5. Suggested solution to exercises 習題建議題解

1. A

Coordinates of centre are $(-2, 0)$.

$$\text{Radius} = \sqrt{2^2 + 0^2 + 3}$$
$$= \sqrt{7}$$

2. C

$$\text{Radius} = \frac{8\pi}{2\pi} = 4$$
$$4 = \sqrt{\left(\frac{D}{2}\right)^2 + 3^2 + 3}$$
$$16 = \frac{D^2}{4} + 12$$
$$D = \pm 4$$

3. D

I. Radius $= \sqrt{3^2 + 4^2 - 10} = \sqrt{15}$

II. Radius $= \sqrt{4^2 + 3^2 - 10} = \sqrt{15}$

III. Radius $= \sqrt{5^2 - 10} = \sqrt{15}$

All circles have the same area.

The answer is D.

4. D

A. ✗. Coefficients of x^2 and y^2 must equal.

B. ✗. $\left(\frac{2}{2}\right)^2 + 0^2 - 3 = -2 < 0$. It is an imaginary circle.

C. ✗. $\left(\frac{2}{2}\right)^2 + \left(\frac{4}{2}\right)^2 - 5 = 0$. It is a point circle.

D. ✓. $x^2 + y^2 + \frac{x}{2} - \frac{y}{2} = 0$

$\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2 - 0 = \frac{1}{8} > 0$. It is a real circle.

5. C

Coordinates of centre are $(4, -2)$.

$$4^2 + 2^2 + 4k > 0$$
$$k > -5$$

6. B

I. ✗. $0 + (-14)^2 + 0 + 16(-14) + 28 = 0$
It lies on the circle.

II. ✗. $(-4)^2 + 2^2 + 4(-4) + 16(2) + 28 = 64 > 0$
It lies outside the circle.

III. ✓. $(-3)^2 + (-4)^2 + 4(-3) + 16(-4) + 28 = -23 < 0$
It lies inside the circle.

IV. ✗. $(-4)^2 + (-2)^2 + 4(-4) + 16(-2) + 28 = 0$
It lies on the circle.

7. C

Centre $(1, -2)$; radius $= \sqrt{1^2 + 2^2 - 4} = 1$
Distance from A to centre $= \sqrt{(5-1)^2 + (-5+2)^2} = 5$
Required distance $= 5 - 1$
$= 4$

8. B

Centre $(3, -2)$
Slope of line joining centre and M is $\dfrac{-2-2}{3-1} = -2$
Slope of chord $= \dfrac{-1}{-2} = \dfrac{1}{2}$
Required equation is

$$y - 2 = \frac{1}{2}(x - 1)$$
$$x - 2y + 3 = 0$$

9. (a) Substitute $(-2,\ 9)$ and $(-5,\ 8)$,

1M $$\begin{cases} 4 + 81 - 2d + 9e + 24 = 0 \\ 25 + 64 - 5d + 8e + 24 = 0 \end{cases}$$

1A Solving, we have $d = 5$ and $e = -11$.

(b) Let C be the centre of circle S and the coordinates of R be $(h,\ k)$.

1M Coordinates $C = \left(-\frac{5}{2},\ \frac{11}{2}\right)$

As C is the midpoint of PR,

1M $\frac{-2+h}{2} = -\frac{5}{2}$ and $\frac{9+k}{2} = \frac{11}{2}$

$h = -3$ $\quad$ $k = 2$

1A The coordinates of R are $(-3,\ 2)$.

1M 10. (a) Coordinates of centres of C_1 and C_2 are $(5,\ -3)$ and $(2,\ -4)$ respectively.

1A Required distance $= \sqrt{(5-2)^2 + (-3+4)^2} = \sqrt{10}$

(b) The radius of C_1 and C_2 are 2 and 3 respectively.

1M Since $(3-2) <$ distance between two centres $< (3+2)$,

1A the two circles intersect at two points.

1M 11. (a) Coordinates of centres of C_1 and C_2 are $(2,\ -1)$ and $(-2,\ -3)$ respectively.

1A Required distance $= \sqrt{(2+2)^2 + (-1+3)^2} = 2\sqrt{5}$

1M (b) $\sqrt{2^2 + (-1)^2} + \sqrt{(-2)^2 + (-3)^2 - k} = 2\sqrt{5}$

$\sqrt{13 - k} = \sqrt{5}$

1A $k = 8$

12. (a) Coordinates of $C = \left(\frac{k}{2},\ -\frac{k+2}{2}\right)$

1M $$3\left(\frac{k}{2}\right) + \frac{k+2}{2} - 9 = 0$$

1A $k = 4$

(b) Coordinates of $C = (2,\ -3)$ and $PC = \sqrt{(2+2)^2 + (-3+4)^2} = \sqrt{17}$

1M Radius of circle $= \sqrt{2^2 + (-3)^2 + 10} = \sqrt{23} > \sqrt{17}$.

1A Thus, P lies inside the circle.

13. A

Required equation is

$$(x+2)^2+(y-5)^2=(2+2)^2+(1-5)^2$$
$$x^2+y^2+4x-10y-3=0$$

14. C

Centre = midpoint of $AC=(7,\ 5)$. The equation is in the form $x^2+y^2-14x-10y+F=0$, where F is a constant.

$$8^2+8^2-14(8)-10(8)+F=0$$
$$F=64$$

15. A

y-coordinate of centre $=\dfrac{(-1)+(-9)}{2}=-5$

Radius of circle $=0-(-5)=5$

Let the coordinates of centre be $(h,\ -5)$.

$$\sqrt{(h-0)^2+(-5+1)^2}=5$$
$$h=-3 \quad \text{or} \quad \text{(rejected)}$$

16. C

Centre is the midpoint of AB.

Coordinates of centre are $\left(-\frac{3}{2},\ 2\right)$.

Required equation is

$$\left(x+\frac{3}{2}\right)^2+(y-2)^2=\left(0+\frac{3}{2}\right)^2+(1-2)^2$$
$$x^2+y^2+3x-4y+3=0$$

17. C

Centre is the midpoint of $(4,\ 0)$ and $(0,\ -6)$.

Coordinates of centre are $(2,\ -3)$.

Required equation is

$$(x-2)^2+(y+3)^2=(0-2)^2+(0+3)^2$$
$$x^2+y^2-4x+6y=0$$

18. B

Let the coordinates of centre be $(h,\ 0)$.

$$\sqrt{(h-0)^2+(5-0)^2}=\sqrt{(h-6)^2+(1-0)^2}$$
$$h^2+25=h^2-12h+37$$
$$h=1$$

Required equation is

$$(x-1)^2+y^2=(0-1)^2+5^2$$
$$x^2+y^2-2x-25=0$$

19. (a) Let the equation of circle be $x^2+y^2+Dx+Ey+F=0$, where D, E and F are constants.

1M
$$\begin{cases} 1-D+F=0 & (5.1) \\ 4+1+2D+E+F=0 & (5.2) \\ 1+16+D+4E+F=0 & (5.3) \end{cases}$$

Consider (5.3) − (5.2) and (5.3) − (5.1), we have

1M
$$\begin{cases} 12-D+3E=0 \\ 16+2D+4E=0 \end{cases}$$

1A Solving, we have $D=0$ and $E=-4$.

Put $D=0$ into (5.1), we have $F=-1$.

1A The equation of the circle is $x^2+y^2-4y-1=0$.

(b) Substitute $(-2,\ 3)$,

1M
$$\text{LHS}=(-2)^2+3^2-4(3)-1$$
$$=0=\text{RHS}$$

1A $D(-2,\ 3)$ lies on the circle. Thus, A, B, C and D are concyclic.

1M 20. (a) The slope of $AB=\dfrac{1-0}{2-1}=1$

The slope of $BD=\dfrac{6-1}{-3-2}=-1$

1 Since the product of the slope of AB and the slope of BD is -1, $AB\perp BD$.

1A (b) The coordinates of the centre of the circle are $\left(\dfrac{1-3}{2},\ \dfrac{0+6}{2}\right)=(-1,\ 3)$.

The equation of the circle is

1M $(x+1)^2+(y-3)^2=(1+1)^2+(0-3)^2$

1A $(x+1)^2+(y-3)^2=13$

21. (a) Slope of AC = slope of $L = -\frac{1}{2}$.

Let $(h,\ k)$ be the coordinates of C.

$$\frac{k-8}{h+5} = -\frac{1}{2}$$ 1M

$$h + 2k = 11$$

Since AC is a diameter, $AB \perp BC$

$$\frac{10-8}{1+5} \times \frac{k-10}{h-1} = -1$$ 1M

$$3h + k = 13$$

Solving, we have $h = 3$ and $k = 4$ 1M

The coordinates of C are $(3,\ 4)$. 1A

(b) Centre of circle $= \left(\frac{-5+3}{2},\ \frac{8+4}{2}\right) = (-1,\ 6)$. 1M

The equation of circle is

$$(x+1)^2 + (y-6)^2 = (1+1)^2 + (10-6)^2$$ 1M

$$(x+1)^2 + (y-6)^2 = 20$$ 1A

22. (a) Let the coordinates of centre be $(h,\ 0)$.

$$\sqrt{(h+1)^2 + (3-0)^2} = \sqrt{(h-1)^2 + (0-1)^2}$$ 1M

$$h^2 + 2h + 10 = h^2 - 2h + 2$$

$$h = -2$$ 1A

The coordinates of C are $(-2,\ 0)$.

The equation of the circle is

$$(x+2)^2 + y^2 = (-1+2)^2 + 3^2$$ 1M+1A

$$x^2 + y^2 + 4x - 6 = 0$$ 1A

(b) $QC = \sqrt{(-2+3)^2 + (0-4)^2} = \sqrt{17} > \sqrt{10}$ = radius of circle. 1M

Thus, Q lies outside the circle. 1A

23. Let the coordinates of C be $(c,\ 0)$.

1M $$\sqrt{(6-c)^2+(3-0)^2}=\sqrt{(7-c)^2+(4-0)^2}$$

$$c^2-12c+45=c^2-14c+65$$

1A $$c=10$$

The equation of circle is

1M $$(x-10)^2+y^2=(6-10)^2+3^2$$

1A $$(x-10)^2+y^2=25$$

24. Let the coordinates of centre be $(h,\ k)$.

1M $$\sqrt{(h-6)^2+(k+2)^2}=\sqrt{(h-2)^2+(k-2)^2}$$

$$h^2+k^2-12h+4k+40=h^2+k^2-4h-4k+8$$

$$8h-8k=32$$

1M Since $(h,\ k)$ lies on $x+2y-1=0$, we have $h+2k-1=0$.

1A Solving, we have $h=3$ and $k=-1$.

The equation of the circle is

$$(x-3)^2+(y+1)^2=(2-3)^2+(2+1)^2$$

1A $$x^2+y^2-6x+2y=0$$

25. (a) Let $PO=x$. Then $OQ=7-x$.

1M $$\frac{1}{2}(6)(7-x)=4\times\frac{1}{2}(2)(x)$$

1A $$x=3$$

1M+1A The coordinates of centre are $\left(\frac{-3+4}{2},\ \frac{-2+6}{2}\right)=\left(\frac{1}{2},\ 2\right)$

(b) The equation of circle is

1M $$\left(x-\frac{1}{2}\right)^2+(y-2)^2=\left(0-\frac{1}{2}\right)^2+(6-2)^2$$

1A $$\left(x-\frac{1}{2}\right)^2+(y-2)^2=\frac{65}{4}$$

1M 26. $(2y)^2+y^2+2(2y)-4y-20=0$

1A $$5y^2-20=0$$

$$y=\pm 2$$

When $y=2$, $x=4$; when $y=-2$, $x=-4$.

1A Required coordinates are $(-4,\ -2)$ and $(4,\ 2)$.

27. $$x^2 + \left(\frac{3x}{4} - \frac{13}{4}\right)^2 + 2x + 8\left(\frac{3x}{4} - \frac{13}{4}\right) - 83 = 0$$ 1M

$$\left(1 + \frac{9}{16}\right)x^2 + \left(-\frac{39}{8} + 2 + 6\right)x + \left(\frac{169}{16} - 26 - 83\right) = 0$$

$$\frac{25}{16}x^2 + \frac{25}{8}x - \frac{1575}{16} = 0$$ 1A

$$x = -9 \quad \text{or} \quad 7$$

When $x = -9$, $y = \frac{3(-9) - 13}{4} = -10$; when $x = 7$, $y = \frac{3(7) - 13}{4} = 2$.

Required coordinates are $(-9,\ -10)$ and $(7,\ 2)$. 1A

28. $$x^2 + \left(4 - \frac{3x}{4}\right)^2 - 2x + 6\left(4 - \frac{3x}{4}\right) - 15 = 0$$ 1M

$$\left(1 + \frac{9}{16}\right)x^2 + \left(-6 - 2 - \frac{9}{2}\right)x + (16 + 24 - 15) = 0$$

$$\frac{25}{16}x^2 - \frac{25}{2}x + 25 = 0$$ 1A

$$x = 4$$

When $x = 4$, $y = 4 - \frac{3(4)}{4} = 1$.

Required coordinates are $(4,\ 1)$. 1A

29. (a) $$x^2 + (-2x + 3)^2 + x - 3(-2x + 3) - 10 = 0$$ 1M

$$(1 + 4)x^2 + (-12 + 1 + 6)x + (9 - 9 - 10) = 0$$

$$5x^2 - 5x - 10 = 0$$ 1M

$$x = -1 \quad \text{or} \quad 2$$

When $x = -1$, $y = -2(-1) + 3 = 5$; when $x = 2$, $y = -1$.

The coordinates of the intersections are $(-1,\ 5)$ and $(2,\ -1)$. 1A+1A

(b) Coordinates of midpoint of intersections are $\left(\frac{-1 + 2}{2},\ \frac{5 - 1}{2}\right) = \left(\frac{1}{2},\ 2\right)$ 1M

Coordinates of centre $= \left(-\frac{1}{2},\ \frac{3}{2}\right)$.

Required distance $= \sqrt{\left(\frac{1}{2} + \frac{1}{2}\right)^2 + \left(2 - \frac{3}{2}\right)^2}$ 1M

$= \frac{\sqrt{5}}{2}$ 1A

1M 30. $(m-2y)^2+y^2+4(m-2y)-8y+15=0$

$(4+1)y^2+(-4m-8-8)y+m^2+4m+15=0$

1A $5y^2-(4m+16)y+m^2+4m+15=0$

1M $\Delta=(-4m-16)^2-4(5)(m^2+4m+15)=0$

$-4m^2+48m-44=0$

1A $m=1\quad\text{or}\quad 11$

1M 31. (a) $x^2+(-x-c)^2-4x+8(-x-c)-52=0$

$(1+1)x^2+(2c-4-8)x+(c^2-8x-52)=0$

1A $2x^2+(2c-12)x+c^2-8c-52=0$

1M $\Delta=(2c-12)^2-4(2)(c^2-8x-52)=0$

$-4c^2+16c+560=0$

1A $c=-10\quad\text{or}\quad 14$

(b) We have $c=14$.

1M $2x^2+[2(14)-12]x+(14)^2-8(14)-52=0$

$2x^2+16x+32=0$

$x=-4$

When $x=-4$, $y=4-14=-10$.

1A Required coordinates are $(-4,\ -10)$.

32. (a) $(3y-k)^2+y^2-10(3y-k)+2y+16=0$ 1M

$(9+1)y^2+(-6k-30+2)y+(k^2+10k+16)=0$

$10y^2+(-6k-28)y+(k^2+10k+16)=0$

Since L is a tangent to C,

$(-6k-28)^2-4(10)(k^2+10k+16)=0$ 1M

$(36-40)k^2+(336-400)k+(784-640)=0$

$-4k^2-64k+144=0$ 1M

$k=-18 \quad \text{or} \quad 2$ 1A

(b) When $k=-18$, we have

$10y^2+80y+160=0$ 1M

$y=-4$

When $y=-4$, $x=3(-4)+18=6$. The coordinates of the intersection are $(6,\ -4)$. 1A

When $k=2$, we have

$10y^2-40y+40=0$ 1M

$y=2$

When $y=2$, $x=3(2)-2=4$. The coordinates of the intersection are $(4,\ 2)$. 1A

33. $x^2+(3x+8)^2-5x-4(3x+8)+k=0$ 1M

$(1+9)x^2+(48-5-12)x+(64-32+k)=0$

$10x^2+31x+32+k=0$ 1A

$\Delta=31^2-4(10)(32+k)<0$ 1M

$-319-40k<0$

$k>-\dfrac{319}{40}$

Required value is -7. 1A

1M 34. (a) Let the equation of S be $x^2 + y^2 + Dx + Ey + F = 0$, where D, E and F are constants.

1M
$$\begin{cases} 1^2 + 2^2 + D + 2E + F = 0 \\ 5^2 + 6^2 + 5D + 6E + F = 0 \\ 5^2 + 8^2 + 5D - 8E + F = 0 \end{cases}$$

Solve the last two equations, we have

$$(81 + 5D - 8E + F) - (61 + 5D + 6E + F) = 0$$

1A $$E = 2$$

We have
$$\begin{cases} 5 + D + 2(2) + F = 0 \\ 61 + 5D + 6(2) + F = 0 \end{cases}$$

Solving, we have $D = -16$ and $F = 7$.

1A Required equation is $x^2 + y^2 - 16x + 2y + 7 = 0$.

1M (b) (i) $x^2 + (mx)^2 - 16x + 2mx + 7 = 0$

$(1 + m^2)x^2 + (2m - 16)x + 7 = 0$

1 Since x_1 and x_2 are roots of the equation, we have $x_1x_2 = \dfrac{7}{1 + m^2}$.

1M (ii) $OP = \sqrt{x_1^2 + y_1^2}$

$= \sqrt{x_1^2 + (mx_1)^2}$

1A $= x_1\sqrt{1 + m^2}$

$OQ = \sqrt{x_2^2 + y_2^2}$

$= \sqrt{x^2 + m^2x_2^2}$

$= x_2\sqrt{1 + m^2}$

$OP \times OQ = (x_1\sqrt{1 + m^2})(x_2\sqrt{1 + m^2})$

$= (1 + m^2)x_1x_2$

1M $= (1 + m^2) \times \dfrac{7}{1 + m^2}$

1A $= 7$

35. (a) Coordinates of centre are $(4,\ 1)$. 1A

Radius $= \sqrt{4^2 + 1^2 - 8}$

$= 3$ 1A

(b) $AB = \sqrt{(7-4)^2 + (-3-1)^2} = 5$ 1M

Radius of $C_2 = 5 - 3 = 2$ 1M

Required equation is $(x-7)^2 + (y+3)^2 = 4$. 1A

(c) Note that A, M, B are collinear and $AM : MB = 3 : 2$.

Let the coordinates of M be $(a,\ b)$.

$$\frac{7-a}{a-4} = \frac{2}{3} \quad \text{and} \quad \frac{1-b}{b+3} = \frac{3}{2}$$ 1M

$$21 - 3a = 2a - 8 \qquad 2 - 2b = 3b + 9$$

$$a = \frac{29}{5} \qquad b = -\frac{7}{5}$$

Required coordinates are $\left(\frac{29}{5},\ -\frac{7}{5}\right)$. 1A

(d) Slope of $AB = \frac{-3-1}{7-4} = -\frac{4}{3}$

Slope of $L = \frac{3}{4}$ 1M

Required equation is

$$y + \frac{7}{5} = \frac{3}{4}\left(x - \frac{29}{5}\right)$$ 1M

$$3x - 4y - 23 = 0$$ 1A