Dexter Wong & His Mathematics Team

Summer Course 2022 – 2023

MATHEMATICS Extended Part

S5 – S6 M2 Differentiation Assignment Set 1

Name: _____ Telephone: _____

Student number: _____ Centre: _____

Lesson date: SUN/MON/TUE/WED/THU/FRI/SAT

INSTRUCTIONS

1. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.
2. Unless otherwise specified, all workings must be clearly shown.
3. Unless otherwise specified, numerical answers must be exact.
4. The diagrams in this paper are not necessarily drawn in scale.

Suggested solution

Distributed in summer course
S5 – S6 M2 Differentiation
Phase 1 – Lesson 1

1. $f(x) = A(2x+4)^{\frac{5}{2}} + B(2x-4)^{-\frac{3}{2}}$

$f'(x) = \dfrac{5A}{2}(2x+4)^{\frac{3}{2}}(2) - \dfrac{3B}{2}(2x-4)^{-\frac{5}{2}}(2)$ 1A

$\begin{cases} 500 = A(5+4)^{\frac{5}{2}} + B(5-4)^{-\frac{3}{2}} \\ 228 = 5A(5+4)^{\frac{3}{2}} - 3B(5-4)^{-\frac{5}{2}} \end{cases}$ 1M

Solving, we have $A = 2$ and $B = 14$. 1A+1A

2. $f(x) = A(x^2+3)^2 + B(2x-3)^{-1}$

$f'(x) = 2A(x^2+3)(2x) - B(2x-3)^{-2}(2)$ 1A

$\dfrac{1}{9} = 0 - 2B(-3)^{-2}$ 1M

$B = \dfrac{-1}{2}$ 1A

$5 = 2A(1+3)(2) + (-1)^{-2}$

$A = \dfrac{1}{4}$ 1A

3. $\dfrac{dy}{dx} = \dfrac{x\sec^2 x - \tan x}{x^2}$ 1M+1A

4. $\dfrac{dy}{dx} = 3(2x+1)^2(2)(x-4)^5 + (2x+1)^3(5)(x-4)^4$ 1M+1A

$= (2x+1)^2(x-4)^4[6(x-4) + 5(2x+1)]$

$= (2x+1)^2(x-4)^4(16x-19)$ 1A

5. $y = (3x-2)^3(3x+2)^{-3}$

$\dfrac{dy}{dx} = 3(3x-2)^2(3)(3x+2)^{-3} + (3x-2)^3(-3)(3x+2)^{-4}(3)$ 1M+1A

$= (3x-2)^2(3x+2)^{-4}[9(3x+2) - 9(3x-2)]$

$= 36(3x-2)^2(3x+2)^{-4}$ 1A

6. $f(x) = (x-4)^{\frac{3}{2}}(x^2+4x+16)^{\frac{3}{2}}$

$f'(x) = \dfrac{3}{2}(x-4)^{\frac{1}{2}}(x^2+4x+16)^{\frac{3}{2}} + (x-4)^{\frac{3}{2}}\left(\dfrac{3}{2}\right)(x^2+4x+16)^{\frac{1}{2}}(2x+4)$ 1M+1A

$= \dfrac{3}{2}(x-4)^{\frac{1}{2}}(x^2+4x+16)^{\frac{1}{2}}[(x^2+4x+16) + (x-4)(2x+4)]$

$= \dfrac{9x^2}{2}(x-4)^{\frac{1}{2}}(x^2+4x+16)^{\frac{1}{2}}$

$\dfrac{9a^2}{2}(a-4)^{\frac{1}{2}}(a^2+4a+16)^{\frac{1}{2}} = 0$

$a = 0$ (rejected) or 4 or $a^2+4a+16 = 0$

For $a^2+4a+16 = 0$, $\Delta = 4^2 - 4(1)(16) = -48 < 0$. 1M

The equation has no real roots.

Thus, $a = 4$. 1A

7. $y = \dfrac{\ln(x^2 + 2^x + e^{2x})}{2\ln 2}$

$\dfrac{dy}{dx} = \dfrac{1}{2\ln 2} \times \dfrac{1}{x^2 + 2^x + e^{2x}} \times (2x + 2^x \ln 2 + 2e^{2x})$ 1M+1A

$= \dfrac{2x + 2^x \ln 2 + 2e^{2x}}{2\ln 2(x^2 + 2^x + e^{2x})}$

$= \dfrac{x + 2^{x-1} \ln 2 + e^{2x}}{\ln 2(x^2 + 2^x + e^{2x})}$ 1A

8. $\dfrac{dy}{dx} = -\sin x(-\sin x) - \cos x(\cos x)$ 1M

$= \sin^2 x - \cos^2 x$ 1A

9. $g'(x) = f'(\sin x)\cos x + f'(\cos x)(-\sin x)$ 1M+1A

$g'\left(\dfrac{\pi}{2}\right) = f'\left(\sin\dfrac{\pi}{2}\right)\left(\cos\dfrac{\pi}{2}\right) - f'\left(\cos\dfrac{\pi}{2}\right)\left(\sin\dfrac{\pi}{2}\right)$ 1M

$= f'(1)(0) - f'(0)(1)$

$= -2$ 1A

10. $\dfrac{dy}{dx} = \dfrac{(4x+1)(3x^2+1) - (2x^2+x-6)(6x)}{(3x^2+1)^2}$ 1M+1A

$= \dfrac{-3x^2 + 40x + 1}{(3x^2+1)^2}$ 1A

11. $\dfrac{dy}{dx} = \dfrac{(-2)(3x - x^2 - 1) - (3-2x)(3-2x)}{(3x - x^2 - 1)^2}$ 1M+1A

$= \dfrac{-2x^2 + 6x - 7}{(3x - x^2 - 1)^2}$ 1A

12. (a) $f'(x) = \dfrac{6x^5}{81}$ 1A

$\dfrac{a^6}{81} = \dfrac{2a^5}{27}$ 1M

$a^5\left(\dfrac{a}{81} - \dfrac{2}{27}\right) = 0$

$a = 0 \quad \text{or} \quad 6$ 1A

(b) $g'(x) = \dfrac{20x^4}{15} - 36x^2$

$= \dfrac{4x^4}{3} - 36x^2$ 1A

$\dfrac{b^6}{81} = \dfrac{4b^4}{3} - 36b^2$ 1M

$b^2\left(\dfrac{b^4}{81} - \dfrac{4b^2}{3} + 36\right) = 0$

$b^2 = 0 \quad \text{or} \quad 54 \quad \text{or}$

$b = 0 \quad \text{or} \quad \pm\sqrt{54}$ 1A

13. $\dfrac{dy}{dx} = \dfrac{(1+\cos x)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$ 1M

$= \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$= \dfrac{1}{1+\cos x}$ 1A

14. $\dfrac{dy}{dx} = 2\cot x \dfrac{d}{dx}\left(\dfrac{\cos x}{\sin x}\right)$ 1M

$= 2\cot x \left(\dfrac{-\sin x(\sin x) - \cos x(\cos x)}{\sin^2 x}\right)$ 1A

$= -2\cot x \csc^2 x$ 1A

15. $\dfrac{dy}{dx} = \dfrac{3x^2(5x^6+6)^{\frac{1}{2}} - (x^3+1)\left(\frac{1}{2}\right)(5x^6+6)^{-\frac{1}{2}}(30x^5)}{5x^6+6}$ 1M

$= \dfrac{3x^2(5x^6+6) - 15x^5(x^3+1)}{(5x^6+6)^{\frac{3}{2}}}$

$= \dfrac{-15x^5 + 18x^2}{(5x^6+6)^{\frac{3}{2}}}$ 1A

16. $y = \dfrac{x^3+3x}{(x^2+13)^{\frac{1}{2}}}$

$\dfrac{dy}{dx} = \dfrac{(3x^2+3)(x^2+13)^{\frac{1}{2}} - (x^3+3x)\left(\frac{1}{2}\right)(x^2+13)^{-\frac{1}{2}}(2x)}{x^2+13}$ 1M

$= \dfrac{(3x^2+3)(x^2+13) - x(x^3+3x)}{(x^2+13)^{\frac{3}{2}}}$

$= \dfrac{2x^4+39x^2+39}{(x^2+13)^{\frac{3}{2}}}$ 1A

17. $y = \dfrac{2x^2+5x-3}{(x^3+3x^2+1)^{\frac{1}{3}}}$

$\dfrac{dy}{dx} = \dfrac{(4x+5)(x^3+3x^2+1)^{\frac{1}{3}} - (2x^2+5x-3)\left(\frac{1}{3}\right)(x^3+3x^2+1)^{-\frac{2}{3}}(3x^2+6x)}{(x^3+3x^2+1)^{\frac{2}{3}}}$ 1M

$= \dfrac{(4x+5)(x^3+3x^2+1) - (2x^2+5x-3)(x^2+2x)}{(x^3+3x^2+1)^{\frac{4}{3}}}$

$= \dfrac{2x^4+8x^3+8x^2+10x+5}{(x^3+3x^2+1)^{\frac{4}{3}}}$ 1A

18. $\dfrac{dy}{dx} = 2(3x+\sqrt{18x^2+7})\left(3 + \dfrac{1}{2}(18x^2+7)^{-\frac{1}{2}}(36x)\right)$ 1M

$= 2(3x+\sqrt{18x^2+7})\left(3 + \dfrac{18x}{\sqrt{18x^2+7}}\right)$ 1A

19. $\dfrac{dy}{dx} = \dfrac{1}{2}\left(2x + (x^2+9)^6 + \dfrac{4}{x}\right)^{-\frac{1}{2}}\left(2 + 6(x^2+9)^5(2x) - \dfrac{4}{x^2}\right)$ 1M

$= \dfrac{1 + 6x(x^2+9)^5 - 2x^{-2}}{\sqrt{2x + (x^2+9)^6 + 4x^{-1}}}$ 1A