Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Compulsory Part

S5 – S6 Core Assignment Set 3

Name: ____________________ Telephone: ____________

Student number: ____________________ Centre: ____________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S5 – S6 Core
Phase 1 – Lesson 3

1. B

Discriminant $= (k-3)^2 - 4(1)(2k)$

$= k^2 - 14k + 9$

Required value $= -\dfrac{-14}{2} = 7$

2. D

Let the vertex of the graph of $y = f(x)$ be $(h,\ 6)$.

$h = -\dfrac{-12}{2(2-q)} = \dfrac{6}{2-q}$

$$6 = (2-q)\left(\frac{6}{2-q}\right)^2 - 12\left(\frac{6}{2-q}\right) - 3$$

$$9 = \frac{36-72}{2-q}$$

$$2-q = -4$$

$$q = 6$$

3. C

When the height is maximum, $t = -\dfrac{12}{2(-5)} = 1.2$.

Maximum height $= 4 + 12(1.2) - 5(1.2)^2$

$= 11.2\text{ m}$

4. D

$y = x^2 - 2ax + 8$

$= (x-a)^2 + 8 - a^2$

So, $a = 3$ and $k = 8 - a^2 = -1$.

5. D

When y is maximum, $x = -\dfrac{5}{2(1)} = -\dfrac{5}{2}$.

$$-\frac{7}{4} = \left(-\frac{5}{2}\right)^2 + 5\left(-\frac{5}{2}\right) + k$$

$$k = \frac{9}{2}$$

6. D

$x^2 - 2x + 10 = (x-1)^2 + 9$. The minimum value is 9 when $x = 1$.

By considering the shape of the curve $y = x^2 - 2x + 10$, the greatest value must occur at either ends.

When $x = -2$, $x^2 - 2x + 10 = 18$; when $x = 3$, $x^2 - 3x + 10 = 13$.

So, the maximum value is 18.

7. $\boxed{D}$

When T attains its minimum, $x = -\dfrac{-3}{2\left(\frac{1}{8}\right)} = 12$

Required time is 12 minutes after adding the cubes.

8. $\boxed{B}$

$f(x)$ attains minimum when $x = -\dfrac{(-3)}{2(4)} = \dfrac{3}{8}$.

9. $\boxed{B}$

A. ✗. Linear graph has no maximum point.

B. ✓. The graph opens downwards.

C. ✗. Linear graph has no maximum point.

D. ✗. The graph opens upwards.

10. $\boxed{B}$

x-intercepts are -1 and 3. The equation is in the form $y = a(x+1)(x-3)$.

$$6 = a(0+1)(0-3)$$
$$a = -2$$

$y = -2(x+1)(x-3) = -2x^2 + 4x + 6$.

x-coordinate of vertex $= -\dfrac{4}{2(-2)} = 1$, and y-coordinate of vertex $= -2(1)^2 + 4(1) + 6 = 8$.

Maximum area $= \dfrac{(3+1)(8)}{2} = 16$.

11. $\boxed{A}$

$$\begin{aligned} C &= \frac{1}{4}v^2 + \left(\frac{1}{2}v - 50\right)^2 \\ &= \frac{1}{2}v^2 - 50v + 2500 \\ &= \frac{1}{2}(v-50)^2 + 1250 \end{aligned}$$

Minimum cost = \$1250.

12. $\boxed{D}$

Rewrite the equation as $4x^2 + kx + (k-3) = 0$.

$$\begin{aligned} \Delta = k^2 - 4(4)(k-3) &= 0 \\ k^2 - 16k + 48 &= 0 \\ k &= 4 \text{ or } 12 \end{aligned}$$

When $k = 4$, $x = -\dfrac{1}{2}$; when $k = 12$, $x = -\dfrac{3}{2}$.

13. B

When $f(x)$ attains its maximum, $x = -\frac{4}{2(-2)} = 1$

Maximum value $= -2(1)^2 + 4(1) - 1$

$= 1$

14. D

Area of $\triangle ABC = \frac{(6-x)(3x+2)}{2}$

$= -\frac{3x^2}{2} + 8x + 6$

Required value $= -\frac{8}{2\left(-\frac{3}{2}\right)}$

$= \frac{8}{3}$

15. C

I. ✓.

II. ✓.

III. ✗. Axis of symmetry is $x = 3$.

16. D

$f(x) = x^2 + 10x + 8 = (x+5)^2 - 17$

I. ✗. Should be $x = -5$ instead.

II. ✗. $f(3 + x^2) = (8 + x^2)^2 - 17$.
Since $x^2 + 8 \geq 8$, minimum value is $8^2 - 17 = 47$.

17. C

x-coordinate of vertex $= -\frac{4}{2(-2)} = 1$.

$f(1) = k + 4 - 2 = 7$

$k = 5$

18. D

$A\,(2,\ 0)$ and $B\,(4,\ 0)$.

x-coordinate of $C = \frac{2+4}{2} = 3$

When $x = 3$, $y = -(3-2)(3-4) = 1$.

Required area $= \frac{(4-2)(1)}{2} = 1$ sq. units

19. D

When y attains its maximum, $x = -\dfrac{c}{2(-1)} = \dfrac{c}{2}$.

$$7 = -\left(\frac{c}{2}\right)^2 + c\left(\frac{c}{2}\right) + 3$$

$$4 = \frac{c^2}{4}$$

$$c = \pm 4$$

20. C

When P attains its maximum, $x = -\dfrac{1200}{2(-3)} = 200$.

Required profit $= -3(-200)^2 + 1200(200) - 50\,000$

$= \$70\,000$

21. (a) $3x^2 - 12x + 14 = 3[x^2 - 2(2)(x) + 2^2] + 2$ 1M+1A

$= 3(x-2)^2 + 2$

Thus, $a = -2$ and $b = 2$. 1A

(b) $\dfrac{6}{3x^2 - 12x + 14} = \dfrac{6}{3(x-2)^2 + 2}$

The function has a maximum value $\dfrac{6}{2} = 3$ when $x = 2$. 1M

The claim is disagreed. 1A

22. (a) Number of cups sold $= 400 - 20x$ 1A

Total income $= \$\,[(30 + x)(400 - 20x)]$

$= \$\,[-20x^2 - 200x + 12\,000]$ 1A

(b) $y = \left(-20x^2 - 200x + 12\,000\right) - 20(400 - 20x)$ 1M

$= -20x^2 + 200x + 4000$ 1A

(c) $y = -20x^2 + 200x + 4000$

$= -20[x^2 - 2(5)(x) + 5^2] + 4500$ 1M

$= -20(x-5)^2 + 4500$ 1M

Thus, the maximum profit is \$4500. 1A

23. (a) $A = w(100 - 2w)$ 1M

$= -2w^2 + 100w$ 1A

(b) When $w = 20$, $A = 1200$.

Thus, the required area is $1200\,\text{m}^2$. 1A

(c) When $A = 912$,

$912 = -2w^2 + 100w$ 1M

$0 = -2w^2 + 100w - 912$

$x = 12 \quad \text{or} \quad 38$

Thus, the required width is 12 m or 38 m. 1A

(d) $A = -2w^2 + 100w$

$= -2[w^2 - 2(25)(w) + 25^2] + 1250$ 1M

$= -2(w - 25)^2 + 1250$ 1M

Thus, the maximum area of the piece of land is $1250\,\text{m}^2$. 1A

24. (a) $2x^2 - 9x - 5 = 0$

$x = 5 \quad \text{or} \quad -\frac{1}{2}$

$g(5) = m - 2(5)^2 = 0$ 1M

$m = 50$ 1A

(b) $f(x) = 2x^2 - 9x - 5$

$= 2\left[x^2 - 2\left(\frac{9}{4}\right)x + \left(\frac{9}{4}\right)^2\right] - \frac{121}{8}$ 1M

$= 2\left(x - \frac{9}{4}\right)^2 - \frac{121}{8}$ 1A

(c) The graph of $y = g(x)$ is first reflected about the x-axis 1A

then is translated $\frac{279}{8}$ units upwards and $\frac{9}{4}$ units rightwards. 1A+1A

Alternative solution

The graph of $y = g(x)$ is first translated $\frac{279}{8}$ units downwards and $\frac{9}{4}$ units rightwards. 1A+1A

then is reflected about the x-axis. 1A

25. (a) $f(x) = ax^2 + 8a^2x + 16a^3 + a$

$= a(x^2 + 8ax + 16a^2) + a$ 1M

$= a(x + 4a)^2 + a$

The coordinates of vertex are $(-4a,\ a)$. 1A

(b) (i) The graph of $y = f(x)$ is translated rightwards by $5a$ units 1A

and then enlarged along the y-axis to 4 times the original 1A

to become the graph of $y = g(x)$.

(ii) $(a,\ 4a)$ 1A

(iii) $P(-4a,\ a)$ and $Q(a,\ 4a)$

(Slope of OP) $\times$ (slope of OQ) $= \frac{a-0}{-4a-0} \times \frac{4a-0}{a-0}$ 1M

$= -1$

So, $\angle POQ = 90°$ and the orthocentre is at O. 1M

Thus, the coordinates of orthocentre are $(0,\ 0)$. 1A

 END OF PAPER