^2+1$ 1M

> 0 1

2. $\begin{vmatrix} 2 & \frac{1}{x} \\ -1 & x \end{vmatrix} = 3$

$2x+\frac{1}{x} = 3$ 1M

$2x^2-3x+1 = 0$ 1M

$x = \frac{1}{2}$ or 1 1A

3. $\begin{vmatrix} \cos x & -\tan x \\ \sin x\cos x+\cos^2 x & \cos x-\sin x \end{vmatrix}$

$= \cos x(\cos x-\sin x)+\tan x(\sin x\cos x+\cos^2 x)$ 1M

$= \cos^2 x-\sin x\cos x+\sin^2 x+\sin x\cos x$

$= 1$ 1

4. $\begin{vmatrix} (-1)^n & -1 \\ 1 & (-1)^n \end{vmatrix} = (-1)^{2n}-(-1)$ 1M

$= 1+1$

$= 2$ 1A

5. $\begin{vmatrix} \sin\theta & 1 \\ \sin 2\theta & \cos\theta \end{vmatrix} = \sin\theta\cos\theta-\sin 2\theta$ 1M

$= \sin\theta\cos\theta-2\sin\theta\cos\theta$

$= -\sin\theta\cos\theta$ 1A

$\begin{vmatrix} (\sin\theta-\cos\theta)^2 & \sin 2\theta-2 \\ -\frac{1}{2} & 1 \end{vmatrix} = (\sin\theta-\cos\theta)^2+\frac{1}{2}(\sin 2\theta-2)$

$= \sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta+\frac{1}{2}(2\sin\theta\cos\theta)-1$

$= -\sin\theta\cos\theta$

Thus, $\begin{vmatrix} \sin\theta & 1 \\ \sin 2\theta & \cos\theta \end{vmatrix} = \begin{vmatrix} (\sin\theta-\cos\theta)^2 & \sin 2\theta-2 \\ -\frac{1}{2} & 1 \end{vmatrix}$. 1

6. $\begin{vmatrix} x & x+1 & x(1+yz) \\ y & y+1 & y(1+zx) \\ z & z+1 & z(1+xy) \end{vmatrix}$

$= x(1+yz)\begin{vmatrix} y & y+1 \\ z & z+1 \end{vmatrix} - y(1+zx)\begin{vmatrix} x & x+1 \\ z & z+1 \end{vmatrix} + z(1+xy)\begin{vmatrix} x & x+1 \\ y & y+1 \end{vmatrix}$ 1M

$= x(1+yz)[y(z+1) - z(y+1)] - y(1+zx)[x(z+1) - z(x+1)]$

$\quad + z(1+xy)[x(y+1) - y(x+1)]$

$= x(1+yz)(y-z) - y(1+zx)(x-z) + z(1+xy)(x-y)$

$= xy - xz + xy^2z - xyz^2 - xy + yz - x^2yz + xyz^2 + xz - yz + x^2yz - xy^2z$

$= 0$ 1A

7. $\begin{vmatrix} x & 0 & x \\ y & x & x \\ x & y & y \end{vmatrix} = x\begin{vmatrix} x & x \\ y & y \end{vmatrix} + x\begin{vmatrix} y & x \\ x & y \end{vmatrix}$ 1M

$= x(xy - xy) + x(y^2 - x^2)$

$= x(y+x)(y-x)$ 1A

8. $\begin{vmatrix} \cos\theta & -\sin\theta & \tan 2\theta \\ \sin\theta & \cos\theta & -1 \\ \cos\theta & \sin\theta & 1 \end{vmatrix}$

$= \tan 2\theta\begin{vmatrix} \sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{vmatrix} - (-1)\begin{vmatrix} \cos\theta & -\sin\theta \\ \cos\theta & \sin\theta \end{vmatrix} + (1)\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ 1M

$= \tan 2\theta(\sin^2\theta - \cos^2\theta) + (\sin\theta\cos\theta + \sin\theta\cos\theta) + (\cos^2\theta + \sin^2\theta)$

$= \tan 2\theta(-\cos 2\theta) + \sin 2\theta + 1$ 1M

$= -\sin 2\theta + \sin 2\theta + 1$

$= 1$ 1A

9. (a) $0 = \begin{vmatrix} 8-x^2 & 0 & 4-x \\ 4-x & 1 & x^2 \\ 2-3x & 0 & x-4 \end{vmatrix}$

$= \begin{vmatrix} 8-x^2 & 4-x \\ 2-3x & x-4 \end{vmatrix}$ 1M

$= (8-x^2)(x-4) - (4-x)(2-3x)$

$= (x-4)(-x^2 - 3x + 10)$ 1M

$= -(x-4)(x-2)(x+5)$

$x = 4$ or 2 or -5 1A

10. $0 = \begin{vmatrix} x-1 & -1 & x+1 \\ 2x+1 & 1 & x \\ -1 & 2 & 2x \end{vmatrix}$

$= (x-1)\begin{vmatrix} 1 & x \\ 2 & 2x \end{vmatrix} - (-1)\begin{vmatrix} 2x+1 & x \\ -1 & 2x \end{vmatrix} + (x+1)\begin{vmatrix} 2x+1 & 1 \\ -1 & 2 \end{vmatrix}$ 1M

$= (x-1)(2x-2x) + [2x(2x+1)+x] + (x+1)[2(2x+1)+1]$

$= 8x^2 + 10x + 3$ 1A

$x = -\dfrac{3}{4} \quad \text{or} \quad -\dfrac{1}{2}$ 1A

11. (a) $\det B = \begin{vmatrix} 1 & x & x+1 \\ 1 & x-1 & x+2 \\ 1 & 2x & x^2 \end{vmatrix}$

$= \begin{vmatrix} x-1 & x+2 \\ 2x & x^2 \end{vmatrix} - \begin{vmatrix} x & x+1 \\ 2x & x^2 \end{vmatrix} + \begin{vmatrix} x & x+1 \\ x-1 & x+2 \end{vmatrix}$ 1M

$= [x^2(x-1) - 2x(x+2)] - [x^3 - 2x(x+1)] + [x(x+2) - (x+1)(x-1)]$

$= 1 - x^2$

$= (1+x)(1-x)$ 1A

(b) $\det B = 0$

$1 - x^2 = 0$

$x = \pm 1$ 1A

12. (a) $M + N = \begin{pmatrix} 6 & 2 & k+1 \\ k-1 & 3 & k \\ k & 1 & -1 \end{pmatrix}$ 1A

(b) $-14 = \begin{vmatrix} 6 & 2 & k+1 \\ k-1 & 3 & k \\ k & 1 & -1 \end{vmatrix}$

$= 6\begin{vmatrix} 3 & k \\ 1 & -1 \end{vmatrix} - 2\begin{vmatrix} k-1 & k \\ k & -1 \end{vmatrix} + (k+1)\begin{vmatrix} k-1 & 3 \\ k & 1 \end{vmatrix}$ 1M

$= 6(-3-k) - 2(-k+1-k^2) + (k+1)(k-1-3k)$

$= -7k - 21$

$k = -1$ 1A

13. $0 = \begin{vmatrix} x & x+1 & 1 \\ x-1 & 2 & -3 \\ 1 & -1 & 4 \end{vmatrix}$

$= (1)\begin{vmatrix} x-1 & 2 \\ 1 & -1 \end{vmatrix} - (-3)\begin{vmatrix} x & x+1 \\ 1 & -1 \end{vmatrix} + (4)\begin{vmatrix} x & x+1 \\ x-1 & 2 \end{vmatrix}$ 1M

$= [(1-x)-2] + 3[-x-(x+1)] + 4[2x-(x+1)(x-1)]$

$= -4x^2 + x$ 1A

$x = 0$ or $\frac{1}{4}$ 1A

14. (a) $\det\left(A^2\right) = \begin{vmatrix} 8 & 6 \\ -2 & 3 \end{vmatrix}$

$= 36$ 1M

$(\det A)^2 = 36$ 1M

$\det A = \pm 6$ 1A

(b) $\det B = \begin{vmatrix} b & 1 \\ 2b & b \end{vmatrix}$

$= b^2 - 2b$

$18 = \det(AB)$

$= (\det A)(\det B)$ 1M

$= \pm 6(b^2 - 2b)$

$6b^2 - 12b = 18$ or $6b^2 - 12b = -18$ 1M

$6b^2 - 12b - 18 = 0$ $6b^2 - 12b + 18 = 0$ (rejected)

$b = -1$ or 3 1A

15. (a) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (1)\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - (1)\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + (1)\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$ 1M

$= (bc^2 - b^2c) - (ac^2 - a^2c) + (ab^2 - a^2b)$

$= c^2(b-a) - c(b^2 - a^2) + ab(b-a)$

$= (a-b)(-c^2) + c(a-b)(a+b) - ab(a-b)$

$= (a-b)[-c^2 + c(a+b) - ab]$ 1M

$= (a-b)[c(b-c) - a(b-c)]$

$= (a-b)(b-c)(c-a)$ 1

(b) $20 = \begin{vmatrix} 1 & x-1 & (x-1)^2 \\ 1 & 3 & 9 \\ 1 & x-5 & (x-5)^2 \end{vmatrix}$

$20 = [(x-1)-3][3-(x-5)][(x-5)-(x-1)]$ 1M

$20 = (x-4)(8-x)(-4)$

$0 = -x^2 + 12x - 27$ 1M

$x = 3$ or 9 1A

16. (a) $P(x) = -2(x+\lambda)^2 - (x+\lambda)^3 - 4$ 1M

Consider the coefficient of x.

$-2(2\lambda) - C_1^3\lambda^2 = 1$ 1M

$-3\lambda^2 - 4\lambda - 1 = 0$

$\lambda = -1$ or $-\frac{1}{3}$ 1A

(b) $P'(x) = -4(x+\lambda) - 3(x+\lambda)^2$ 1A

When $\lambda = -1$, $P'(1) = -4(1-1) - 3(1-1)^2 = 0$. 1A

When $\lambda = -\frac{1}{3}$, $P'(1) = -4\left(1 - \frac{1}{3}\right) - 3\left(1 - \frac{1}{3}\right)^2 = -4$. 1A

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