Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Extended Part

S5 – S6 M2 Integration Assignment Set 1

Name: _____ Telephone: _____

Student number: _____ Centre: _____

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S5 – S6 M2 Integration
Phase 1 – Lesson 1

1. $\displaystyle\int \frac{3+7\sqrt{t}-\sqrt[3]{t}}{\sqrt[3]{t}}\,\mathrm{d}t = \int (3t^{-\frac{1}{3}} + 7t^{\frac{1}{6}} - 1)\,\mathrm{d}t$ 1M

$\displaystyle = \frac{9}{2}t^{\frac{2}{3}} + 6t^{\frac{7}{6}} - t + \text{constant}$ 1A

2. (a) $\displaystyle\frac{1}{x} - \frac{1}{x+5} = \frac{(x+5)-x}{x(x+5)} = \frac{5}{x^2+5x}$ 1A

(b) $\displaystyle\int \frac{\mathrm{d}x}{x^2+5x} = \frac{1}{5}\int \left(\frac{1}{x} - \frac{1}{x+5}\right)\mathrm{d}x$ 1M

$\displaystyle = \frac{\ln|x|}{5} - \frac{\ln|x+5|}{5} + \text{constant}$ 1A

$\displaystyle = \frac{1}{5}\ln\left|\frac{x}{x+5}\right| + \text{constant}$

3. $\displaystyle\int \cos^4 2x\,\mathrm{d}x = \frac{1}{4}\int (1+\cos 4x)^2\,\mathrm{d}x$ 1M

$\displaystyle = \frac{1}{4}\int (1 + 2\cos 4x + \cos^2 4x)\,\mathrm{d}x$ 1M

$\displaystyle = \frac{1}{4}\int \left[1 + 2\cos 4x + \frac{1}{2}(1+\cos 8x)\right]\mathrm{d}x$ 1M

$\displaystyle = \frac{3x}{8} + \frac{\sin 4x}{8} + \frac{\sin 8x}{64} + \text{constant}$ 1A

4. $\displaystyle\int \frac{4x+9}{8x^2+38x+45}\,\mathrm{d}x = \int \frac{1}{2x+5}\,\mathrm{d}x$ 1M

$\displaystyle = \frac{\ln|2x+5|}{2} + \text{constant}$ 1A

5. $\displaystyle\int (\cos 2x - \sin 6x)^2\,\mathrm{d}x = \int (\cos^2 2x - 2\cos 2x \sin 6x + \sin^2 6x)\,\mathrm{d}x$ 1M

$\displaystyle = \int \left[\frac{1}{2}(\cos 4x + 1) - (\sin 8x + \sin 4x) + \frac{1}{2}(1 - \cos 12x)\right]\mathrm{d}x$ 1M

$\displaystyle = \frac{\sin 4x}{8} + x + \frac{\cos 8x}{8} + \frac{\cos 4x}{4} - \frac{\sin 12x}{24} + \text{constant}$ 1A

6. $\displaystyle\int \sin 8x \sin 5x \sin x\,\mathrm{d}x = \frac{1}{2}\int \sin x(\cos 3x - \cos 13x)\,\mathrm{d}x$ 1M

$\displaystyle = \frac{1}{4}\int [(\sin 4x - \sin 2x) - (\sin 14x - \sin 12x)]\,\mathrm{d}x$ 1M

$\displaystyle = -\frac{\cos 4x}{16} + \frac{\cos 2x}{8} + \frac{\cos 14x}{56} - \frac{\cos 12x}{48} + \text{constant}$ 1A

7. (a) $\displaystyle P + \frac{Q}{x+4} + \frac{R}{2x-5} = \frac{P(x+4)(2x-5) + Q(2x-5) + R(x+4)}{(x+4)(2x-5)}$

$\displaystyle = \frac{2Px^2 + (3P+2Q+R)x + (-20P-5Q+4R)}{(x+4)(2x-5)}$

So, $2P = 4$, $3P + 2Q + R = 11$ and $-20P - 5Q + 4R = -59$. 1M

Solving, we have $P = 2$, $Q = 3$ and $R = -1$. 1A+1A

(b) $\displaystyle\int \frac{4x^2+11x-59}{2x^2+3x-20}\,\mathrm{d}x = \int \left(2 + \frac{3}{x+4} - \frac{1}{2x-5}\right)\mathrm{d}x$ 1M

$\displaystyle = 2x + 3\ln|x+4| - \frac{\ln|2x-5|}{2} + \text{constant}$ 1A

8. $\displaystyle\int \cos 3x \sin x \sin 4x \, dx = \frac{1}{2}\int \sin x(\sin 7x + \sin x)\, dx$ 1M

$\displaystyle = \frac{1}{4}\int [(\cos 6x - \cos 8x) + (1 - \cos 2x)]\, dx$ 1M

$\displaystyle = \frac{x}{4} - \frac{\sin 8x}{32} + \frac{\sin 6x}{24} - \frac{\sin 2x}{8} + \text{constant}$ 1A

9. $\displaystyle\int \frac{2u^8 + 4u^2 - 10}{u^3}\, du = \int (2u^5 + 4u^{-1} - 10u^{-3})\, du$ 1M

$\displaystyle = \frac{u^6}{3} + 4\ln|u| + 5u^{-2} + \text{constant}$ 1A

10. (a) $\displaystyle\frac{dy}{dx} = e^x + xe^x$ 1A

(b) By (a), $\displaystyle xe^x = \frac{dy}{dx} - e^x$

$\displaystyle\int xe^x\, dx = \int \left(\frac{dy}{dx} - e^x\right) dx$ 1M

$\displaystyle = y - e^x + \text{constant}$ 1A

$\displaystyle = xe^x - e^x + \text{constant}$

11. $\displaystyle\int \frac{x+2}{\sqrt[3]{x} + \sqrt[3]{2}}\, dx = \int (x^{\frac{2}{3}} - 2^{\frac{1}{3}}x^{\frac{1}{3}} + 2^{\frac{2}{3}})\, dx$ 1M

$\displaystyle = \frac{3x^{\frac{5}{3}}}{5} - \frac{3\sqrt[3]{2}x^{\frac{4}{3}}}{4} + 2^{\frac{2}{3}}x + \text{constant}$ 1A

12. $\displaystyle\int \frac{x+8}{x+6}\, dx = \int \left(1 + \frac{2}{x+6}\right) dx$ 1M

$\displaystyle = x + 2\ln|x+6| + \text{constant}$ 1A

13. $\displaystyle\int \sin^2 x \cos^2 x\, dx = \int \left(\frac{\sin 2x}{2}\right)^2 dx$ 1M

$\displaystyle = \frac{1}{4}\int \frac{1}{2}(1 - \cos 4x)\, dx$ 1M

$\displaystyle = \frac{x}{8} - \frac{\sin 4x}{32} + \text{constant}$ 1A

14. $\displaystyle\int \sin\frac{7\theta}{2} \sin\frac{\theta}{2}\, d\theta = \frac{1}{2}\int (\cos 3\theta - \cos 4\theta)\, d\theta$ 1M

$\displaystyle = \frac{\sin 3\theta}{6} - \frac{\sin 4\theta}{8} + \text{constant}$ 1A

15. $\displaystyle\int (6^{x+1} - 5x^6)\, dx = \int (e^{(x+1)\ln 6} - 5x^6)\, dx$ 1M

$\displaystyle = \frac{e^{(x+1)\ln 6}}{\ln 6} - \frac{5x^7}{7} + \text{constant}$

$\displaystyle = \frac{6^{x+1}}{\ln 6} - \frac{5x^7}{7} + \text{constant}$ 1A

16. $\displaystyle\int \frac{\sin^2 t + 2\cos^3 t}{\cos^2 t}\,\mathrm{d}t = \int (\tan^2 t + 2\cos t)\,\mathrm{d}t$ 1M

$\displaystyle= \int (\sec^2 t - 1 + 2\cos t)\,\mathrm{d}t$ 1M

$= \tan t - t + 2\sin t + \text{constant}$ 1A

17. (a) $\displaystyle\frac{Px+Q}{x^2-2} + \frac{R}{x+3} = \frac{(Px+Q)(x+3) + R(x^2-2)}{(x^2-2)(x+3)}$

$\displaystyle= \frac{(P+R)x^2 + (3P+Q)x + (3Q-2R)}{(x^2-2)(x+3)}$

So, $P+R=6$, $3P+Q=3$ and $3Q-2R=-10$. 1M

Solving, we have $P=1$, $Q=0$ and $R=5$. 1A+1A

(b) $\displaystyle\int \frac{6x^2+3x-10}{x^3+3x^2-2x-6}\,\mathrm{d}x = \int \frac{6x^2+3x-10}{(x^2-2)(x+3)}\,\mathrm{d}x$

$\displaystyle= \int \left(\frac{x}{x^2-2} + \frac{5}{x+3}\right)\mathrm{d}x$

Let $u = x^2 - 2$. Then $\mathrm{d}u = 2x\,\mathrm{d}x$. 1M

$\displaystyle\int \frac{6x^2+3x-10}{x^3+3x^2-2x-6}\,\mathrm{d}x = \frac{1}{2}\int \frac{\mathrm{d}u}{u} + \int \frac{5}{x+3}\,\mathrm{d}x$ 1M

$\displaystyle= \frac{\ln|x^2-2|}{2} + 5\ln|x+3| + \text{constant}$ 1A

18. (a) $\displaystyle\frac{1}{x-2} - \frac{1}{x+2} = \frac{(x+2)-(x-2)}{(x-2)(x+2)}$

$\displaystyle= \frac{4}{x^2-4}$ 1

(b) $\displaystyle\int \frac{\mathrm{d}x}{x^2-4} = \frac{1}{4}\int \left(\frac{1}{x-2} - \frac{1}{x+2}\right)\mathrm{d}x$ 1M

$\displaystyle= \frac{\ln|x-2|}{4} - \frac{\ln|x+2|}{4} + \text{constant}$ 1A

19. $\displaystyle\int (e^x+1)^3\,\mathrm{d}x = \int (e^{3x} + 3e^{2x} + 3e^x + 1)\,\mathrm{d}x$ 1M

$\displaystyle= \frac{e^{3x}}{3} + \frac{3e^{2x}}{2} + 3e^x + x + \text{constant}$ 1A

20. $\displaystyle\int (1 - 2\sin^2 4x)\,\mathrm{d}x = \int \cos 8x\,\mathrm{d}x$ 1M

$\displaystyle= \frac{\sin 8x}{8} + \text{constant}$ 1A

21. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{2-2x} - \sqrt{4-2x}} = \int \frac{\sqrt{2-2x} + \sqrt{4-2x}}{(2-2x)-(4-2x)}\,\mathrm{d}x$ 1M

$\displaystyle= -\frac{1}{2}\int (\sqrt{2-2x} + \sqrt{4-2x})\,\mathrm{d}x$

$\displaystyle= \frac{(2-2x)^{\frac{3}{2}}}{6} + \frac{(4-2x)^{\frac{3}{2}}}{6} + \text{constant}$ 1A

22. $\int 9^x \cdot 2^x \, dx = \int 18^x \, dx$

$= \int e^{x \ln 18} \, dx$ 1M

$= \dfrac{e^{x \ln 18}}{\ln 18} + \text{constant}$

$= \dfrac{18^x}{\ln 18} + \text{constant}$ 1A