Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Compulsory Part

S4 – S5 Core Assignment Set 3

Name: ______________________ Telephone: ____________

Student number: ______________________ Centre: ____________

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

**INSTRUCTIONS**

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings (except for multiple choice questions) must be clearly shown.
3. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
4. The diagrams in this paper are not necessarily drawn in scale.

**Suggested solution**

Distributed in summer course
S4 – S5 Core
Phase 1 – Lesson 3

1. $\boxed{\text{A}}$

$\angle OBD = 90°$, $\angle ACD = 90°$ and $AB = AC = 4\,\text{cm}$.
$\triangle OBD \sim \triangle ACD$ and $AC = \dfrac{AC}{\cos 60°} = 8\,\text{cm}$.

$$\frac{OB}{AC} = \frac{BD}{CD}$$
$$\frac{OB}{4} = \frac{8-4}{\sqrt{8^2-4^2}}$$
$$OB = \frac{4\sqrt{3}}{3}\text{ cm}$$

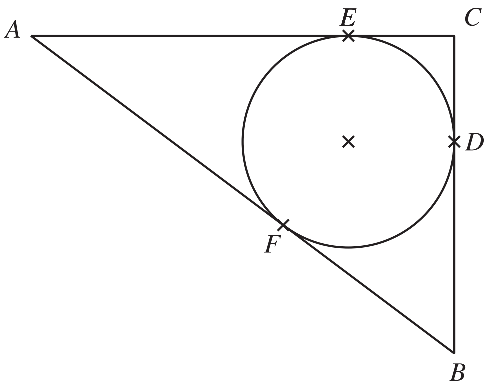
2. $\boxed{\text{D}}$

$AB = AE$ and $\angle ABE = \dfrac{180° - 54°}{2} = 63°$
$\angle CBF = 180° - 63° = 117°$
$\angle BFD = 117° + 27° = 144°$

3. $\boxed{\text{C}}$

Let $D$, $E$, $F$ be the point of contact of the inscribed circle and $BC$, $AC$, $AB$ respectively.

Let $CE = x$. Then $AE = 12 - x$, $CD = x$ and $BD = 5 - x$.
By tangent properties, $AF = 12 - x$ and $BF = 5 - x$.
Since $AB = \sqrt{5^2 + 12^2} = 13$, we have $x = 2$.
Thus, the coordinates of incentre are $(10,\ 3)$.




4. $\boxed{\text{A}}$

$CB = CD$ and $\angle CDB = \angle DBC = 34°$.
$\angle BDF = 180° - 100° = 80°$.
$\angle FDE = 180° - 80° - 34° = 66°$.
$ED = EF$, $\angle EFD = \angle EDF = 66°$ and $\angle DEF = 180° - 66° - 66° = 48°$.

5. $\boxed{\text{C}}$

Let $O$ be the centre of circle $DPQ$. Then $O$, $P$, $C$, $Q$ are concyclic.
$\angle PCQ = 90°$ and $\angle POQ = 180° - 90° = 90°$.
$\angle PDQ = \dfrac{\angle POQ}{2} = 45°$.

6. $\boxed{\text{D}}$

Let $\angle CAP = x$.

$\angle QCP = \angle CAP + \angle CPA = 27° + x$

$PC = PQ$ and $\angle PQC = \angle QCP = 27° + x$

$AP = AQ$ and $\angle APQ = \angle PQC = 27° + x$

Consider $\triangle APQ$.

$$\begin{aligned} 2(27° + x) + x &= 180° \\ x &= 42° \end{aligned}$$

7. $\boxed{\text{B}}$

$\angle OBC = 90°$ and $\angle AOB = 90° - 28° = 62°$.

$\angle DOE = \angle AOB = 62°$ and $\angle OED = \dfrac{180° - 62°}{2} = 59°$.

8. $\boxed{\text{B}}$

$\angle ACB = 90°$ and $\angle ABD = 90°$

$\angle BAC = \angle DAB$

$\triangle ABC \sim \triangle ADB$

Let the radius of the circle be $r$.

$$\begin{aligned} \frac{AC}{AB} &= \frac{AB}{AD} \\ \frac{6}{2r} &= \frac{2r}{6+2} \\ r &= 2\sqrt{3} \end{aligned}$$

Required radius is $2\sqrt{3}$ cm.

9. $\boxed{\text{D}}$

Let $\angle ATB = a$. Then $\angle BTC = 50° - a$.

$TA = TB$ and $\angle ABT = \dfrac{180° - a}{2}$.

$TB = TC$ and $\angle TBC = \dfrac{180° - (50° - a)}{2}$.

$\angle ABC = \dfrac{180° - a}{2} + \dfrac{180° - (50° - a)}{2} = 155°$.

10. $\boxed{\text{A}}$

Let $\angle RAO = a$ and $\angle RBO = b$.

Then $\angle PBO = \angle RBO = b$ and $\angle RAO = \angle QAO = a$.

$$\begin{aligned} (2a) + (2b) + 52° &= 180° \\ a + b &= 64° \end{aligned}$$

$\angle AOB = 180° - (a + b) = 116°$.

11. $\boxed{\text{C}}$

$\angle ACB = \angle QAB = 64°$
$\angle AOB = 2\angle ACB = 128°$
$OA = OB$ and $\angle OBA = \dfrac{180° - 128°}{2} = 26°$
$\angle BOC = \angle OBA = 26°$
Reflex $\angle AOC = 360° - 26° - 128° = 206°$
$\angle ABC = \dfrac{206°}{2} = 103°$

12. $\boxed{\text{D}}$

Let $O$ be the centre of circle.
$\angle OPB = \angle OQB = \angle OSL = \angle ORL = 90°$
$\angle HAC = \angle HCA = \dfrac{108°}{2} = 54°$ and $\angle BCH = \angle HCA = 54°$
In $OQCS$, $\angle QOS = 360° - 54° - 90° \times 2 = 126°$ and $\angle ROS = 126° \times \dfrac{4}{4+5} = 56°$
In $OPHS$, $\angle POS = 360° - 108° - 90° \times 2 = 72°$
In $OPAR$, $\angle BAK = 360° - 56° - 72° - 90° \times 2 = 52°$

13. $\boxed{\text{C}}$

$PA = PB$ and $\angle PAB = \dfrac{180° - 46°}{2} = 67°$
$\angle ACB = \angle PAB = 67°$
$\angle ABC = 180° - 67° - 28° = 85°$
$\angle ADC = 180° - 85° = 95°$

14. $\boxed{\text{B}}$

$\angle ABC = \dfrac{82°}{2} = 41°$
Consider $\triangle ABP$.

$$\begin{aligned} 38° + 41° + (90° + \angle BAO) &= 180° \\ \angle BAO &= 11° \end{aligned}$$

15. $\boxed{\text{C}}$

Let $O$ be the centre. Then $O$, $A$, $C$, $T$ are concyclic.
$\angle AOC = 180° - y$ and
$$x = \frac{180 - y}{2}$$
$2x + y = 180°$

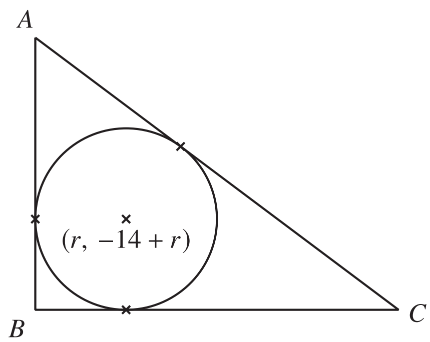
16. $\boxed{\text{A}}$

$\angle YOZ = 360° - 126° - 134° = 100°$
$\angle OZP = \angle OYP = 90°$
$\angle QPR = 360° - 90° - 90° - 100° = 80°$

17. $\boxed{\text{A}}$

$\angle ABC = 90°$.

Let the radius be $r$ cm.
The coordinates of centre will be $(r,\ -14+r)$.
Only option A has the format of $(r,\ -14+r)$.




18. $\boxed{\text{D}}$

$\angle TAC = \angle ACE = 57°$
$TA = TC$ and $\angle TCA = \angle TAC = 57°$
$\angle ECD = 180° - 57° - 57° = 66°$

19. $\boxed{\text{B}}$

Let $\angle ACB = 2\theta$. Then $\angle BAC = 3\theta$.
$\angle ABC = 90°$.
$$2\theta + 3\theta + 90° = 180°$$
$$\theta = 18°$$
$\angle ACP = 90°$ and $\angle BQC = 90° - 3\theta = 36°$.

20. $\boxed{\text{C}}$

Let the radius be $r$ cm.

$$15^2 + r^2 = (r+9)^2$$
$$225 + r^2 = r^2 + 18r + 81$$
$$r = 8$$

Required radius is 8 cm.

21. (a) $\angle OAQ = 90°$ 1M

$\angle OAD = 90° - 36° = 54°$

Since $OD = OA$, $\angle ADO = \angle OAD = 54°$ 1A

(b) In $\triangle ODA$, $\angle AOB = 54° + 54° = 108°$ 1M

$\angle OAP = \angle OBP = 90°$ 1M

In quadrilateral $OAPB$,

$$\angle APB + 90° + 90° + 108° = 360° \quad \text{1M}$$

$$\angle APB = 72° \quad \text{1A}$$

22. $\angle FBP = 90°$ 1M

In $\triangle ABF$, $\angle BAF = \angle FBP - \angle AFB = 90° - 26° = 64°$

$\angle CAO = \angle BAO = \frac{1}{2}\angle BAF = 32°$ 1M

In $\triangle OF$, $\angle AOB = \angle CAO + \angle F = 32° + 26° = 58°$

$\angle AOC = \angle AOB = 58°$ 1M

$\angle OGE = \frac{1}{2}\angle AOC = 29°$

$\angle GOE = \angle AOB = 58°$

In $\triangle OGE$, $\angle CEO = \angle OGE + \angle GOE = 29° + 58° = 87°$ 1M+1A

23. (a) $\angle OAC = \angle OBC = 90°$ 1M

In quadrilateral $AOBC$,

$$\angle AOB + 90° + 46° + 90° = 360° \quad \text{1M}$$

$$\angle AOB = 134°$$

$\angle AEB = \frac{1}{2}\angle AOB = 67°$ 1M

$\angle BED = 67° - a$ 1A

(b) $\angle DAE = 90°$ 1M

In $\triangle AEF$,

$$\angle F + 67° + 90° = 180° \quad \text{1M}$$

$$\angle F = 23° \quad \text{1A}$$

24. (a) Since $OC = OB$, $\angle OCB = \angle OBC = 62°$.

$\angle BOC = 180° - 62° - 62° = 56°$ 1M

$\angle BAC = \frac{1}{2}\angle BOC = 28°$ 1M+1A

(b) In $\triangle AQC$,

$$\angle ACQ + 90° + 28° = 180° \quad \text{1M}$$

$$\angle ACQ = 62°$$

Since $PA = PC$, $\angle CAP = \angle ACQ = 62°$ 1M

In $\triangle APQ$, $\angle APQ = 180° - 62° - 62° = 56°$ 1M+1A

25. (a) Let $\angle ABC = a$.

$\angle BAF = 90°$ 1M

In $\triangle ABF$, $\angle AFB = 180° - 90° - a = 90° - a$

Since $CA = CF$, $\angle CAF = \angle CFA = 90° - a$

$\angle BAC = \angle BAF - \angle CAF = 90° - (90° - a) = a$

$BC = BA$ 1M

$\angle BCA = \angle BAC = a$

In $\triangle ABC$,

$$a + a + a = 180° \quad \text{1M}$$

$$a = 60°$$

So, $\angle ABC = 60°$ 1A

(b) $BA = BD$ and $BC = BA$ 1M

So, we have $BA = BD = BC$, $\angle DAB = \angle ADB$ and $\angle BDC = \angle BCD = 45°$

In $\triangle BCD$, $\angle CBD = 180° - 45° - 45° = 90°$ 1M

$\angle ABD = \angle CBD - \angle ABC = 90° - 60° = 30°$

In $\triangle BDA$,

$$\angle ADB + \angle DAB + 30° = 180° \quad \text{1M}$$

$$2\angle ADB + 30° = 180°$$

$$\angle ADB = 75° \quad \text{1A}$$

26. (a) $\angle POA = \angle POQ$ and $\angle ROB = \angle ROQ$ 1M

$64° = \angle POQ + \angle ROQ = \angle POA + \angle ROB$ 1M

$\angle OAP = \angle OBR = 90°$

In quadrilateral $AOBT$,

$$90° + (64° + 64°) + 90° + \angle ATB = 360° \quad \text{1M}$$

$$\angle ATB = 52° \quad \text{1A}$$

(b) $PA = PQ$ and $RB = RQ$ 1M

$16\text{ cm} = TP + PA = TP + PQ$

$TB = TA = 16\text{ cm}$ 1M

$16\text{ cm} = TR + RB = TR + RQ$

Perimeter of $\triangle TPR = 16 + 16 = 32\text{ cm}$ 1A

27. (a) $\angle OCD = 90°$ *(tangent $\perp$ radius)* 1M

$\angle OAB = \angle OCD$ *(ext. $\angle$, cyclic quad.)* 1M

$= 90°$

Thus, $BA$ is the tangent to the smaller circle at $A$. *(converse of tangent $\perp$ radius)* 1A

(b) Since $\angle OAB = 90°$, $OB$ is a diameter of the larger circle. 1M

$OB = 2 \times 13 = 26\text{ cm}$.

In $\triangle AOB$, $AB = \sqrt{26^2 - 10^2} = 24\text{ cm}$ 1M+1A



**END OF PAPER**